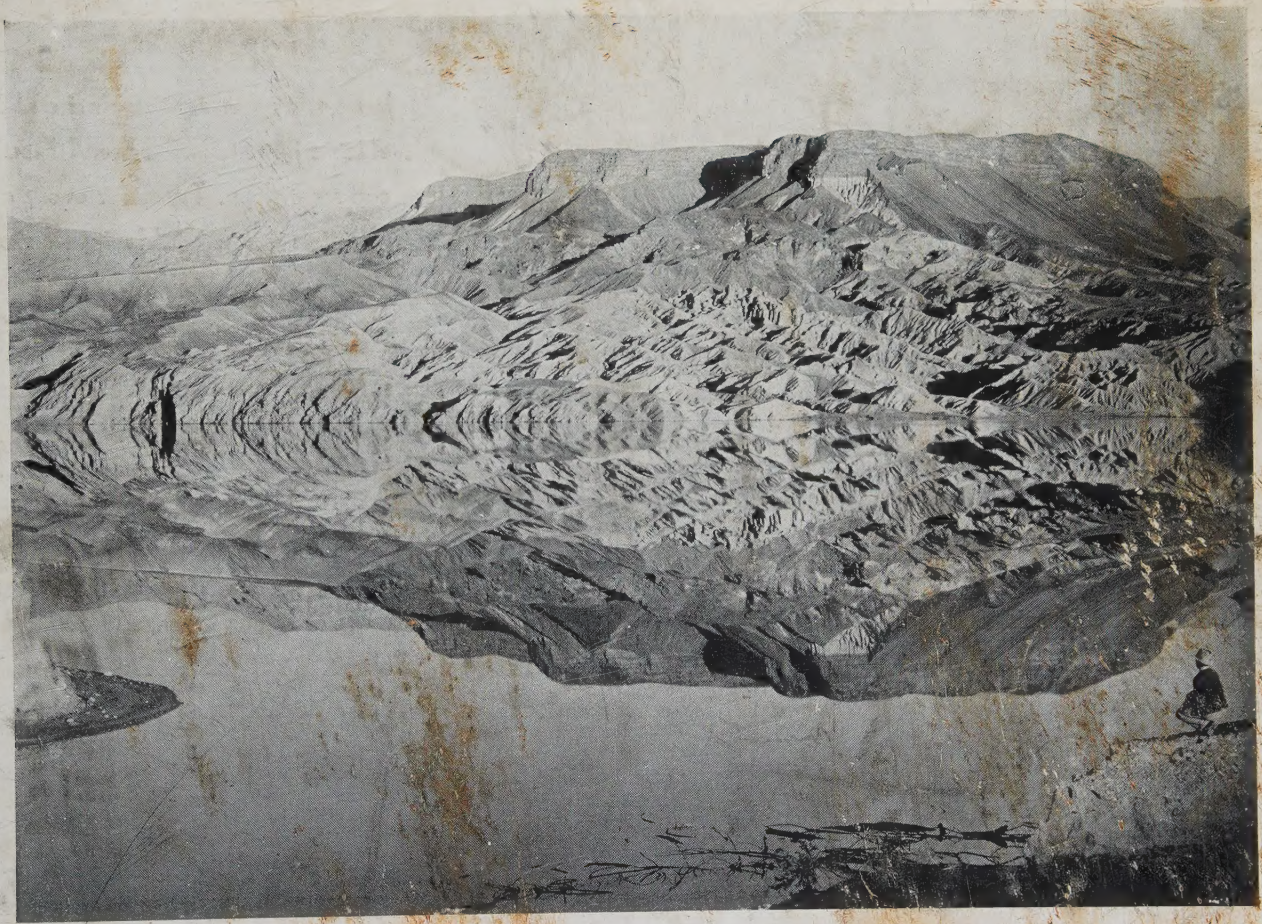
Electrical Engineering

November
1935



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Front Cover

Fortification Hill as seen from the Nevada shore of Black Canyon reservoir above Boulder Dam. Photo courtesy U.S. Bureau of Reclamation

Special Articles

A.I.E.E. Papers

—Turn to next page

In This Issue—

HEATING by electricity is a subject of ever-increasing interest. Two types of electric heating are described in this issue. One involves induction heating of iron parts at low temperature. It is useful in such operations as baking paint or rapidly drying wet metal parts (*pages 1210–2*). The other is an electric furnace in which a carbon electrode raised to a very high temperature is used in metallurgical operations (*pages 1195–9*).

MATHEMATICAL ANALYSIS of electric circuits and machinery is featured in 3 papers in this issue: One embraces certain extensions to Heaviside's operational methods (*pages 1222–7*); one presents an analysis of an inverter circuit (*pages 1227–35*); one gives a method for analyzing the performance of capacitor motors (*pages 1235–9*).

THE electrical control scheme adopted for a Diesel electric train of the light weight streamlined type which was placed in operation a few months ago, enables the greatest possible use to be made of the available engine power (*pages 1240–5*).

KNOWLEDGE of the resistivity of the earth's crust and the relation it bears to geological structure is useful in several types of electrical calculations. Data on this subject obtained through numerous experiments are available. (*pages 1153–61*).

COMMUTATION by means of harmonically vibrating contacts actuated by a "rotating" electromagnetic field is the principal novel feature of a method of converting alternating to direct current described in this issue (*pages 1213–21*).

ARC FURNACES are highly desirable loads for electric power companies, but may disturb other customers unless certain precautions are observed in connecting such loads to power systems (*pages 1173–8*).

PILOT wire schemes for relay protection of transmission lines have recently undergone changes which enhance the advantages of this type of protection considerably (*pages 1262–9*).

TWO immense projects on the Columbia River, Bonneville, and Grand Coulee, have been made the subject of an interesting group of photographs taken early in September (*pages 1272–3*).

DISCUSSIONS have been omitted from this issue to provide additional space for papers to be presented at the forthcoming 1936 A.I.E.E. winter convention.

FERROMAGNETISM has resisted the attack of theorists for many years. Recent advances in this field are summarized in this issue (*pages 1251–61*).

LOAD RATINGS for underground electric power cables may be computed by applying the method of harmonic analysis of load cycles (*pages 1166–72*).

WELDING research shows that a stable arc cannot be maintained in highly purified argon gas under ordinary welding conditions (*page 1144–9*).

PHOTOELECTRIC cells of the dry disk type have characteristics which make them desirable for use in many applications (*pages 1186–90*).

1936 TRANSACTIONS

The A.I.E.E. TRANSACTIONS for 1936 will be produced only for those having subscription cards on file at A.I.E.E. headquarters not later than Friday, December 20, 1935. Present production methods require determination, in advance of the printing of the January 1936 issue of ELECTRICAL ENGINEERING of the exact number of TRANSACTIONS volumes to be provided for. For the convenience of members desiring to maintain in this permanent form a file of A.I.E.E. papers and discussions, important additional information and a convenient order blank are provided on page 10 of the advertising section of this issue.

Opportunity and the Young Engineer

—A Message From the President

COLLEGE STUDENTS and young engineers have been confronted during the past few years with the perplexing problem "What opportunities are there in the engineering profession?"

In my opinion, there are greater opportunities in the engineering field today than have ever existed before. Perhaps more than at any previous time in the last few decades industry has been marking time while scientific development has advanced at an ever-increasing rate, leaving a vast field open to the application of science's discoveries and to modernization. Furthermore, I also believe that no matter what specific line of activity offers itself or may be decided upon for a life work, the young engineer will find that his engineering training has furnished the best possible foundation, whether he becomes a scientist, professional engineer, educator, businessman, lawyer, or banker.

It is true that the present economic disorder has afforded limited opportunity for useful employment, but this is only a temporary condition, and the young engineer has had extra time in which to strengthen his life equipment.

A man's education is not completed when he receives his degree at graduation. Many men in the business world consider that it only begins at that time. Graduation is just the beginning of an engineering career, the college course being the foundation for the superstructure which is to follow.

The most that can be expected of any engineering course is that it teaches the student to think. Training for the practice of engineering is similar to the training of a law student. The law student does not have to know all of the law, but he does learn where he can find the principles, and must be able to reason from these principles to particular cases.

After graduation, the engineer does not have to remember all of his analytical geometry, or his thermodynamics, or his calculus, but he will be expected to know what principles to apply and how to apply them in attempting the solution of a particular problem. If he knows these 2 things he need never hesitate to tackle any engineering problem presented to him.

The words "successful career" have been variously defined. Some measure success by the ratio of money received to the time and effort spent; others think that success is measured by the kind of service rendered and the degree of its benefit to mankind. The young engineer has a choice. It has been said that to achieve what the world calls success a man must attend strictly to business and keep a little in advance of the times. In any event, success has never been handed out to man, but has had to be earned by perseverance, hard work, energy, patience, and singleness of purpose.

The pages of history are filled with the names of persons who have attained renown through some distinguished service contributing to the welfare of mankind. These are the Isaac Newtons, Robert Fultons, James Watts, and many others. Throughout the whole of history we read also of those who have used their talents to break down and destroy civilization—the Caesars and Napoleons who disrupted the natural progress of the world to satisfy their personal ambitions and selfish desires. Both groups have received the plaudits of the multitude. They worked for a cause—creation, preservation, or destruction.

Another group includes the Benjamin Franklins and Thomas A. Edisons who rose to fame through the fields of invention. They were purely individualists who assumed their tasks, not in response to popular demand, but inspired only by their own visions. They had the conviction that there was some specific object which they could accomplish, and they carried their work to a successful conclusion without outside advice or assistance.

In general, engineering education and training have effectively developed a man's power of observation, his analytical ability, and his desire and capacity to learn, but all too frequently have failed to develop in the engineer an ability to express himself effectively. The highest measure of engineering success often is won by the man who possesses, in addition to his technical knowledge, the ability to deal with people and to "sell" himself and his ideas. Leadership must first express itself in speech. One must know how to ask for things, how to explain things, and how to speak persuasively enough to win the active support of others. Doing business is chiefly talking business. Engineering is a business and one of the most remarkable things about it is that there is scarcely a region of the world's activity into which the engineer's direct interest does not extend.

The American Institute of Electrical Engineers through its many Sections and Student Branches presents an excellent means to all young electrical engineers to take an active part in the discussions of current engineering problems. This feature aids the engineer materially in developing the essential qualities of clear thought and effective speech. This is but one of the many benefits derived from membership in the Institute.

The question of opportunity for the young engineer is simply the question of his scientific training, plus initiative, aggressiveness, character, and personality. Good men are needed now as they always will be.

Arc Welding in Argon Gas

During the course of the extensive research in arc welding being conducted at Lehigh University under joint sponsorship of the A.I.E.E. and several other organizations, apparatus for experimental welding at atmospheric pressure in controlled pure gas atmospheres has been designed and constructed. Tests in argon gas reveal 3 phenomena not previously associated with the welding arc: (1) the impossibility of maintaining a stable arc in highly purified argon under ordinary conditions; (2) the absence of all crater formation under pure iron in pure argon; and (3) the absence of all observable "pinch effect" accompanying the detachment of the globules from the electrode wire.

By
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AN EXTENSIVE PROGRAM of investigation is being carried out at Lehigh University in the study of arcs and arc welds of pure iron. These studies have been sponsored by A.I.E.E. and aided by The Engineering Foundation. Several reports on the progress of these studies have been issued previously.[1–4] The present report discloses the design and construction of an apparatus for experimental welding at atmospheric pressure in controlled pure gas atmospheres, using automatic feeds for both electrode and work piece. Auxiliary apparatus is provided for purifying the gases and for analyzing them, both before welding begins and after it is completed. This apparatus design may be useful to other investigators in the study of welding atmospheres.

Briefly summarized, the more important results obtained so far are as follows:

1. Stable welding arcs cannot be maintained in argon of 99.5 per cent purity, when the electrode is clean, unless the open-circuit voltage is greater than 62 volts and the short-circuit current greater than 110 amperes. In air or in impurer argon stable arcs form readily at currents and voltages far less than these minima.

2. No crater is formed under the iron arc in argon. This condition results in complete lack of penetration into the base metal, and thus renders welding impossible.

3. The globules at the end of the electrode grow gradually to about $^1/_4$ inch in diameter and detach under no apparent force except that of gravitation.

4. The melting rates per kilowatt-hour in argon and in air are approximately equal.

Among the most important metallurgical aspects of the investigation are the high ductility of the pure iron welds made in argon: 90 per cent reduction of area and 30 per cent elongation. For pure iron welds made in air the reduction of area was only 3.5 per cent instead of 90, and the elongation only 4 per cent instead of 30. These results indicate the great effect of air in lowering the ductility of the metal obtained in the arc welding operation. The strength of the pure iron welds on the contrary was 40,000 pounds per square inch for those made in argon and rose to 65,000 for those made in air. The effect of the air thus is to raise the strength of the weld greatly. Some experiments made with cellulose coating on the electrode gave results intermediate with respect to those in argon and those in air. Other purely metallurgical aspects of the investigation are to be reported elsewhere.

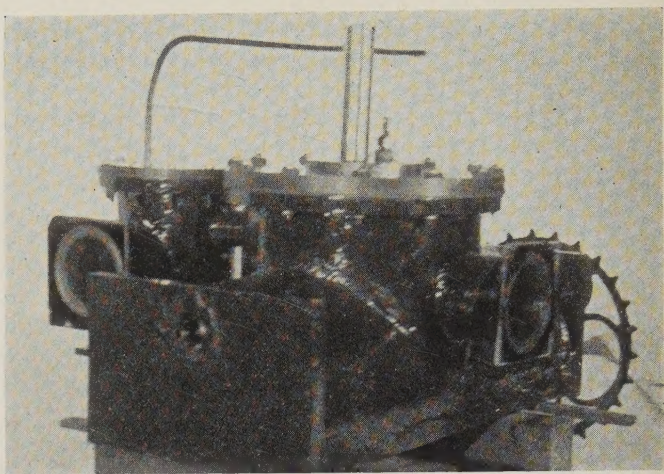

Fig. 1. Exterior view of chamber for experimental welding in pure gases

Material Used

The rods and plates used in this program were of carbonyl iron which had been treated in a stream of undried hydrogen for 72 hours at 1,375 to 1,420 degrees centigrade, by a co-operating laboratory, that of the General Electric Company at Schenectady, N. Y. The wire was 4 millimeters ($^5/_{32}$ inch) in diameter and the base metal plates were 6 and 2 millimeters thick. These plates were used to line a 45 degree V groove between 2 $^1/_2$ inch steel plates, the plates serving as backing to prevent the

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1. For all numbered references see list at end of article.

arc from melting through the pure iron "liners" and from "blowing" magnetically. This arrangement is shown at *V* in figure 3. The chemical analysis of the carbonyl iron after hydrogen treatment was as follows:

Carbon	0.009	Sulphur	0.005
Manganese	0.046	Nitrogen	0.002
Silicon	0.002	Oxygen	0.002
Phosphorus	0.01	Hydrogen	0.0002

The analyses were supplied by another co-operating laboratory, that of the American Rolling Mill Company; all values given are percentages.

Most of the tests were made using bare wire. In a few instances, the wire was coated to produce a "shielded arc" such as has recently yielded commercially a weld metal of markedly improved properties. The coating contained cellulose with sodium silicate as a binder and some titanium oxide to reduce porosity and act as an arc stabilizer. These rods were coated by courtesy of the welding research laboratory of the A. O. Smith Corporation.

The argon used was supplied by the incandescent lamp department of the General Electric Company. As received, the gas contained approximately 1 per cent nitrogen. All carbonaceous gas impurities were less than 30 parts per million. The argon was given final purification after being admitted to the welding chamber and when analyzed just before using, it showed a purity of 99.3 to 99.6 per cent argon.

Description of Apparatus

Figure 1 shows the chamber in which the tests in pure argon were made. Figures 2 and 3 show the main details of construction. The welding chamber is built of copper tubing $7^1/_2$ inches in diameter and $^3/_{32}$ inch thick to provide good heat conduction. It consists of a vertical tube *A* 6 inches high cut to fit into a horizontal half cylinder *B* 15 inches long. The bottom and ends of the horizontal part are made from copper plates. The top of the vertical tube is fitted with a flat cover plate *C* $^1/_2$ inch thick. The cover plate can be bolted securely to the top of the vertical tube with a rubber gasket between to prevent leakage. A suitable window is provided for viewing the welding operation.

Specimens to be welded are placed on a movable table *D* on the bottom of the chamber. The table is moved by a lead-screw *E* which is driven through a stuffing box *F* by a $^1/_4$-horsepower variable-speed motor.

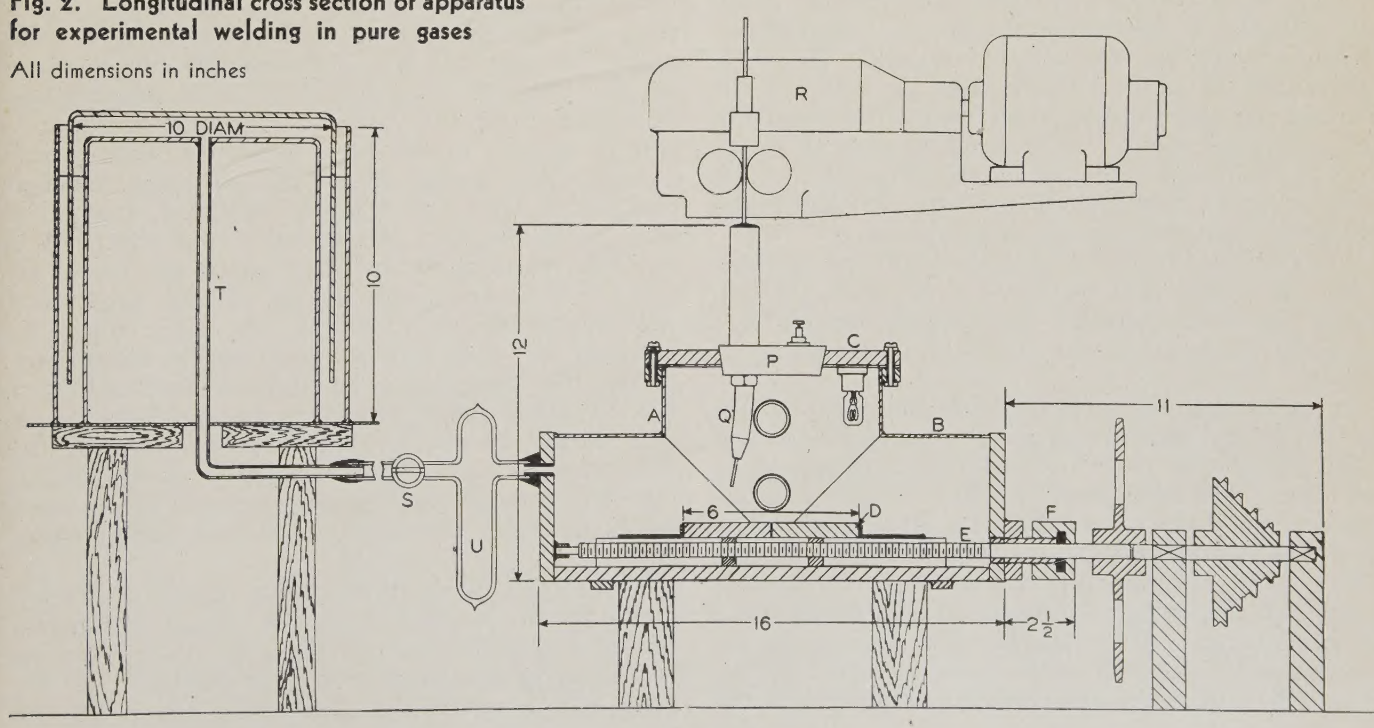
An auxiliary chamber 4 inches in diameter and 6 inches high is connected to the rear of the welding chamber with 2 horizontal tubes *G* each 1 inch in diameter. This auxiliary chamber contains a "misch-metal arc" (*H–I*), a glow discharge between an iron anode (*I*) and a "misch-metal" cathode (*H*). ("Misch metal" is a pyrophoric alloy containing about 65 per cent of rare earth metals, such as cerium and lanthanum, the balance being iron.) Such a discharge activates the cathode so that it combines with any chemically active gas surrounding it, thus purifying the inert atmosphere used in these experiments. The connection to the argon tank and the vacuum pump is made to the top of this auxiliary chamber so that all the argon admitted is made to pass the misch-metal arc before passing into the welding chamber.

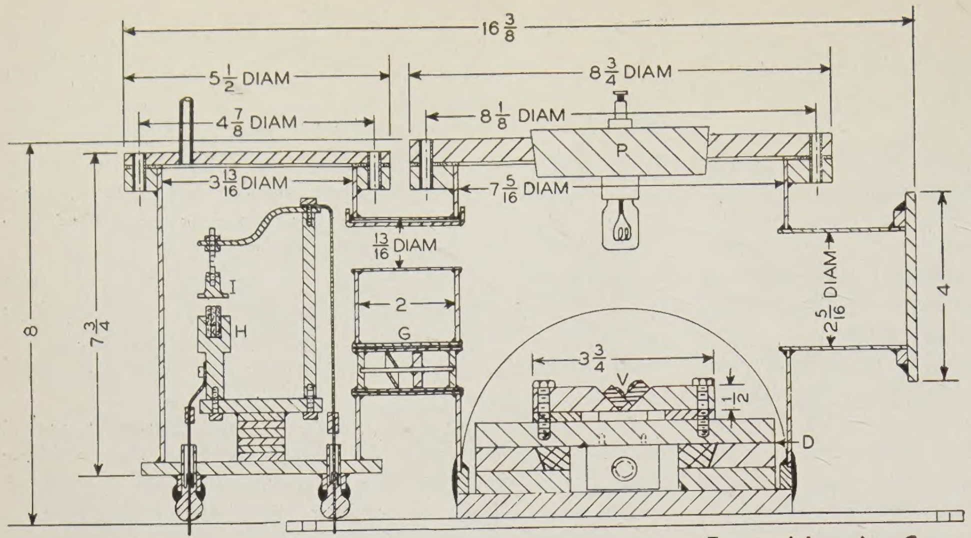
Figure 4 shows the seal that allows the welding rod to pass into the chamber and yet not allow air to leak in or argon to leak out. Two soft rubber washers *J* are compressed between parts *K* and *L* by turning *K*. The soft rubber fits tightly around the rod and against the walls of the tube *M*.

To evacuate the system tube *N* is sealed to the

Fig. 2. Longitudinal cross section of apparatus for experimental welding in pure gases

All dimensions in inches





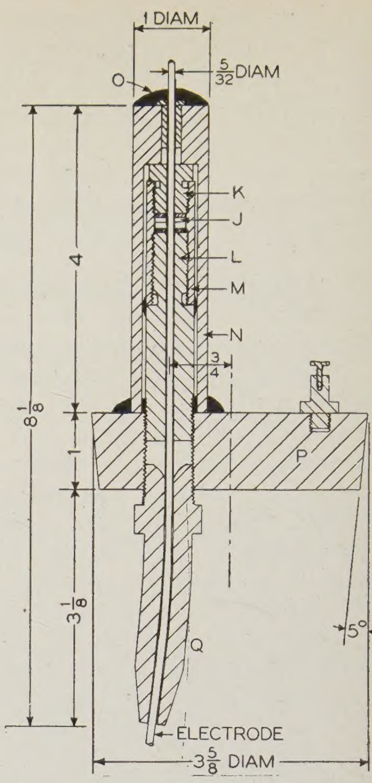


Fig. 3 (above). Cross-sectional view of welding chamber

Fig. 4 (right). Cross-sectional view of welding electrode seal

All dimensions in inches

copper plug P. A split collar O fits tightly around the rod and into the small hole on the top of the tube N. This joint is entirely sealed with "picein," a black wax commonly used in high vacuum work for sealing joints in the apparatus. After argon has been admitted and is up to atmospheric pressure, the picein and split collar are removed and the rod thoroughly cleaned of picein with ether.

The copper plug P fits into a tapered hole in the cover plate of the main arc chamber, and the tapered joint is sealed with stopcock grease. The rod passes through this copper plug $^3/_4$ inch off center, thus providing a means for shifting the electrode from one side to another.

The current pick-up nozzle Q is fitted to the bottom of the tapered copper plug. The rod is fed into the chamber with an automatic welding head R.

During welding, the argon expands considerably because of the heat of the arc. To accommodate this expansion, a gas reservoir was built which consists of a gas tight container floating over vacuum pump oil. Figure 2 shows the details of construction. When evacuating the system, stopcock S is left open until the oil level is raised on the inside of the floating container and oil flows down tube T and into the glass reservoir U. The stopcock then is closed and is left closed until the chamber has been filled with argon to atmospheric pressure and the argon has been cleaned by the misch-metal arc.

For those welds that were made in air, the movable table, of course, was mounted in the open directly under the welding head. The welding current was obtained from a Lincoln "stable-arc" welding generator.

In order to know quantitatively the purity of the gas before and after welding in it, a gas analyzer was built which was patterned after that proposed by Severyns, Wilkinson, and Schum.[5] Figure 5 shows the construction. It consists of a small glass vessel in which a small quantity of lithium can be heated. The mixture of gases to be tested is admitted to the vessel and the pressure is measured with an open-end U manometer. After heating the lithium for 45 minutes in the presence of the gas to be tested the gas is cooled to the same temperature as before and the pressure measured again. The percentage of impurity is expressed as:

$$\frac{P_1 - P_2}{P_1} \times 100 = \text{per cent impurity}$$

where P_1 is the pressure before heating and P_2 is the pressure after heating.

Method of Welding

After the system has been evacuated, it was flushed once with argon and again evacuated. The argon for the final filling was admitted very slowly. After an argon pressure of approximately 2 to 4 millimeters of mercury was obtained, the misch-metal arc was started. A period of 30 to 45 minutes was allowed to bring the system to an atmospheric pressure of argon. It has been found by other experimenters that misch-metal arcs are more efficient scavangers when the argon is admitted slowly. It was found also that if the argon was admitted in less than 15 minutes the misch-metal arc would become very unstable and go out. If this occurred it was impossible to start the arc again until the pressure had been reduced to 10 to 20 millimeters.

After the argon had been admitted and the system was at atmospheric pressure, the misch-metal arc was allowed to run for 6 to 8 hours. During this time the gas was given a positive circulation by means of a small fan and motor placed in the lower tube connecting the misch-metal and welding chambers.

At the end of the purification period the misch-metal arc was turned off and the picein seal around the welding rod was removed. The welding in argon was done with a current of approximately 180 amperes and an arc voltage of 10 to 12 volts. The arc

length could be controlled very closely and was kept at approximately $^1/_8$ to $^3/_{16}$ inch at all times by means of the automatic feed of the welding head. For the welds made in air with the bare pure iron electrodes a current of 150 amperes was used with an arc of 15 to 17 volts. When welding with the heavily coated pure iron electrode 130 to 140 amperes was used with 25 to 28 volts. These conditions were the necessary ones for each type of weld in order to obtain sound nonporous welds.

Welding Characteristics Observed in Argon

Deposition of metal in an atmosphere of argon is strikingly different in appearance, for both pure iron and steel, from what it is in air. While the melting rates in argon and in air are similar (argon 0.922 pound per kilowatt-hour, air 0.895 pound), yet the transfer of metal in argon takes the form of very large globules approximately $^7/_{32}$ inch in diameter at the rate of about 1 every 3 seconds, whereas in air the globules are quite small and fall at the rate of about 10 to 15 per second. The globules in argon cross the arc calmly and under no apparent force except that of gravitation. Quite frequently they grow so large as to bridge the arc and then they free themselves from the electrode quite rapidly. The so-called "pinch effect," proposed by Carl Hering and studied by E. F. Northrup, is not manifest in promoting globule detachment in the iron arc in argon, even when the drops grow to a very large size before detaching. The arc is quiet, with a complete absence of the steady sharp crackling sound that is characteristic of a good welding arc in air. When viewed through the dark yellowish-green welding glass, the arc core appears bright yellow with successive sheaths of greenish-blue, black, and orange.

Another striking and significant difference when welding in argon is the complete absence of the crater in the liquefied base metal or previous weld bead that is so important for welding in air. This results, of course, in a complete lack of penetration into the parts. Figure 6 shows the appearance of the welding arc in argon and the lack of crater formation. Figure 7 is a macrograph of a section of a multipass weld of ordinary steel made in pure argon. The lack of penetration is clearly evident since the contour of each individual layer easily can be traced.

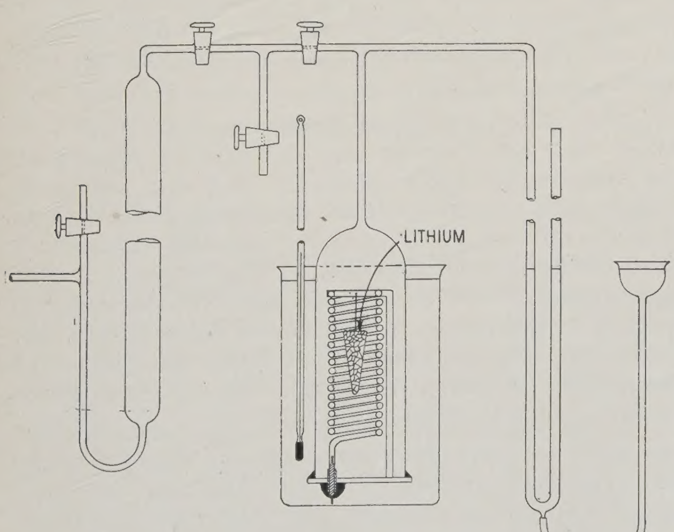


Fig. 5. Diagram of gas analyzer

Thus one of the most basic and essential features of the commercial arc welding process, namely, penetration and crater formation, is not an inherent characteristic of the iron arc, but, fortunately for the engineer, it is one that appears when arc welds are made in air. This absence of crater formation is of peculiar interest since numerous theories have been advanced to explain the nature of the force in the arc that creates this crater, and yet none of these theories considered the atmosphere as an important factor. One theory, proposed by P. P. Alexander and G. Donath, is that a pressure within the arc is created by a gas or electron steam and that this pressure blows the liquid metal outward from the center of the molten puddle toward its edges. One of the authors in an earlier study[6] has attributed the forces of crater formation partially to the differences of surface tension of the molten metal at the center of the puddle from that at the edges. Neither of these theories would account for the lack of crater formation found for iron arcs in argon.

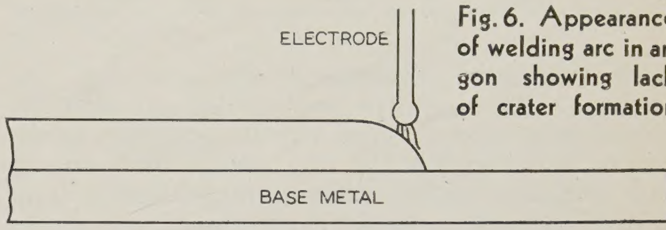


Fig. 6. Appearance of welding arc in argon showing lack of crater formation

An attempt was made to find out what percentage admixture of air was necessary in the argon in order to obtain penetration and fusion. When welding in 50 per cent air and 50 per cent argon, the penetration was about equal to that obtained in air. Welds then were made in 90 per cent argon and 10 per cent air. Figure 8 is a macrograph of a section of such a weld. A slight amount of penetration can be noted, and there was a very shallow crater at the end of the weld bead. Hence it can be seen that crater formation is dependent on the presence of air and that very little air is required to produce penetration and crater formation. The experiments in penetration were carried out by an expert in experimental arc welding, and every possible combination of current, voltage, arc length, and rate of travel was tried until the best result achievable was obtained in each gas mixture. Figures 7 and 8 show a single cross section of the results. They are, however, entirely representative of the welds as a whole.

Unstable Arcs in Argon

One of the authors and J. Leland Myer have reported[2] that an arc discharge could not be obtained under ordinary conditions in pure argon gas. Further studies with Albert M. Thorne[3] have confirmed this fact for other inert gases and have yielded knowledge

Table I—Results of Welding Tests in 99.3 to 99.5 Per Cent Pure Argon Gas

Rod material: commercial steel wire *E*-No. 1-*A* $^1/_8$ inch in diameter

Weld No. and Pass	Welding Current Amperes	Welding Volts	Open-Circuit Volts	Short-Circuit Amperes	Rod Polarity	Rod Surface	Stability of Arc
14–1	40–60	26	58–60	110–120	−	Clean	Fairly stable
14–2	40–60	26	58–60	110–120	−	Clean	Unstable
14–3	40–60	26	58–60	110–120	−	Clean	Unstable
14–4	40–60	26	58–60	110–120	−	Clean	Unstable
14–5	40–60	26	62–64	110–120	−	Clean	2 to 10 seconds
14–6	40–60	26	62–64	110–120	−	Clean	2 to 10 seconds
15–1	70	14	60	110–120	+	Clean	Unstable
15–2	70	14	60	110–120	+	Clean	Unstable
15–3	70	14	65	110–120	+	Clean	5 to 10 seconds
15–4	70	20	80	110–120	+	Clean	Stable
15–5	50–70	14–17	58–60	110–120	−	Clean	Unstable
15–6	120–160	24–30	63	200	−	Clean	Stable
15–7	70	14	60	110–120	−	Sull	Stable
15–8	70	14	60	110–120	−	Sull	Stable
15–9	120	18	63	200	−	Sull	Stable

of the boundary conditions of the nonarcing phenomenon. These studies were carried out with low current arcs, that is, from 1 to 10 amperes. In the present investigation the arc voltage and current were within an entirely different range, that is, from 40 to 180 amperes. In this range also nonarcing was observed under certain conditions both with commercial steel wire and with pure iron wire.

Negative Electrode Coating Removed

When welding with commercial steel wire of $^1/_8$ inch diameter with the "sull" coating (the light sulphate coating left on the wire after pickling in sulphuric acid) removed, and with a welding current of 40 to 60 amperes and an arc voltage of 26 volts, after the arc had been maintained for a short time (30 to 50 seconds) it became very unstable. The electrode would be fed down by the automatic device, make contact with the plate, and then be drawn away. When the contact with the plate was broken a flash of an arc would occur lasting for only a fraction of a second and then would go out. The

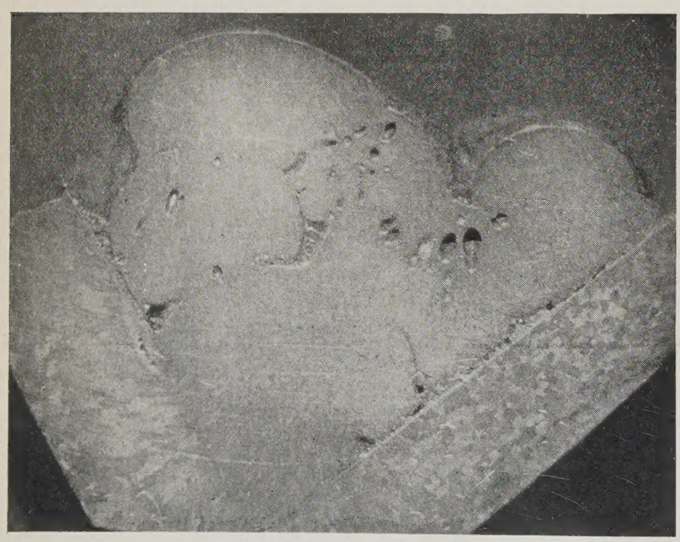


Fig. 7. Cross section of multipass weld of ordinary steel in pure argon gas. Note lack of penetration into plate and into early deposits

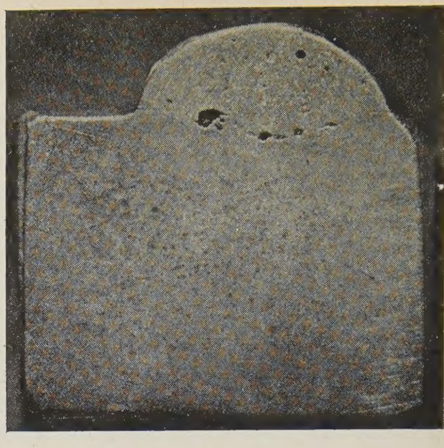


Fig. 8. Same welding conditions as figure 7, but in a mixture of 90 per cent argon and 10 per cent air. Note slight but definite penetration into plate

welding head automatically would move the electrode down again and the same process would be repeated. This would occur 8 or 10 times until finally an arc could be struck that would last for a few seconds, and when it went out the same cycle would be repeated. This took place when the open circuit voltage of the welding generator was 58 to 60 volts with momentary short-circuit currents of 110 to 120 amperes and the polarity of the electrode negative. The instability was not attributable to the commonly observed magnetic effect encountered at the ends of the plate, for it would take place in the center portion of the weld as well as toward the ends. When the open circuit voltage was increased slightly above 60 volts (62 to 64) the arc would last for several seconds before going out. When making the test welds in argon this unstable range was avoided by increasing the open circuit voltage of the generator to 65 volts or more and by increasing the welding current.

Positive Electrode Coating Removed

When the polarity was reversed, that is, with the electrode positive (and cleaned of all drawing compound as before), the arc was again slightly unstable with open circuit voltages of 58 to 60 volts and momentary short-circuit currents of 110 to 120 amperes. During the short welding periods the arc voltage was 17 volts with a welding current of 50 to 70 amperes. When the open circuit voltage was increased to 65 volts there still were periods of nonarcing, but when increased beyond this the arc persisted until stopped.

Negative Electrode Coating Remaining

When the sull coating was allowed to remain on the commercial steel electrode and the electrode polarity was negative, the arc was fairly stable with an open circuit voltage of 58 to 60 volts and momentary short-circuit currents of 110 to 120 amperes. This gave values of 14 to 17 volts and 70 amperes for welding conditions. When these values were obtained the arc would emanate from the surface of the molten globule at the end of the electrode only.

The condition of the surface of the electrode thus had a pronounced effect upon the arc characteristics when welding in argon. When the rod has been cleaned of its sull coating and drawing compound, and when the electrode polarity is made negative, the arc will travel up the side of the electrode for a

distance of $^1/_2$ to $^3/_4$ of an inch. This behavior made welding very difficult, for while the arc would be emanating from the side of the electrode, the arc voltage would be high and the welding head automatically would feed the rod down to lower the voltage, until finally the end of the rod would ram into the base metal and cause a complete short circuit. However, this same phenomenon did not take place when the electrode polarity was positive, but the arc then would jump around on the plate in a very erratic manner. When the electrode was negative and the sull coating remained on the rod, the arc emanated steadily from the end of the electrode only. Evidently the cathode is unstable, whether it be formed of the wire or the plate, unless the wire is coated. In this event the cathode is stable. Results of these tests are summarized in table I.

These observations on lack of penetration and crater formation and lack of arc stability indicate that intrinsically the high current arc is something quite different from that with which we have become acquainted by observing only the arc between steel electrodes in air. Other purely metallurgical aspects of the investigation are to be reported elsewhere.

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Bridge Measurement of Electromagnetic Forces

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A simple impedance bridge may be used to measure the variation of inductance of a disconnecting switch of practical dimensions as the switch blade is displaced by slight amounts from the closed position. The electromagnetic force tending to open the switch blade, calculated from the observed variation of inductance, checks the theoretical value obtained from Dwight's formula. The bridge may be used to determine the electromagnetic forces acting on current carrying members the configuration of which is such as to make accurate theoretical calculation impossible.

CONSIDERABLE work has been done in the mathematical treatment of electromagnetic forces between members of an electric circuit. The actual measurement of these forces has received comparatively little attention; and as far as the authors could ascertain, with the exception of torque measurements in electrical instruments,[1] all such measurements have been direct physical measurements of the forces involved.

It is well known from fundamental electromagnetic theory that the electromagnetic force in any given direction on a member of an electric circuit is proportional to the square of the current flowing through that member and the rate of change of self-inductance with respect to the deformation of the circuit in that direction. This can be expressed as

$$F = \frac{1}{2} I^2 \frac{dL}{dS} \tag{1}$$

The form of this equation immediately suggests a very obvious method of measurement of electromagnetic force, namely, the determination of the self-inductance as a function of the desired deformation. Indeed, several writers, Dwight,[2] Hague,[3] Karapetoff,[4] and others, have suggested this measurement of inductance as a function of deformation as being the most convenient means of determining electromagnetic forces.

The forces under consideration here are those existing in simple types of circuits, such as bus bars, switches, and circuit breakers. To make direct measurements of these forces involves heavy currents and means of measuring the actual resulting forces. A method of measuring such forces in this manner is described in a paper by J. Walter Roper[5] in which he has investigated forces in a circular and

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1. For all numbered references see list at end of paper.

a rectangular circuit. If the values of self-inductance be plotted versus the deformation in the direction of the force to be measured, the slope of the resulting curve at the point of zero deformation gives the quantity $\Delta L/\Delta S$, from which the force in that direction can be calculated.

Since the force on a member is proportional to the derivative of self-inductance with respect to deformation, it is not necessary to determine the absolute value of the self-inductance of that member, but only small increments of inductance of the system including the member. This peculiar advantage enables one to use a bridge with "impure" arms so long as the portions of the arms that must be changed, to balance the bridge as the circuit is deformed, are sufficiently "pure." In view of the fact that the changes of inductance in circuit members of practical interest are extremely minute, this is a very important consideration.

Strangely enough the authors have not been able to find in technical literature any report of the measurement of the variation of self-inductance for the purpose of determining electromagnetic forces on circuit members of the types mentioned in the beginning of this paper. The purpose of this paper is to show that it is entirely feasible to measure such electromagnetic forces by means of an impedance bridge. Briefly:

1. An impedance bridge is described that measures the change of inductance in a disconnecting switch of practical dimensions; measurements made with this bridge check the theoretical value computed from Dwight's formula.
2. The bridge method actually measures the $\Delta L/\Delta S$ of the structure being studied and thereby makes possible the calculation of electromagnetic forces prevailing under actual operating conditions with ample accuracy for design purposes.
3. The bridge method does not measure physical forces and therefore requires only that the structure being studied carry currents of magnitude sufficient to obtain adequate bridge sensitivity.

Disconnecting Switch

A circuit was chosen of simple configuration so as to be readily amenable to mathematical analysis in order to have a check between theoretical and measured values. For this purpose a disconnecting switch was constructed similar to the one described by Dwight, for which he calculated the force at the break jaw tending to open the blade.[2] A diagram of the switch is shown in figure 1. To complete the circuit, a parallel return conductor was added and placed adjacent to the connecting leads of the switch, as shown. The return conductor circuit was opened at one point and connected to the bridge with a pair of twisted leads.

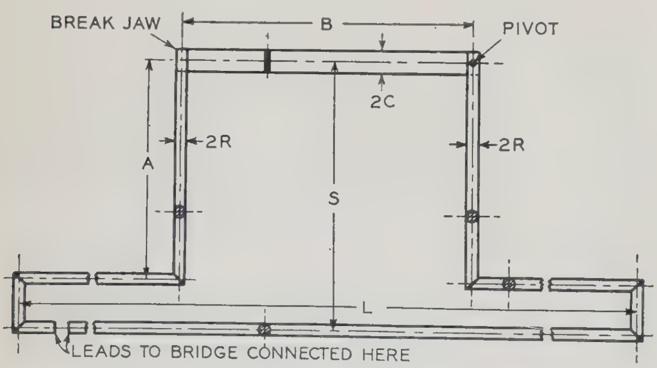

Fig. 1. Diagram of disconnecting switch

A = 30.48 cm (12.00 in.) R = 0.794 cm (0.3125 in.)
B = 40.64 cm (16.00 in.) S = 35.56 cm (14.00 in.)
C = 1.587 cm (0.625 in.) L = 208.3 cm (82.0 in.)

Dwight's formula assumes connecting leads and a return conductor of infinite length. By placing the return conductor comparatively close to the connecting leads of the switch (see figure 1), the value of $\Delta L/\Delta S$ can be made to approach the ideal value to a fair degree of approximation without involving excessively large circuit dimensions. The actual values of the dimensions indicated in figure 1 are given in the caption, the symbols corresponding to those used in Dwight's formula.

The force at the break jaw tending to open this switch as developed by Dwight is as follows:

$$F = \frac{I^2}{4.45 \times 10^7}\left[2.3026 \log_{10}\left(\frac{2A}{R}\right) - \frac{2}{3} - \frac{1}{2}\frac{A}{B} - \frac{1}{6}\frac{C^2}{A^2} + \frac{3}{20}\frac{R^2}{A^2} + \frac{1}{24}\frac{A^3}{B^3} + \frac{1}{24}\frac{AC^2}{B^3} + \frac{B}{S}\right] \text{pounds} \quad (2)$$

The current is expressed in amperes. Substituting the dimensions of the switch in figure 1 in equation 2,

$$F = \frac{I^2}{4.45 \times 10^7}\left[2.3026 \log_{10}\left(\frac{2 \times 12.00}{0.3125}\right) - \frac{2}{3} - \frac{12.00}{2 \times 16.00} - \frac{1}{6}\left(\frac{0.625}{12.00}\right)^2 + \frac{3}{20}\left(\frac{0.3125}{12.00}\right)^2 + \frac{1}{24}\left(\frac{12.00}{16.00}\right)^3 + \frac{12.00 \times (0.625)^2}{24 \times (16.00)^3} + \frac{16.00}{14.00}\right] \text{pounds}$$

giving

$$F = 1.002 \times 10^{-7} I^2 \text{ pounds} = 4.54 \times 10^{-8} I^2 \text{ kilograms}$$

Substituting this value of force into equation 1, the change of inductance with distance should be

$$\frac{\Delta L}{\Delta S} = \frac{2F}{I^2} = 8.92 \text{ abhenries per centimeter} \quad (3)$$

The Impedance Bridge

The bridge method of measuring these forces presents several advantages. It is necessary to pass only a small current through the switch circuit, sufficient to give adequate bridge sensitivity. No special connecting devices are needed inasmuch as it is not necessary to open the switch blade beyond a point at which a satisfactory friction contact can be maintained at the break jaw. The configuration of the circuit, whether or not it is susceptible to mathematical analysis, is immaterial to the operation of the bridge.

It was necessary to develop a bridge that could detect a change in inductance of 1×10^{-9} henry. A Maxwell bridge was chosen to measure this change in inductance. As explained before, the problem does not involve the measurement of the absolute values of the impedances in any of the bridge arms; this simplifies matters considerably. The complete bridge circuit is shown in figure 2.

The resistances in the various arms were equal (15 ohms), with the exception of a small increment of resistance added to R_2 to balance the resistance of the switch circuit. To secure a final resistance balance a 0.1-ohm constant-inductance slide-wire was employed in series with R_3. Blocks of high grade mica capacitors were connected in parallel with a variable air capacitor to balance out the residuals of the bridge and of the switch circuit. A precision type of variable air capacitor of 1,500 micromicrofarads maximum capacitance was used for the final reactance balance. The change of resistance occasioned by the maximum opening of the switch blade necessitated only a very small change in the slide-wire setting (less than 0.002 ohm). Because of this small change in resistance and also because the resistances in the remaining arms were not disturbed and were small in value, it was not necessary to shield the members of the bridge electrostatically from each other. The switch was removed from the immediate vicinity of the bridge to minimize any mutual effects. The entire bridge was enclosed in a metallic screen, and a frequency of 56 cycles was used to eliminate disturbing effects of 60 cycle circuits in the laboratory. All bridge connections were made with heavy bus bar, with the exception of the twisted leads connecting the switch to the bridge; these were enclosed in a small pipe for purposes of rigidity.

To secure phase selectivity of balance a moving-coil a-c galvanometer was used as a null detector. This instrument had a sensitivity of 1.3×10^{-9} ampere per millimeter. The resistance of the moving coil was 20 ohms, thus affording a good impedance match with the impedance of the bridge circuit at the galvanometer terminals.

To prevent a negligibly small resistance unbalance from affecting the reactive balance, it was necessary to keep the phase of the galvanometer field very closely along the quadrature component. Instead of carefully shifting the phase of the galvanometer field from one component to another with a single phase-shifting transformer, it was found to be more convenient to use 2 phase-shifting transformers; one was set permanently for the reactive balance, and the other for the resistance balance. These were connected alternately to the galvanometer field by means of a double-pole double-throw switch as indicated in figure 2.

Analysis of Bridge Circuit

In developing the expression for the increase of inductance of the switch circuit, it is necessary to consider the effect of residual inductance in each resistance element. The impedances of the several arms are then as follows:

$$Z_1 = R_1 + j\omega L_1 \qquad Z_3 = \frac{R_3 + j\omega L_3}{1 - \omega^2 C_3 L_3 + j\omega R_3 C_3}$$
$$Z_2 = R_2 + j\omega L_2$$
$$Z_4 = R_4 + j\omega L_4$$

Here the R's are the resistances of the various arms, C_3 is the shunt capacitance, L_2, L_3, and L_4 are the residual inductances of the corresponding resistances, and L_1 is the residual inductance of R_1 plus the inductance of the switch circuit in the closed position. At balance,

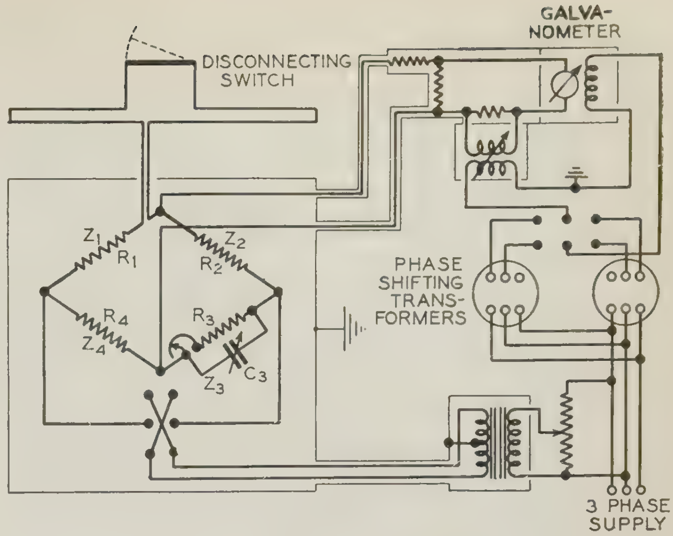

Fig. 2. Circuit diagram of complete bridge

$$\frac{(R_1 + j\omega L_1)(R_3 + j\omega L_3)}{1 - \omega^2 C_3 L_3 + j\omega R_3 C_3} = (R_2 + j\omega L_2)(R_4 + j\omega L_4) \tag{4}$$

Equating quadrature components and solving for L_1,

$$L_1 = C_3 R_2 R_4 - C_3 \omega^2 L_2 L_4 + \frac{R_3(L_2 R_4 + L_4 R_2) - L_3(R_2 R_4 - \omega^2 L_2 L_4)}{R_3^2 + \omega^2 L_3^2} \tag{5}$$

Here the first term is the normal balance condition for inductance in a "pure" Maxwell bridge, and the remaining terms are corrections engendered by the residual inductances of the resistances.

When the blade of the switch is opened by a slight amount, the impedance in the Z_1 arm changes from $R_1 + j\omega L_1$ to $(R_1 + \Delta R_1) + j\omega(L_1 + \Delta L_1)$. This small change in impedance is balanced by changing C_3 to $(C_3 + \Delta C_3)$ and the constant-inductance slide-wire from $R_3 + j\omega L_3$ to $(R_3 + \Delta R_3) + j\omega(L_3 + \Delta L_3)$. Since the slide-wire undergoes only a minute change, the denominator of 2 of the correction terms may be written

$$(R_3 + \Delta R_3)^2 + \omega^2(L_3 + \Delta L_3)^2 \cong R_3^2$$

The expression for inductance change of the switch circuit then becomes

$$\Delta L_1 \cong \Delta C_3 R_2 R_4 - \Delta C_3 \omega^2 L_2 L_4 + \frac{\Delta R_3(L_2 R_4 + L_4 R_2) - \Delta L_3(R_2 R_4 - \omega^2 L_2 L_4)}{R_3^2} \tag{6}$$

The resistances of all arms being maintained substantially equal, the following substitution may be made in the correction terms:

$$R = R_2 = R_3 = R_4$$

and then obtain, upon neglecting $(\omega^2 L_2 L_4)/R_3^2$ in comparison with unity,

$$\Delta L_1 \cong \Delta C_3(R_2 R_4 - \omega^2 L_2 L_4) + \frac{\Delta R_3}{R}(L_2 + L_4) - \Delta L_3 \tag{7}$$

Consideration of the residual inductances of the

resistance boxes used as well as a generous allowance for the change in inductance ΔL_3 of the constant-inductance slide-wire revealed that the correction terms were less than 0.1 per cent of the main term $\Delta C_3 R_2 R_4$. It should be noted that balancing the bridge with the Z_3 arm brings only the variation of capacitance ΔC_3 into the main term, the resistances R_2 and R_4 remaining undisturbed.

The product term $\Delta C_3 R_2 R_4$ dictates the use of a low impedance bridge in order that increments of capacitance may be read conveniently on standard types of precision variable air capacitors. Too large values of resistances necessitate an inconveniently small variable capacitor, whereas too small values of resistances tend to make the resistance balance of the bridge difficult for convenient operation.

Results

A typical set of measurements on the disconnecting switch shown in figure 1 is given in table I. The increments of inductance ΔL are plotted against increments of displacement ΔS in figure 3. The points approximate a straight line of slope 9.10 abhenries per centimeter. An idea of the consistency of performance of the bridge may be obtained from a series of such measurements. The resultant slopes obtained are:

Test number	1	2	3	4	Mean
Slope of line (abhenries per centimeter)	9.28	9.19	9.10	9.31	9.22

The spread between maximum and minimum values expressed as a percentage of the mean is 2.3 per cent. The chief factor contributing to this spread is believed to be the difficulty of obtaining a resistance balance because of variation of contact resistance in the constant-inductance slide-wire.

Although the increment of resistance ΔR_3 appears only in a correction term in the expression for ΔL_1 the variation of contact resistance prevented as accurate a resistance balance as was maintained for the inductance component. However, considering the magnitude of the inductance measured, the accuracy of the bridge is ample for the determination of forces in current carrying members found in practice.

Table I—Typical Measurements on Switch Shown in Figure 1

Bridge input current = 0.50 ampere
Frequency = 56 cycles per second
C_3 = 11.965 × 10^{-9} farad (switch closed)
$R_1 = R_4$ = 15.0 ohms
R_2 = 15.2 ohms
R_3 = 15.018 ohms*

ΔS** Centimeters	ΔC_3 Micromicrofarads	ΔL Abhenries
0.00	0.0	0.0
0.96	40.8	9.3
1.58	59.4	13.5
2.17	85.8	19.6
0.69	27.6	6.3
1.83	71.4	16.3
1.20	51.6	11.8

* For balance the resistance change throughout a run was too small to be read on the scale of the 0.1-ohm slide-wire.

** ΔS was measured with a cathetometer along the arc passing through the center of the break jaw.

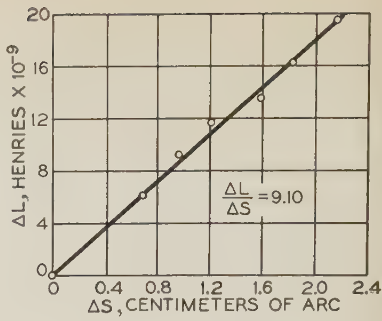

Fig. 3. Variation of inductance with displacement, plotted from data of table I

In every measurement listed in this paper, the value of $\Delta L/\Delta S$ exceeded the theoretical value computed from Dwight's formula by about 3 per cent. The mean of the observed values, 9.22, exceeds the theoretical value, 8.92, by 3.25 per cent. There are several differences between conditions as measured and those assumed by the formula. The switch, as measured, was connected to leads and return conductor of finite length with end connections as shown in figure 1, whereas the formula is developed on the basis of infinite leads and return conductor. There is also a possibility of discrepancy caused by the assumptions in the contribution to inductance of the switch blade within the jaws and, to some extent, in the limit of integration for current flow when calculating the effect of the jaw member. On page 1343 in Dwight's paper[2] it is stated that: "Now the flux in the jaw does not produce the full amount of mechanical force on the blade, because the current in the blade tapers off gradually from the full value to zero in the distance $2R$ in the jaw." Further, on pages 1339 and 1340: "The current in the arm, which tapers off gradually for the distance $2C$ at the jaw, may be assumed to be full strength as far as the middle of the blade."

It is difficult to see how any other assumptions could be made, while keeping the analysis within reasonable limits of practical calculation. However, it is obvious that as the switch blade is withdrawn from the closed position, these assumptions are modified continuously. Dwight states also that in the usual case, the contribution of the switch blade within the jaw constitutes about 8 per cent of the total mechanical force. Thus whatever be the nature of current flow at the jaw, the attempt to reconcile theoretical with observed values would concern only a small fraction of the total force.

The authors hope that engineers will find the method presented in this paper useful for determining forces on current carrying members of various kinds, the configuration of which is such as to preclude mathematical analysis.

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Earth Resistivity and Geological Structure

The relation which the resistivity of the earth's crust bears to the geological structure has been studied with the aid of many experimentally obtained data. Although the resistivity is found to vary between wide limits, and not always to be the same even for the same types of rock formation in different locations, it has been found that the variation for any particular material is within more narrow limits, and that the older materials generally have the higher resistivities. The results of the studies are of value in several types of electrical calculations.

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In connection with inductive coordination problems it is frequently necessary to estimate low-frequency ground-return mutual impedances between power and communication lines. Where both lines are in place the mutual impedance can usually be measured; however, even in such cases preliminary estimates are often wanted. If one of the lines has not been constructed estimates alone must be relied upon.

The distribution of currents in the earth—the extent and character of the spreading of the current filaments downward and outward—is a major factor in the determination of these impedances. This distribution is controlled by the resistivities of the component parts of the earth's crust and the arrangement of these parts. In impedance formulas that are customarily used the effect of the earth is taken care of by the inclusion of a single parameter; the earth resistivity. For a homogeneous earth this would be the actual resistivity of the material composing it. But the crust is nowhere homogeneous; hence, the resistivity used in such formulas is always of the nature of an average of the resistivities of the several parts of the crust—it is termed the effective earth resistivity.

The effective earth resistivities for fundamental power-system frequencies derived from mutual impedance measurements made in many parts of the world, as given in published papers,[1–9] range from 2 to 10,000 meter-ohms. (The unit of resistivity used herein is the meter-ohm. The resistivity of a particular material, expressed in terms of this unit, is equal to the resistance in ohms between opposite faces of a one meter cube of that material. Earth resistivity in meter-ohms may be converted to earth conductivity in abmhos per centimeter by multiplying its reciprocal by 10^{-11}.) Recent information indicates that values considerably above 10,000 meter-ohms may be encountered in exceptional cases. With such a range to contend with, it is to be expected that estimates of ground-return mutual impedances for situations in areas where no earth resistivity data are available may be in error by large factors.

Previous Studies

In an effort to improve upon the accuracy of such estimates a study was begun several years ago of the relation between effective earth resistivity and geology. Consideration was at that time given only to areal geology, the geology of the strata of the crust lying immediately below the soil and other loose surface materials. Earth resistivity data from 25 or 30 tests at different points in the United States were then available.

From this preliminary work, it appeared that the resistivities in areas of very old rocks were high and that, in a general way, decreasing resistivity corresponded to decreasing age of the rocks. There were, however, a number of outstanding discrepancies that could not be satisfactorily explained. Further studies of this character made as additional test data became available provided information of value but the discrepancies were still unexplained.

It then became apparent that consideration of the areal geology alone was not sufficient; instead, that the earth's structure to considerable depths must be taken into account. Data on this stucture and the effective resistivities indicated by mutual impedance measurements at a large number of test sites have now been assembled. Analysis of these data shows a more or less consistent relation between the resistivity at any given point and the age and physical characteristics of the geological formations involved. This relation is such that, in general, decreasing effective resistivity corresponds roughly to decreasing age of the formations, as the earlier study seemed to indicate. However, there are certain exceptions to this rule.

The literature on this subject includes papers giving the results of effective resistivity determinations at various points in Japan[3,4] and Europe.[5–9] These papers include data on the geology of the test sites but these data are in most instances confined to the principal materials composing the uppermost strata or to the geological periods of these strata. None of the published papers giving the results of effective

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1. For all numbered references, see list at end of paper.

resistivity determinations within the United States has included geological information.

Usefulness of the Data

The principal field of usefulness of the data to be presented is in the preliminary estimating of mutual impedances between ground return circuits and in the calculation of the self-impedances of such circuits, where only the order of magnitude of the earth resistivity is required. Where more accurate values of mutual impedance are necessary the type of data presented is useful in planning the character, extent, and arrangements of the needed field tests.

Certain of the data may be also found useful for other purposes, such as the calculation of impedances of lines at higher frequencies, estimating the character and difficulty of obtaining artificial grounds, and in connection with studies of natural earth currents, attenuation of radio signals, the behavior of traveling waves on lines, and potential gradients in the earth resulting from lightning discharges.

It must be recognized, however, that except where the earth's structure is known to be essentially uniform, estimated resistivities based upon these data must be used with especial caution in problems involving the higher frequencies or those concerned with mutual impedances of lines whose end ground connections are so situated that "ground potentials" are of importance.

The prediction of effective resistivities with a high degree of accuracy, even for low frequencies, is not possible. However, with the aid of correlation data such as those to be presented, the accuracy of these predictions can be increased materially. Giving no consideration to such data, estimates of effective resistivity varying over a range of 5,000 or more to 1 might be made for any particular area, whereas, with the information now available, it seems possible to narrow this range to 500 or 1,000 to 1 in areas of highly complicated structure and to much lower ratios under more favorable conditions. In fact there are many areas in which estimates can be made with a fair degree of assurance within a range of 10 or 20 to 1.

Summary of Principal Results

The effective resistivities indicated by the tests included in the study range from 2 to 10,000 meter-ohms, about 10 per cent of the values lying between 2 and 10 meter-ohms, 70 per cent between 11 and 1,000 meter-ohms, and the remaining 20 per cent above 1,000 meter-ohms.

The principal correlation data are summarized in the following tabulation. This is the result of grouping the tests in accordance with the geological periods to which the principal strata comprising the structure in each case belong and noting the ranges within which the resistivities determined by the greater part of the tests of each group lie.

Pre-Cambrian and combinations of pre-Cambrian and Cambrian	1,000–10,000 meter-ohms
Cambrian and Ordovician combinations	100–1,000 meter-ohms
Ordovician to Devonian, inclusive, and combinations of these periods	50–600 meter-ohms
Carboniferous, Triassic, and combinations of Carboniferous and earlier periods	10–300 meter-ohms
Cretaceous, Tertiary, Quaternary and combinations of these periods	2–30 meter-ohms

It would be well to examine briefly the meaning of this summary and to consider its limitations. The geologists tell us that underlying the entire continent of North America are extremely old rocks, extending downward to great depths. Little is known of the relative ages of different parts of this underlying structure. They are here grouped under the general term pre-Cambrian. In some areas pre-Cambrian rocks appear at or near the surface, the only covering being clays, soils, and other loose materials. In other areas they are overlain by rocks and sediments formed during later periods, the total thickness of which ranges up to many thousands of feet. The ages, arrangement, and characteristics of these upper strata are much better known. They are assigned by geologists to various periods in accordance with the ages during which they were formed. These periods appear in the tabulation in order from the oldest to the youngest.

In the case of tests made in areas where the pre-Cambrian rocks are overlain by younger strata it becomes necessary to consider just what portion of the structure probably influenced the test results. For instance, at the test sites included in the second group of the summary the upper part of the structure consists of Ordovician rocks. These are underlain by Cambrian strata which, in turn, lie on the pre-Cambrian base. The question arises whether the results were influenced by the Ordovician strata alone, by both the Ordovician and Cambrian, or by the Ordovician, Cambrian, and pre-Cambrian. The method of approach to this problem is discussed in the appendix. Application of this method discloses that probably only the Ordovician and Cambrian strata were involved to any important extent. The tests in the other groups have been treated in a similar manner.

In the cases considered above, the measurements were apparently influenced largely by strata of a single period or of 2 or more periods of about the same age. The problem is not always as simple as this. For instance, in some areas combinations of very old and very young strata occur. Areas in which the oldest rocks, the pre-Cambrian, lie directly under comparatively thin sediments of the latest periods—the Quaternary, Tertiary, and Cretaceous— are not uncommon. The effective resistivities shown by the tests in such areas range between very wide limits and the tabulation should not be taken as indicative of the values which may prevail under such conditions.

The effects of soils, glacial drift, alluvial deposits along the courses of streams, and other surface materials may also in some instances be such as to result in effective resistivities differing widely from those that would be indicated by the tabulation. However, in the cases included in the study, except in a few instances where alluvial deposits are present, these effects have not been such as to throw the resistivities

outside the ranges given. The effect of local alluvial deposits, where they overlie the older rock strata, is to lower very materially the effective resistivity that would be expected were the deposits not present.

Another limitation which must be considered is concerned with the presence of rocks formed by volcanic action. Apparently such rocks usually have a high resistivity and where they occur in a comparatively young structure, the effective resistivity may be much higher than the tabulation would indicate.

Principles of Correlation

It seems logical that there should be some correlation between earth resistivity and geological structure when the natural processes of formation of the earth's crust and the nature of its materials are considered.

The strata of the earth's crust that are of principal concern in this work are those formed by the deposition in lowlands, seas, and inland lakes of material eroded from strata at higher elevations, and by the accumulation of shells and organic materials on the bottoms of the deeper bodies of water. These materials when first deposited are loosely consolidated; the deposits may later undergo chemical changes or be subjected to great pressure; by these and other means the loose materials may be consolidated to form rocks and the metamorphosis may continue until these rocks become dense crystalline structures. The crust is, however, constantly in a state of flux, and the simple structure formed by sedimentation may, as time goes on, be folded and faulted or the strata tilted. Also, this structure may be elevated by crustal movements and in turn undergo erosion, later to again be lowered relative to the surrounding land and to receive additional sediments.

During the process of formation of sedimentary strata, or after formation, molten rock may be forced up through the structure by volcanic action, thence to spread out over the surface or to be forced between the strata in the form of great sheets. The "igneous" rock thus formed may, when first cooled, be more or less crystalline, the extent of crystallization depending, among other factors, upon the rate of cooling. As with the sedimentary rocks, those formed by volcanic action may later be extensively metamorphosed.

The dry minerals composing rocks and sediments, with the exception of certain ores, are of such high resistivity that they may be considered as complete nonconductors. According to Sundberg,[10] the resistivities of these materials as they occur in nature are determined by the waters contained in them; the waters within the particles of the material and, to a much greater extent, in the pores between particles. For materials below the water table, where the pores are completely filled, the resistivity depends upon the pore volume, the shape and arrangement of the particles, and the composition of the impregnating waters. For soils and other materials above the water table, the pores may be only partly filled with water, thus introducing another variable.

When sediments are first deposited the pore volume is great. As time goes on and they are consolidated and metamorphosed, the pore volume gradually decreases to very low values. Representative values of pore volumes, as given by Meinzer,[11] are 35 to 53 per cent of the total volume for clays and unconsolidated sands, 4 to 16 per cent for sandstones, shales, slates, and limestones, and 0.02 to 1.85 per cent for very old crystalline rocks.

Since the younger rocks and sediments are of comparatively large pore volume, they might be of either low or high resistivity so far as this factor alone is concerned. However, these materials are usually comparatively soluble and the salts dissolved from them form good electrolytes; hence they are usually of low resistivity. Exceptions to this are sands and gravels. They are relatively insoluble and where they are so situated that they do not receive conducting waters from other strata, as where they form the upper portions of structures, they may be of very high resistivity. Old crystalline rocks are not only of low pore volume but they are relatively insoluble and impervious. Consequently, they neither provide salts to form electrolytes nor receive conducting waters from other strata. Hence they are necessarily of high resistivity.

Individual strata of the crust at any point cannot be considered independently. In the case of the more pervious materials an intermingling of the impregnating waters of adjacent strata may take place; also these waters may flow along the strata. Hence the resistivity of a particular stratum may be dependent not only upon its own characteristics but also upon the surrounding conditions, and strata of the same age and of apparently similar materials in different localities may have different resistivities.

Young formations are usually simple in physical structure, and, since they are relatively pervious, the flow of waters between strata should tend to make the structure appear even simpler from a resistivity standpoint. Hence, where such formations prevail it is to be expected that effective resistivities determined from mutual impedances measured between wires at different separations or at different points within the same locality will be comparatively uniform. Very old structures are likely to be complicated by much faulting, crumpling, and folding of the strata, and inclusions of igneous rocks. Very little flow of waters between the parts of such structures is to be expected; hence the resistivities of the several parts may be quite different. Also, where effective resistivities are high the structure to depths of several thousand feet will be involved if the frequency is low; thus the chances for structural irregularities to affect the resistivities are great. Because of these factors wide and erratic variations in effective resistivities are likely to be encountered in regions where high resistivity rocks prevail.

Methods of Determining Earth Resistivities

The greater part of the resistivity data used in the study was derived from measurements of the mutual impedances of existing lines, usually a power transmission line and a telephone line. In certain cases

the measurements were confined to a single section, in others they were made both in the over-all section and in a number of subsections. In some instances either an existing line or a long wire supported in a temporary manner was energized as a ground return circuit and several comparatively short wires, gener-

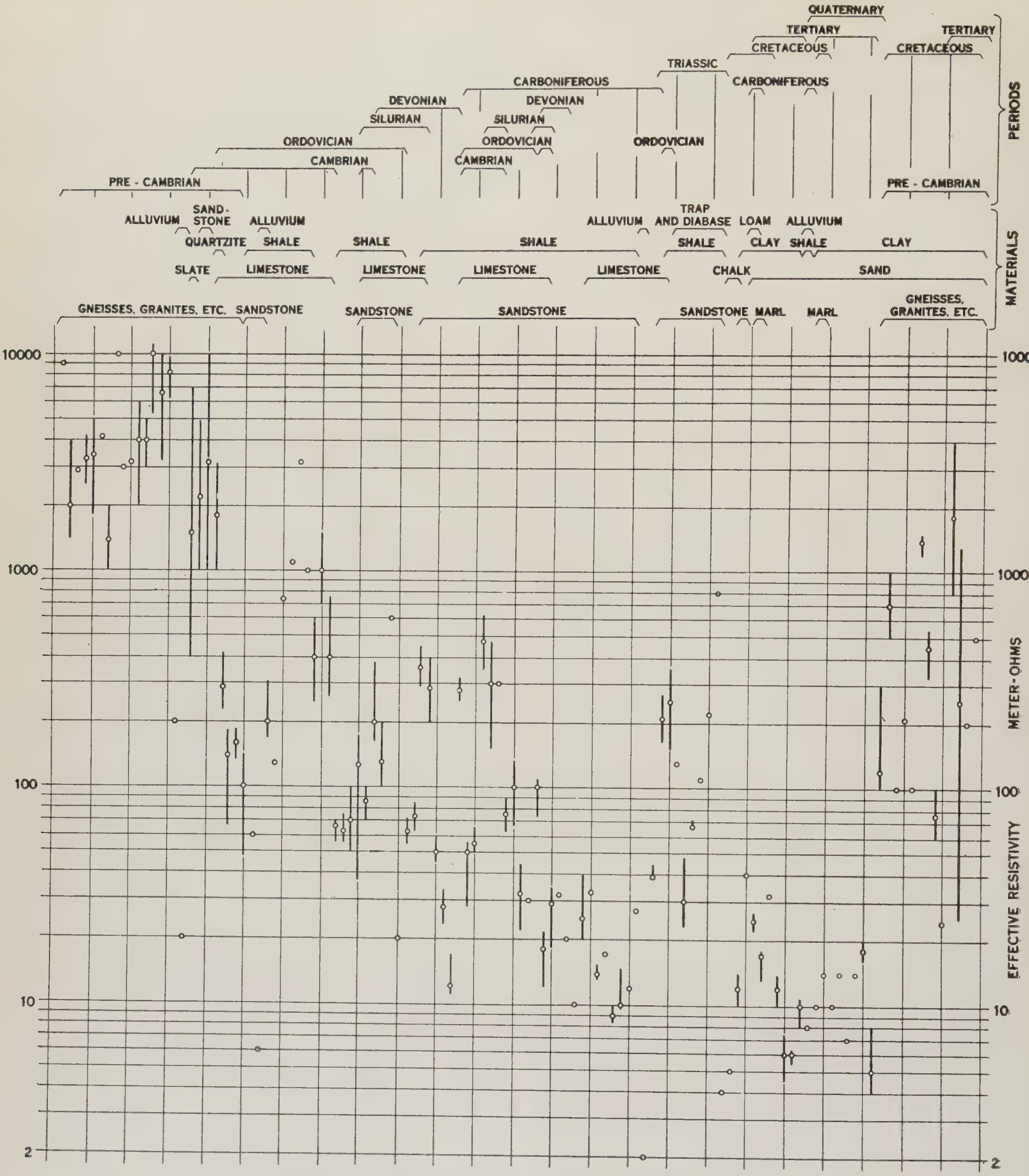


Fig. 1. Correlation of effective resistivities with geological periods and materials

The heavy lines indicate the range of effective resistivities for tests in which measurements were made in several subsections or exploring wires; the mean effective resistivities are indicated by circles. Isolated circles indicate effective resistivities where only one measurement was made

The sequence in which the various types of materials are listed is not necessarily the time sequence in which these materials occur in the geological structures, although the oldest materials are, in general, on the left

The overburden is not indicated except in a few cases where material thicknesses of alluvium are present

"Limestone" as used in connection with silurian and earlier formations includes, in many cases, dolomites as well

ally termed exploring wires, were placed at various separations from the energized circuit and parallel to it. Measurements were then made of the mutual impedance of the energized circuit and each of the exploring wires in turn. With few exceptions the measurements were made at a frequency of 60 cycles. Methods of making such measurements and of deriving from them the effective resistivities have been discussed in published papers.[1,5,6,8,9]

In most cases the resistivities indicated by the measurements in various exploring wires or subsections vary over a considerable range. Where the measurements were made in consecutive subsections the term "mean effective resistivity" is here applied to the value corresponding to the over-all mutual impedance. Where exploring wires were used, this term is applied in most instances to the geometric mean of the maximum and minimum values.

A second method of measurement which has been employed in a few instances involves the exploration of the field between the end electrodes of a ground return circuit energized with direct current. This method is similar to those frequently used in such work as geophysical prospecting, one of which has been described as Gish and Rooney.[12]

General Correlation Data

The basic data included in the study are summarized in figure 1 in a manner to facilitate comparison of the resistivities derived from the results of the several tests with the geological periods and types of rocks involved in each case. For those cases in which only one measurement of mutual impedance was made, the effective resistivities are indicated in this figure by isolated circles. Where measurements were made in several sections of lines or in several exploring wires, the mean effective resistivities are indicated by circles and the range of individual values by the vertical lines drawn through these circles. For any particular test the principal materials composing the structure and the geological periods of the principal strata are indicated by the brackets directly above the resistivity data. From left to right the tests are included in the figure in order of the ages of the predominant strata composing the structures, from the oldest to the youngest, so far as it has been practicable to do so.

This figure shows in striking manner the tendency for effective resistivities to decrease with decreasing age of structure but at the same time it emphasizes the irregularity of this relation.

It does not seem unreasonable that this irregularity should exist when it is considered that each geological period covers a very wide range in age, that the strata of any given period are composed of many different types of materials, and that the impregnating waters of the strata of one period may be affected by neighboring strata of other periods. Where strata of the same period have been deposited in widely separated areas, these factors have a greater opportunity to cause variations in resistivities. The test results confirm this; they indicate that the effective resistivities of structures of given periods within certain large geographical regions are markedly different from those of structures of the same periods in other large regions. Within any one of such regions, excluding areas where igneous and highly metamorphosed sedimentary rocks are involved, the effective resistivities of structures of the same period are encompassed within a comparatively narrow band. This effect is illustrated by figure 2. As

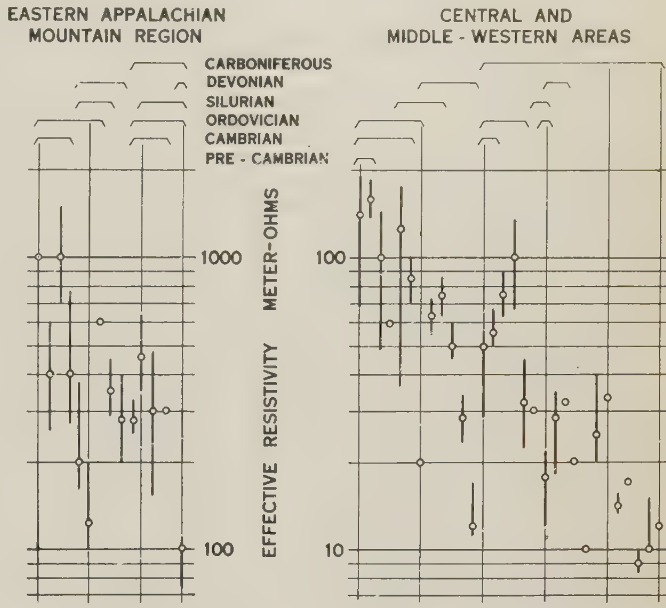


Fig. 2. Correlation of effective resistivities and geological periods—Paleozoic areas

shown by this figure the effective resistivities in the eastern Appalachian Mountain region are higher by ratios of 5 or 10 to 1 than those determined for structures of corresponding periods farther to the west.

Not only is the variation between the effective resistivities at more or less widely separated localities of interest, but also the variation in the resistivities corresponding to mutual impedances for different separations at particular points, or to mutual impedances of consecutive sections of 2 lines, the separation between which may be constant or may vary in an irregular manner. In each of the tests in areas of the older rocks, the pre-Cambrian and combinations of pre-Cambrian and Cambrian, in connection with which measurements were made in several sections or exploring wires, the variations in resistivities were very erratic and in several instances were quite wide, in one case nearly 20 to 1. Such variations are in accord with the expectations for the behavior of these older structures as outlined above.

Considering the tests involving structures composed principally of strata of Cambrian or later periods, it will be noted from figure 1 that the range of variation indicated for any particular test is comparatively narrow; in general, 2 or 3 to 1. The structures in most of these cases are composed of fairly uniform layers, horizontal or inclined at low angles. However, even where these simpler structures are involved there may be considerable vari-

ation in the stratification at different points along the test lines. In the case of some of the tests involving strata of intermediate age, the Cambrian, Ordovician, Silurian, and Devonian, the structures are complicated by faulting, folding, and tilting of the strata. The uniformity of the results shown by individual tivities of structures and the periods to which the component parts of these structures are assigned. The relation between these resistivities and the various types of rocks, the granites, limestones, sandstones, etc., must also receive consideration.

Where recently deposited sedimentary materials,

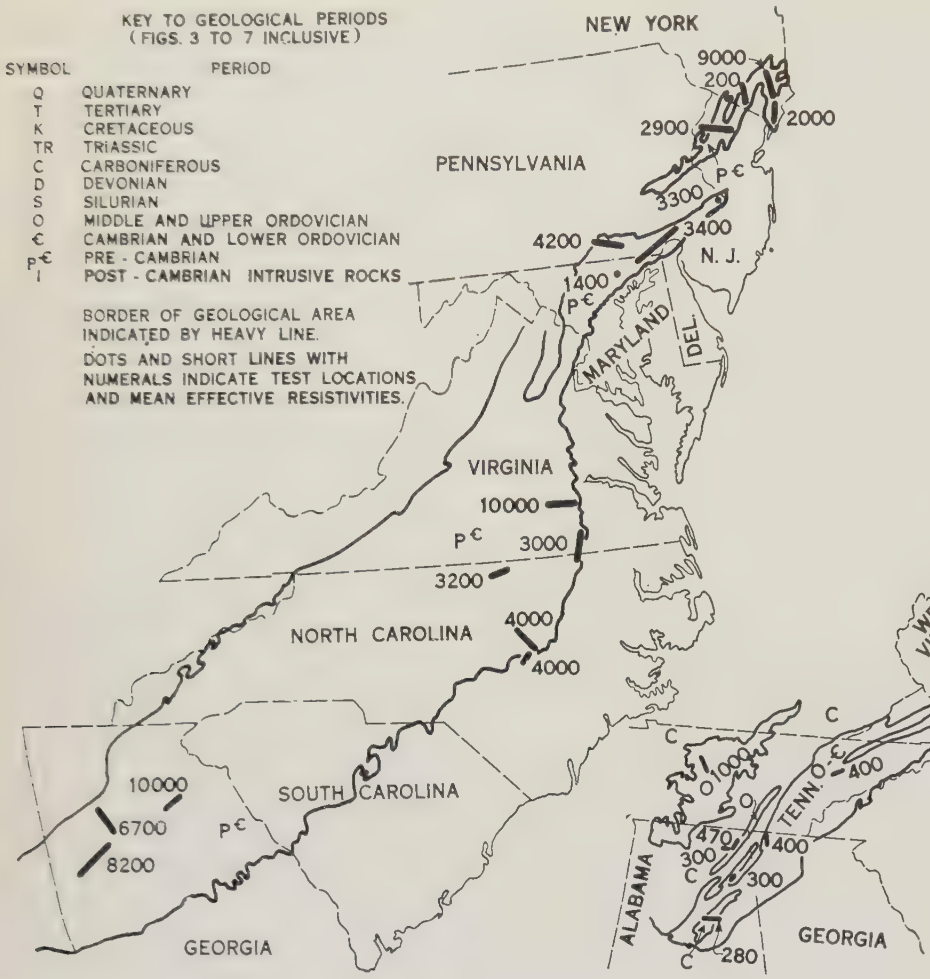


Fig. 3 (left). Areal geology and effective resistivities—pre-Cambrian areas

Fig. 4 (below). Areal geology and effective resistivities—Appalachian Paleozoic areas

Refer to key on figure 3

tests where such structural variations are encountered is perhaps explained, at least in part, by the tendency for the resistivities of the several parts of structures composed of the more pervious materials to merge due to the intermingling of the impregnating waters.

On the right-hand side of figure 1 are shown the data for several tests involving structures composed of materials of 2 of the latest periods, the Cretaceous and Tertiary, directly underlain by pre-Cambrian strata. The upper strata are composed mostly of low resistivity materials and the pre-Cambrian strata are of high resistivity. There is a distinct layer effect in such cases and the effective resistivities may range from the resistivity of the upper materials, where these are very thick, to values approaching the resistivity of the pre-Cambrian rocks as the thickness of the upper strata decreases. As will be noted the mean effective resistivities indicated by these tests range from 24 to 1,800 meter-ohms.

The foregoing discussion has been confined almost entirely to the relation between the effective resis- such as clays, sands, marls, and chalks, predominate, the effective resistivities range from 2 to 32 meter-ohms. Such materials are consolidated in time to form shales, slates, sandstones, limestones, and similar rocks, which in various stages of induration will be found principally in structures ranging in age from the Cretaceous to the Cambrian, inclusive. The range in effective resistivities where these rocks prevail is from 10 to 1,000 meter-ohms. The pre-Cambrian rocks consist mostly of schists, gneisses, granites, quartzites, and other dense, hard varieties. Similar rocks are present in some Cambrian structures, and very hard rocks, such as granites, have been formed by volcanic action in even later periods. The effective resistivities where these types prevail range from 1,000 to 10,000 meter-ohms.

Correlation Data for Specific Areas

To give a general picture of the resistivity characteristics of different areas within which tests have

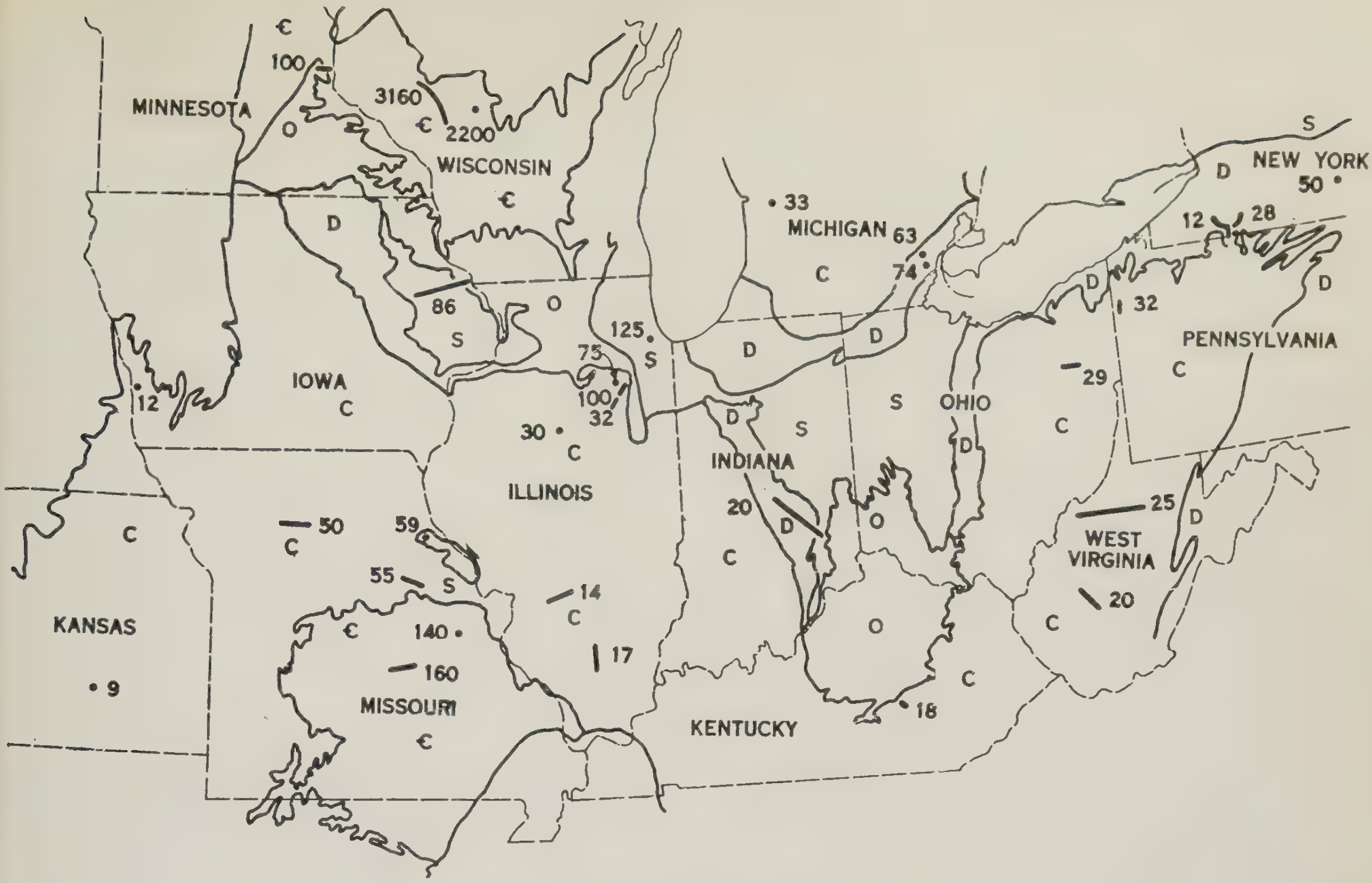


Fig. 5 (above). Areal geology and effective resistivities—Central and Middlewestern Paleozoic areas

Refer to key on figure 3

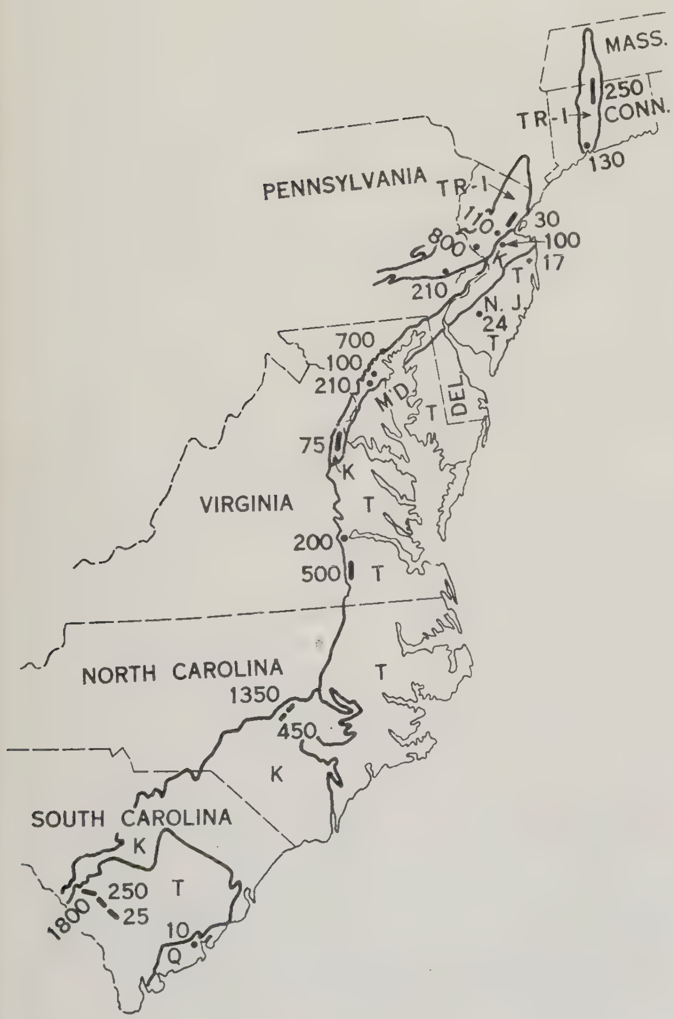
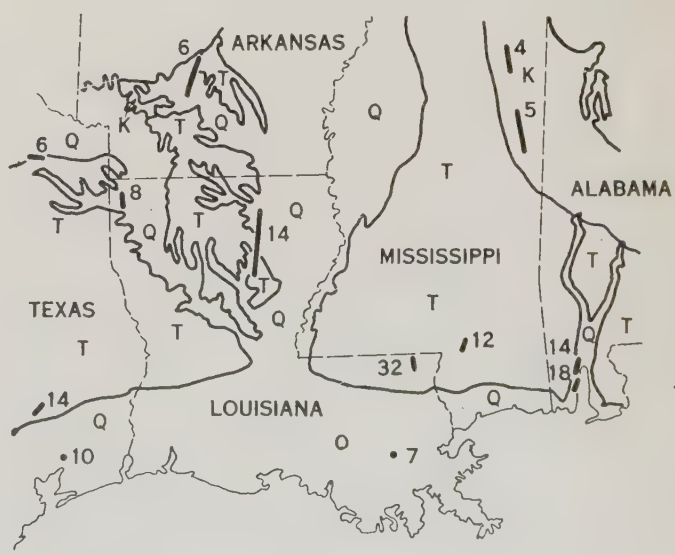


Fig. 6 (left). Areal geology and effective resistivities—Eastern and Atlantic Coastal Plain Mesozoic and Cenozoic areas

Refer to key on figure 3

Fig. 7 (below). Areal geology and effective resistivities—Gulf Coastal Plain Mesozoic and Cenozoic areas

Refer to key on figure 3



been made, the maps of figures 3 to 7, inclusive, are presented. A few areas, within each of which only 1 or 2 tests have been made, are not included. The maps show the geological periods of the upper strata as they would appear were the overlying mantle of soil, glacial drift, local alluvial deposits and such removed. The small scale has necessitated the omission within areas assigned to given periods of many small areas of other periods. For each test the maps show the location, the mean effective resistivity, and roughly the extent of the test section.

To emphasize the characteristics of the different areas they have been divided, in preparing the maps, into 3 groups. These groups are: first, the pre-Cambrian; second, the Paleozoic, which includes the periods from the Cambrian to the Carboniferous; and third, the Mesozoic and Cenozoic, which includes the Triassic to the Quaternary. The Paleozoic areas are further subdivided into 2 subgroups, which, for convenience, are denoted as the Appalachian and the Central and Middlewestern areas. The Mesozoic and Cenozoic areas are likewise subdivided into the Eastern and Atlantic Coastal Plain, and the Gulf Coastal Plain areas.

The structures of the pre-Cambrian areas shown in figure 3 are highly complicated and are composed of dense crystalline rocks, some of igneous origin, others being metamorphosed sedimentary rocks. The tests within these areas indicated mean effective resistivities ranging from 1,400 to 10,000 meter-ohms, the only exception being one test in New York from which a value of 200 meter-ohms was derived. The rocks at the latter test site are partly covered with alluvium.

A wide variety of structures is involved in the tests in the Paleozoic areas shown in figures 4 and 5. Likewise the effective resistivities varied widely, the mean values ranging from 67 to 3,200 meter-ohms. Values above 1,000 meter-ohms were indicated only at points where pre-Cambrian rocks are involved or where rocks of the earlier Paleozoic periods have been partly or wholly metamorphosed and greatly disturbed by crustal movements.

The remaining tests of the Paleozoic group may be divided into 2 classifications; those in the Appalachian areas, which indicated values from 67 to 1,000 meter-ohms, and those in the Central and Middlewestern areas, which indicated values from 12 to 160 meter-ohms. At the sites of the greater part of these tests the structures are relatively simple. However, at many of the Appalachian area test sites the structures are characterized by complicated folding of the strata and by many faults.

Two distinctly different types of structure are of concern in the Mesozoic and Cenozoic areas of figure 6; first, those of the Triassic areas shown in the northern portion of the figure, and second, the younger areas of the Atlantic Coastal Plain. In the Triassic areas, the structures are highly complicated. The strata have been tilted and extensively faulted, and there are many intrusions of igneous rock. The effective resistivities in these areas range from 30 to 800 meter-ohms.

The sediments of the Atlantic Coastal Plain, of Cretaceous, Tertiary and Quaternary periods, lie directly on pre-Cambrian rocks. The sediments are generally very thin along the western edge of the plain and the effective resistivities there range from 75 to 1,800 meter-ohms. Toward the coastline the thickness gradually increases to 2,000 feet or more,

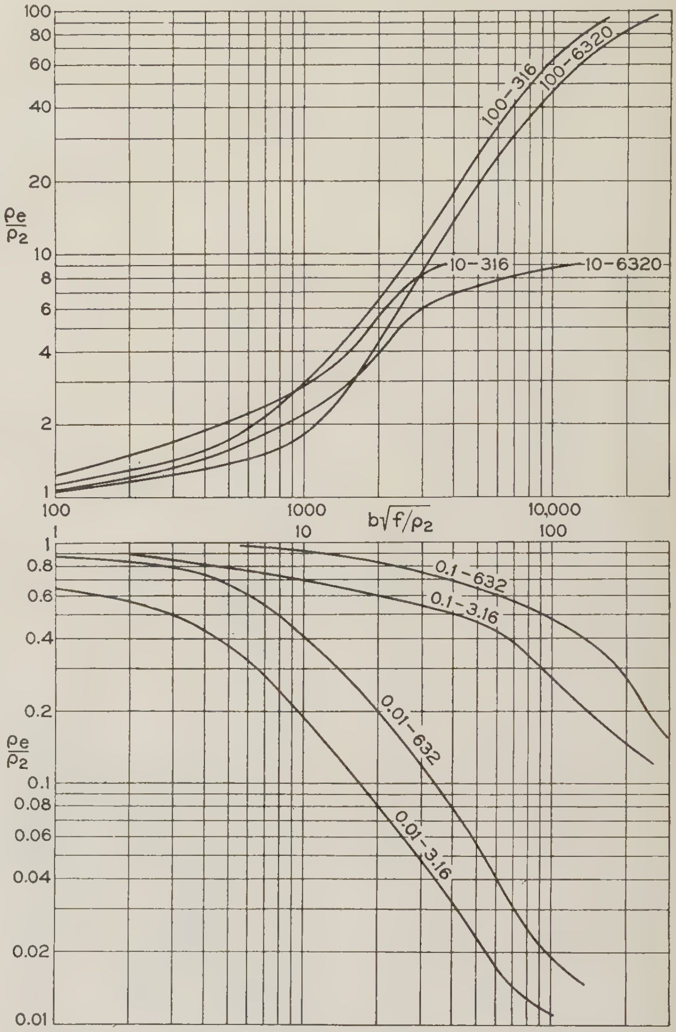


Fig. 8. Two-layer earth—variation of effective resistivity with thickness of upper layer, resistivities of upper and lower layers, and frequency, for wires on the surface of the earth

b = thickness of upper layer
x = separation between wires
ρ_e = effective resistivity
ρ_1 = upper-layer resistivity
ρ_2 = lower-layer resistivity
f = frequency
Resistivities in meter-ohms; distances in feet; frequencies in cycles per second

Figures on each curve are values of $\frac{\rho_1}{\rho_2}$ and $x\sqrt{\frac{f}{\rho_2}}$ respectively

and the effective resistivities decrease to values of 10 to 25 meter-ohms.

At the test sites in the Gulf Coastal Plain areas of figure 7 the Mesozoic and Cenozoic strata are very thick, in most instances more than 2,000 feet. The effective resistivities are uniformly low, most of the mean values lying between 4 and 18 meter-ohms.

Concluding Remarks

Study of the electrical characteristics of the earth's crust is in its infancy. Much remains to be done in the way of determinations in the field of the characteristics of the component parts of structures and of these structures as a whole. Such field studies of electrical characteristics should be paralleled by studies of the physical characteristics of these structures and of the composition of their impregnating waters. Furthermore, the data from many such investigations should be analyzed and correlated.

Further theoretical study of the behavior of various types of nonuniform structures, supplemented possibly by laboratory investigations employing reduced scale models of earth structures, would be of value.

Appendix

In connection with studies of the character of that presented in this paper some idea must be had as to what portion of the structure probably influenced the mutual impedance measurements from which the effective resistivities were derived. The complicated nature of the earth's crust together, in the usual case, with lack of knowledge of the individual resistivities of the parts of the structure precludes a direct solution of this problem. However, in any particular case it is possible to specify a certain depth and to say that it is improbable that the strata below a depth of that order influenced the results to any material extent. With no knowledge of the resistivities of the several parts it is not possible to determine just what portion of the structure above this depth is involved.

This problem may be attacked with the aid of a recently published formula[13] for the mutual impedance of grounded wires on the surface of a 2-layer earth; that is, a structure composed of 2 horizontal layers, each of uniform resistivity, the upper layer of finite thickness, the lower layer extending to an infinite depth. The curves of figure 8, which are based upon this formula, show for any frequency and for selected separations between wires and ratios of upper-layer to lower-layer resistivity, the relation between the effective resistivity, lower-layer resistivity, and thickness of upper layer.

In the usual case the deeper portions of a structure are of higher resistivity than the upper strata. Assuming this to be true for a

Table I—Representative Depths Determined for 60 Cycles

Effective Resistivity	Depth to Lower Layer $\rho_1 < \rho_2$	Depth to Lower Layer $\rho_1 > \rho_2$
3,000 meter-ohms	5,000 feet	12,000 feet
1,000 meter-ohms	3,300 feet	7,400 feet
100 meter-ohms	1,500 feet	2,300 feet
10 meter-ohms	600 feet	600 feet

ρ_1 = resistivity of upper layer
ρ_2 = resistivity of lower layer

particular case, then the resistivity of the upper strata, considering these as forming a single uniform layer, must be less than the effective resistivity if the depth to the lower strata is such that they influence the effective resistivity. If an upper layer having a resistivity somewhat less than that of the effective value and a lower layer having a resistivity which is not likely to be exceeded are assumed, then the corresponding depth as determined from the 2-layer curves will be a value below which the structure may be considered to have had relatively little influence on the measurements. For unusual cases in which there is reason to believe that the resistivity of the deeper strata may be less than that of the upper strata, the depth below which the structure probably had little influence on the measurements can be determined in a similar manner.

Representative depths determined by this method for a frequency of 60 cycles are given in table I. For the condition of a lower layer of higher resistivity than the upper layer, these depths were determined by assuming an upper layer having a resistivity equal to $^2/_3$ of the effective resistivity and a lower-layer resistivity of 10,000 meter-ohms. For the opposite condition, an upper layer having a resistivity 50 per cent greater than the effective resistivity and a lower-layer resistivity of 1 meter-ohm were assumed.

For both types of structure, at frequencies lower than 60 cycles greater depths would be of concern; for higher frequencies, smaller depths.

Other facts of interest concerning the expected behavior of a 2-layer structure can be deduced from the curves of figure 8. For instance, it will be noted that effective resistivities for small and large separations between wires may differ in some instances by factors as great as 2.5 to 1. This difference is greatest where the upper layer is of the lower resistivity.

The possible effects of relatively thin upper layers, such as soils, sands, glacial drift, and alluvial deposits, may also be determined from figure 8. It will be seen that, for low frequencies, where the upper layer is of the higher resistivity, this layer will have little influence on the effective resistivity unless it is very thick. For example, for a frequency of 60 cycles, a lower-layer resistivity of 10 meter-ohms, and a 200 foot upper layer having a resistivity anywhere between 10 and 1,000 meter-ohms, the effective resistivity will not exceed 20 meter-ohms. Where the upper layer is of the lower resistivity the effective resistivity may vary widely with variations in the resistivity of this layer even though it may be very thin. Given a 1,000 meter-ohm lower layer and a 200 foot upper layer, the 60 cycle effective resistivities for wide separations will be around 60 and 600 meter-ohms for upper layer resistivities of 10 and 100 meter-ohms, respectively.

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A Cable Code Translator System

By means of specially devised translators, described in this paper, 3-element cable-code signals may be translated into 2-element signals suitable for transmission over ordinary landline telegraph plants; by means of retranslating circuits, also described, the signals may be restored to their original 3-element form. Signals in the 2-element form may be monitored without the aid of retranslators. Synchronism is not required between the translators and retranslators.

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EVER SINCE its first cables were laid, The Commercial Cable Company, which owns and operates several submarine telegraph cables between North America and Europe, has used a 3-element code for signaling. In this code a dot is represented by a positive potential applied to the cable for one unit of time, a dash by a negative potential applied for one unit of time, and a space between letters by a ground potential applied for a similar length of time. The combinations assigned to the letters of the alphabet are in accordance with the continental code.

This method of signaling is used quite generally by other administrations over long submarine cables. It has, among other advantages, the marked advantage of greater accuracy than the 5-unit 2-element code which is in general use in landline systems and on some cables. In the 3-element cable code practically all mutilations of signals that might occur are detected readily by the receiving operator, and necessary corrections can be made before delivery of the message to the customer. In the 5-unit code, however, it is not possible for the receiving operator to detect most errors, except in plain language messages, and consequently the number of undetected errors is much greater. Because a large proportion of cable traffic is in unpronounceable code words, it is desirable to use a code that offers a means for readily detecting errors.

A paper recommended for publication by the A.I.E.E. committee on communication, and scheduled for discussion at the A.I.E.E. winter convention, New York, N. Y., Jan. 28–31, 1936. Manuscript submitted Aug. 23, 1935; released for publication Sept. 25, 1935.

The disadvantages that previously have been held against the 3-element cable code have been that it would not directly operate an automatic printer and that it was not suitable for transmission over landline systems equipped to repeat only 2-element signals.

An automatic printer operated directly from 3-element cable-code signals is now in general use both in the Commercial Cable and All America Cable systems.[1] It is the purpose of this paper to describe a system of automatically translating 3-element cable-code signals into a special higher-frequency 2-element code suitable for transmission through ordinary landline equipment, and nonsynchronous retranslating circuits for restoring the signals into their original form.

Translator System Comprises 4 Units

The need for a suitable translator has been felt for some time, but the announcement of the British Post Office that it soon would be unable to lease metallic underground conductors between the cable landing at Weston-super-Mare and the main cable office in London made the need more pressing. The British Post Office proposed to lease channels of its voice frequency carrier system in place of the metallic conductors. The voice frequency system, however, was capable of transmitting only a 2-element code and therefore was unsuited for cable code working. Unless a translator were made available it would be necessary to lease 2 channels, one for dots and one for dashes, to transmit the signals from each submarine cable.

In response to this demand a complete translator system has been developed and is now in successful operation in England as well as on an overland line between New York City and Canso, Nova Scotia. It is believed that the present development offers a better solution to the translation problem, especially on landlines of moderate length, than previously developed systems. The various units comprising the complete translator system are as follows:

Type 32*XM*. With this translator a tape perforated in accordance with the 3-element cable-code is used to transmit 2-element signals into a landline.

Type 23*RY*. This unit retranslates the 2-element signal received over a landline into the 3-element cable code. The output of this translator comes from relays and is suitable for direct transmission into a short cable, or may be regenerated and then sent into a long cable.

Type 23*DW*. This unit retranslates the 2-element signals received over a landline into the 3-element cable code in form suitable for recording on an ink writer. It is somewhat simpler than the type 23*RY*, but is not suitable for operating printers or transmitting into another cable.

Type 32*RG*. This translator operates in conjunction with a regular cable code regenerator and translates the 3-element cable-code signals received over a submarine cable into 2-element signals for transmission into a landline.

Two-Element and 3-Element Codes

The 2-and 3-element codes are shown in figure 1. It may be noted that the 2-element signals are not of the standard continental code. In this 2-element

1. See reference at end of paper.

code, each dot, dash, or space occupies the same amount of transmission time. A 2-element dot consists of 1 unit of spacing, 1 unit of marking, and 1 unit of spacing, thus making a total of 3 units. A dash consists of 2 units of marking and 1 unit of spacing, or a total of 3 units. The zero signal consists of 3 units of spacing. In the 2-element code 3 units occupy the same period of time as one "center hole" or unit in the 3-element cable code. There is, therefore, no need of storing any signals at the translator station.

When recorded by an ink writer, the 2-element code may be easily read and therefore may be readily monitored. It is not intended, however, that messages shall be written up regularly from this code. Before recording or writing up the signals they will be retranslated into their original 3-element form. It should be noted that the third or last unit of each 2-element cable code signal is always spacing. It should be noted also that the ending of the marking interval on both dots and dashes occurs at regular intervals. On the letter *C*, for example, the interval of time between the end of the dash marking and the end of the dot marking is exactly 3 units. A similar interval of time separates the end of the dot marking and the end of the dash marking. These equal intervals of time are utilized in translating the signal into the 3-element form.

Transmitting 2-Element Signals From 3-Element Cable-Code Tape

In figure 2 are shown the circuits of translator 32*XM*, a tape controlled automatic transmitter that has been adapted to transmit 2-element code signals from a 3-element cable-code tape. This may be a standard form of automatic transmitter that operates at a constant speed. Dot and dash contacts are shown which are controlled by the dot and dash peckers under the control of the perforated tape. The 2 commutators are mounted on the same shaft of the automatic transmitter and revolve at the rate of one revolution per center hole or feed hole of perforated tape. During the first third of the revolution the circuit through both cams is open and the transmitting relay during this time is invariably on spacing. It is during this interval that the peckers "feel" for and assume their new positions as determined by the holes in the perforated tape. When a dot pecker has been selected, it is obvious that the circuit to the transmitting relay will be open for $^2/_3$ revolution and closed for $^1/_3$ revolution. When a dash pecker has been selected, the circuit to the transmitting relay will be open for only $^1/_3$ revolution and closed for $^2/_3$ revolution. When a zero signal is to be transmitted, neither pecker will be operated and there will be no marking of the transmitting relay for a full revolution of the commutators. The type of signals that will be transmitted, therefore, will be in accordance with the 2-element cable-code signals shown in figure 1.

Translating 2-Element Signals to 3-Element Cable-Code Form

Means for translating the 2-element signal back into its cable code form is shown in figure 3, which is a schematic diagram of translator 23*RY*. Two adjustable timing circuits are required: One of these circuits is for distinguishing between dots and dashes by the duration of the marking time of the received signal; the other is to space the dot and dash transmitting relays when no marking signal has been received for a predetermined time interval.

Each valve together with its anode relay and shunted grid capacitor merely acts as an easily adjusted reliable "slow-operate quick-release" relay. The cathode of each valve is tied to the main power battery in such manner that the cathode potential is approximately midway between the positive and negative potentials of the main power battery shown in the figure. The actual connection usually is made by means of a potential divider. When the control relay is in a marking position as shown, the shunted $^1/_2$-microfarad capacitor has a negative charge and the grid of *V*1 is highly negative with respect to the cathode. When a marking signal is received over the telegraph line the control relay releases and the negative connection is removed from the capacitor. The negative charge leaks off the capacitor through resistance *R*1 and the grid finally becomes positive with respect to the cathode. At some definite time after the marking signal has been received, anode current flows and the anode relay operates. The shunt resistance *R*1 of the capacitor

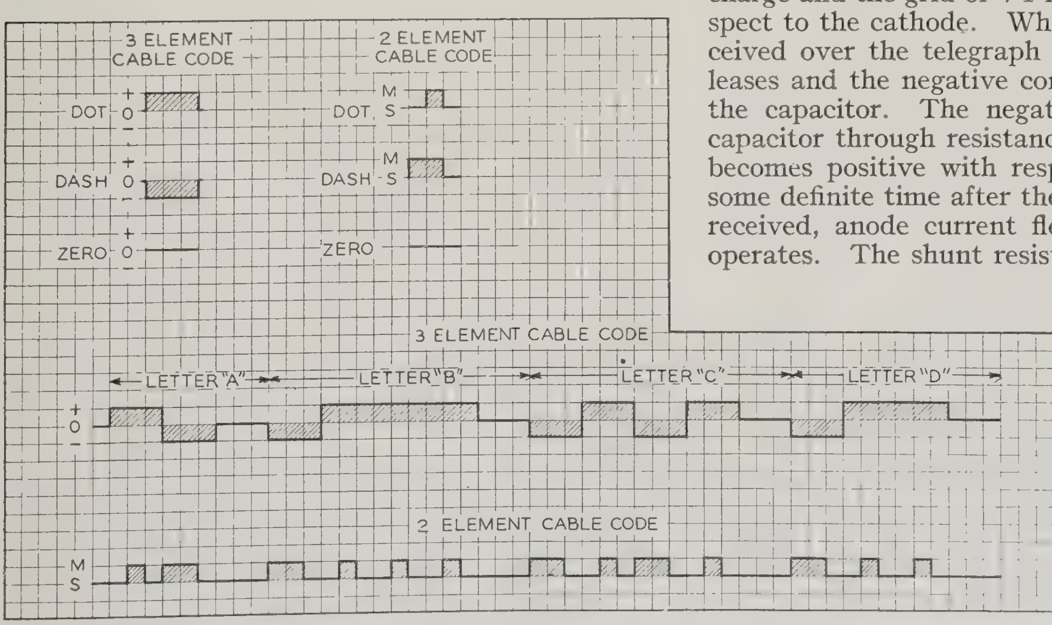

Fig. 1. Comparison of 2-element and 3-element codes

M—Marking S—Spacing

will have been adjusted so that the anode relay is not operated when a dot is received, but is operated when a dash is received. When the line relay restores to the spacing position, a circuit is completed from the line relay tongue, back contact, and thence

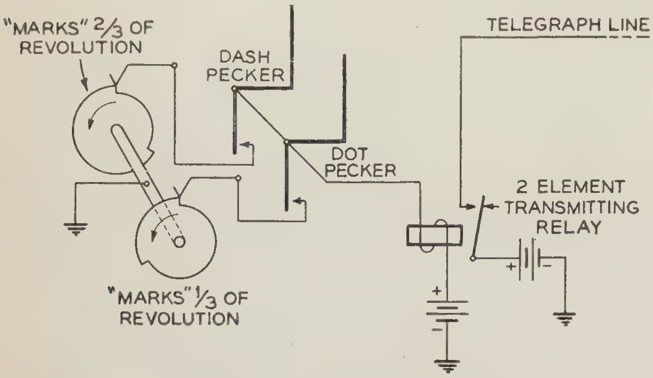

Fig. 2. Schematic diagram of translator for transmitting 2-element signals from 3-element cable-code tape

to the tongue of the anode relay. For a dot signal the anode relay is unoperated and the circuit is completed from the anode relay tongue, through the back contact, through the dot and dash transmitting relays in such direction as to "mark" the dot transmitting relay and "space" the dash relay, thereby sending out a dot cable-code signal. The dot and dash transmitting relays are polarized and remain in the positions in which they have been set. For a dash signal the anode relay will have been operated and the circuit from the plate relay tongue is completed through a holding winding on the anode relay, through the dot and dash transmitting relay, and "marks" the dash transmitting relay and "spaces" the dot transmitting relay, thereby sending out a dash cable-code signal. Each time the line relay restores to the spacing position, a cable code dot or dash is retransmitted. At the same time the control relay operates and negatively charges the capacitor connected to the grid of *V*1. If the line relay has been "marking" for only a short time a dot is retransmitted. If it has been "marking" for a longer period of time a cable code dash is retransmitted.

It should be noted that all the valves used in the different types of translators have a grid resistor of at least one megohm value. The purpose of this grid resistor is to prevent the grid from becoming highly positive with respect to the cathode, which would result in excessive anode current. As soon as the grid becomes positive with respect to the cathode the grid current produces an *IR* drop across the grid resistor of sufficient value to prevent excessive anode current.

The purpose of valve *V*2 and associated circuits is to restore the dot and dash transmitters to their spacing positions so as to send a zero or ground potential signal into the cable whenever no 2-element marking signals have been received for a certain time. While the line relay is not on the marking contact the charge on the capacitor associated with *V*2 slowly is leaking off. The rate at which the charge leaks off is controlled by the adjustable shunt *R*2. When the grid of *V*2 attains a certain potential with respect to the cathode, the space relay is operated. When the tongue of the space relay leaves the back contact the dot and dash operating circuit of the dot and dash transmitting relays is opened, and when the space relay tongue reaches the marking contact a circuit is completed that restores the dot and dash transmitting relays to "spacing" in the event either one had been "marking." The restoration of the dot and dash transmitting relays to "spacing" results in zero or ground potential being applied to the cable, thus indicating a space between letters.

Translating 2-Element Signals to 3-Element Ink-Writer Records

The following is a description of the translator (23*DW*) designed for use at a station where it is desired to produce cable-code ink-writer records only from a received 2-element signal. The circuit is simpler and has some advantages over translator 23*RY*, which produces cable code signals on a pair of relays. The principal advantage is that a small misadjustment of the dot-dash adjusting rheostat will not result in recording dots for dashes or *vice versa,* but instead will result in either the dots or dashes, as the case may be, sometimes failing to reach their full amplitude, thus indicating the necessary adjustment to be made.

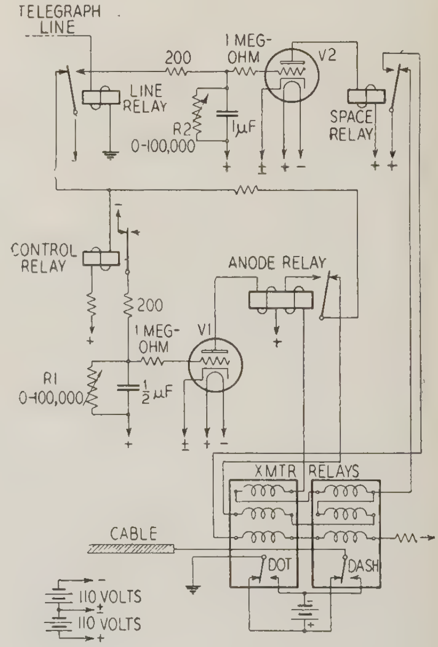

Fig. 3. Translator for converting 2-element signals to 3-element form

Resistances are shown in ohms

Referring to figure 4, it may be noted that the ink writer is connected in the plate circuit of valve *V*1. Being a sensitive device, the writer is shunted so that only a portion of the plate current passes through its operating coil. Resistances *R*1 and *R*2 are used as a voltage divider or potentiometer to apply a re-

duced voltage to the anode of the valve. The resistances in the plate circuit are so proportioned and arranged that when the grid is negative with respect to the cathode, there is no anode current and therefore no deflection of the ink writer pen. This condition is maintained while a dot is being recorded. When the grid is positive with respect to the cathode, there is maximum current in the anode circuit and the ink writer pen has maximum deflection. The ink writer is connected so that this deflection is in the dash or downward direction on the tape. This condition is maintained while dashes are being recorded. In order to record the spaces it is necessary, of course, for the pen to assume a position midway between the dot (zero anode current) and the dash (maximum anode current) line. The grid of $V1$ is maintained at a suitable potential with respect to the cathode so that the ink writer pen assumes a position intermediate to the dot and dash positions and thus records spaces. The zero adjustment potentiometer, shown in the schematic diagram, supplies the required grid potential for recording a space.

Assume that spacing is being received over the line. The line relay tongue remains against its spacing contact (S) thus energizing relays $PF1$ and $PF2$. When the line relay tongue has remained away from its marking contact (M) for sufficient length of time the grid of $V2$ becomes positive with respect to the cathode. The anode current energizes $VR2$ thus rendering the grid of $V1$ the same potential as the zero adjustment potentiometer slider. A zero or spacing signal therefore is indicated on the ink writer. Suppose a marking signal of dot duration now actuates the line relay. When the line relay tongue reaches its marking stop, capacitor $C4$ quickly is charged negatively thus stopping the anode current flow of valve $V2$. Relay $VR2$ leaves its marking contact and disconnects the zero adjustment potentiometer from the grid of $V1$. The small grid capacitor $C3$ maintains the intermediate voltage on the grid for as long as required. The departure of the line relay tongue from its spacing contact results in relays $PF1$ and $PF2$ being deenergized. When the tongue of relay $PF1$ reaches its spacing contact, capacitors $C1$ and $C2$ are paralleled, their charges mix and voltages equalize. Since the capacitance of $C2$ is only a small fraction of that of $C1$, the potential of $C1$ only slightly is changed. Capacitors $C1$ and $C2$ now slowly take up a positive charge through the dot-dash adjustment rheostat. At the end of the marking period of the line relay, relays $PF1$ and $PF2$ again are operated. When relay $PF1$ operates, capacitors $C1$ and $C2$ are separated and $C1$ is charged negatively. The operation of relay $PF2$ joins or parallels capacitors $C2$ and $C3$. The capacitance of $C2$ is much larger than that of $C3$; therefore the potential of $C2$ is altered only slightly when the charges mix. Capacitor $C3$ is now of the same potential as $C2$ and the grid of $V1$ is of a similar potential. For the dot signal just described $C3$ does not have sufficient charge to render the grid positive with respect to the cathode; the grid becomes negative; there is no anode current flow, and the ink writer therefore records a dot.

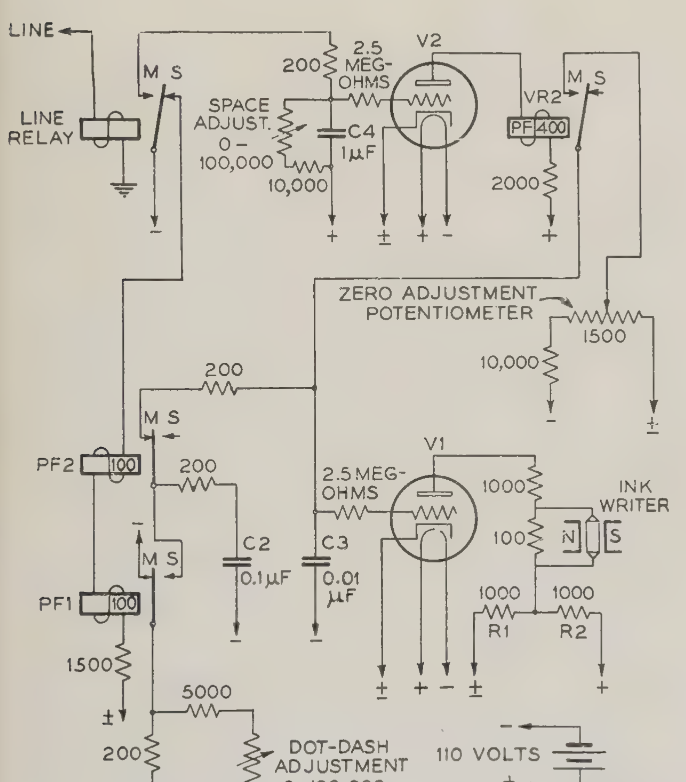

Fig. 4. Translator for converting 2-element signals to 3-element ink-writer records

Resistances are shown in ohms

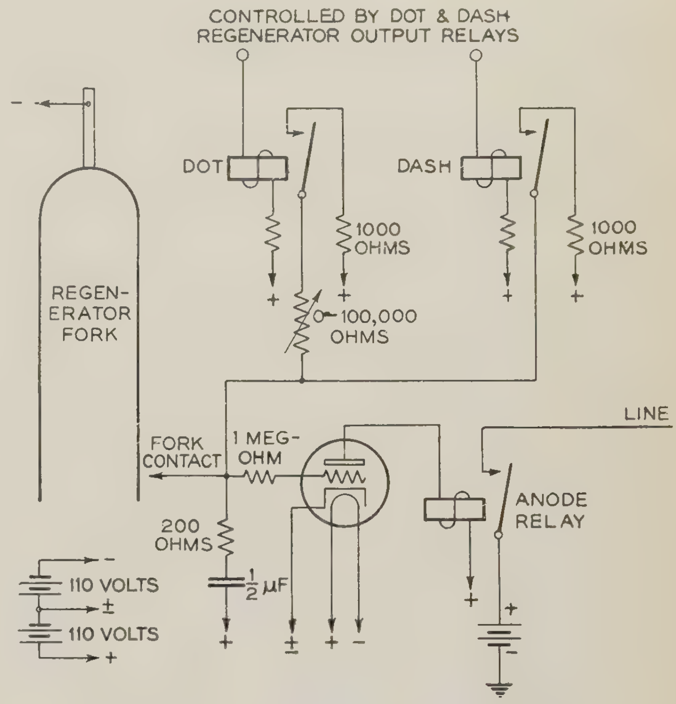

Fig. 5. Translator for converting 3-element signals received over long cables to 2-element form

Had the marking interval of the line relay been of dash length, capacitor $C1$ would have received a greater charge and $C2$ and $C3$ would have had a proportionately greater potential. The potential of $C3$ for a dash would have been sufficient to render the grid positive with respect to the cathode, and therefore maximum anode current would flow and the ink writer would record a dash.

The space adjustment rheostat is adjusted so that if the line relay does not "mark" for an interval of time equal to one "center hole," the grid of $V2$ becomes positive with respect to the cathode and anode current flows. Relay $VR2$ operates and thus makes the grid of $V1$ assume a potential, determined by the setting of the zero adjustment potentiometer, that results in the ink writer recording a cable code space. Several ink writers in parallel may be actuated simultaneously.

Translating Signals Received Over a Long Cable to 2-Element Form

When it is necessary to translate the signals received over a long cable into the 2-element form, a circuit similar to that shown in figure 5 (translator $32RG$) may be used. A standard cable code regenerator, not shown in the figure, is used to regenerate the cable code signals, and the regenerator tuning fork is used also in connection with the translator. Referring to figure 5, the 2 neutral relays, marked "dot" and "dash," are controlled by the output signal of the regenerative repeater. The fork contact shown is one that makes contact at the same time as the fork pick-up contact so that movement of the tongues of the dot and dash relays occurs only while the fork is making contact. The marking contact of the fork is adjusted to make for $^1/_3$ of the time.

While the fork is "marking," the grid of the valve is negative with respect to the cathode and there is no anode current. The anode relay, of course, will be unoperated, and a spacing signal is sent to the line. The capacitor is charged negatively. The cathode of the valve, it should be noted, is connected to the midpoint of the battery. When the fork leaves the contact the negative charge remains on the grid capacitor, provided neither dot nor dash input relay is "marking," and the output relay continues to send "spacing" to the line. If the dash input relay is in an operated position when the regenerator fork leaves its contact, then the grid capacitor quickly is discharged. The resulting anode current operates the output relay thus sending "marking" to the line. If, however, when the regenerator fork leaves its contact the dot input relay is in an operated position, it will take another $^1/_3$ "center hole" time interval, when the high resistance adjustable rheostat is set correctly, before the condenser is discharged sufficiently to operate the output relay and send a marking signal to the line.

Reference

1. Direct Printing Over Long Nonloaded Cables, M. H. Woodward and A. F. Connery. Elec. Engg., v. 51, Feb. 1932, p. 132.

Economical Loading of Underground Cables

Load ratings for underground electric power transmission cables for 2 typical metropolitan load cycles are computed in this paper by the method of harmonic analysis of load cycles. Test data are given which agree reasonably well with theoretical calculations. A logical basis for emergency rating of cables also is demonstrated. A comparison of costs of transmission for various duct conditions indicates that large conduit systems are not economical because of the lower ratings necessitated by the greater heat dissipated.

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In a former paper,[1] harmonic analysis of load cycles is employed in reducing to mathematical basis, the problem of temperature rise of cables above ambient temperature. Temperature rise is computed for each harmonic and the results added; this is permissible as the constants of the cables and duct structures are very nearly linear. It is stated also that the temperature rise for each harmonic may be computed only once for any size and type of cable and the results assembled in a table. Such calculations have been made for various sizes of 15 and 25 kv shielded cables and the results tabulated in table I. The data in this table may be combined to obtain the temperature rise of the conductor above that of the duct air for any load cycle desired, by adding the harmonic temperature rises in their proper phase relations.

In order to compute the temperature rise of the duct air above the base temperature of the earth, the heat flow at the sheath surface must be known. It would not be sufficiently accurate to assume the heat flow cycle at the sheath surface to be the same as at the conductor because there is considerable attenuation in passing through the cable insulation. The attenuation factors and time lags for each harmonic of heat flow are shown in table II.

The temperature rise of the duct air for each har-

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1. For all numbered references see list at end of paper.

monic and various sizes of duct structures is given by table III. These were computed by assuming an equivalent cylindrical conduit structure as described in reference 1. The steady state duct constants are taken from reference 12. These were obtained as a result of averaging data from various operating companies throughout the country. It should be noted that the constants of the conduit structure are such that the harmonic temperature rises are practically independent of the outside radius of the equivalent cylindrical conduit structure. This would be expected since the effect of the variable components of heat flow would die out rapidly at a distance of a few feet. The dimensions of an actual 12 duct conduit and the inside radii of the assumed cylindrical structure are shown in figure 4.

By harmonic analysis, the copper losses corresponding to 2 load cycles most common in metropolitan transmission systems are expressed by the following equations:

Winter load cycle:

$$\text{Copper loss} = 0.440 + 0.330 \sin(\omega t - 5.90 \text{ hr}) + 0.175 \sin(2\omega t + 1.70 \text{ hr}) + 0.073 \sin(3\omega t - 2.40 \text{ hr}) \quad (1)$$

Summer load cycle:

$$\text{Copper loss} = 0.560 + 0.252(\sin \omega t - 3.36 \text{ hr}) + 0.229(\sin 2\omega t - 1.00 \text{ hr}) + 0.045(\sin 3\omega t + 3.37 \text{ hr}) + 0.180(\sin 4\omega t - 1.81 \text{ hr}) \quad (2)$$

Curves of equations 1 and 2 adjusted for 8 per cent dielectric loss are shown in figure 1, together with the corresponding temperature rise curves for 4 400,000-circular mil, 25-kv cables installed in a 4 duct conduit.

All components of equations 1 and 2 are given as ratios to maximum copper loss, this maximum being taken as 1.00. The average copper loss (loss factor) is given by the first term of each equation. Each harmonic component is multiplied by the corresponding factors in tables I, II, and III, obtaining the temperature rises for each harmonic. These are added together in their proper phase relations and maximum temperature rises obtained as shown in table IV, *A* and *B*. These factors, when combined in proportion to the steady state thermal resistances from the earth to duct air and duct air to conductor, give the total maximum temperature rise, ϕ, from earth to conductor caused by the copper loss. Strictly speaking, each harmonic temperature rise for earth to duct and duct to conductor should be combined in its proper magnitude and phase relation and the results totaled to obtain the value of ϕ. It has been found for the usual load cycles, as illustrated in figure 1, that the temperature curves are flat enough at the peaks so that the difference in times of the maxima does not affect the results appreciably. A table of values of ϕ for 2 shielded cables and various numbers of ducts is given in part *C* of table IV.

Table II—Conductor-to-Duct Heat Flow Attenuation Factors, With Time Lags, for Harmonics 1 to 4

Constants of Cables Given in Table I

Rated Kv	Size of Conductor, B & S Gauge or Cir Mils	Attenuation Factors 1	2	3	4	Harmonic Time Lag in Hours 1	2	3	4
14	4/0	0.990	0 910	0.835	0.760	0.825	0.790	0.750	0.705
	350,000	0.980	0.890	0.795	0.710	0.960	0.910	0.800	0.790
	500,000	0.965	0.860	0.765	0.670	1.075	1.005	0.925	0.845
25	4/0	0.980	0.890	0.800	0.720	0.975	0.940	0.890	0.840
	300,000	0.970	0.865	0.770	0.680	1.080	1.025	0.965	0.895
	500,000	0.950	0.825	0.720	0.625	1.245	1.145	1.055	0.965

Steady and Cyclic Loading Ratings

The rating that may be given a cable for cyclic loading may be expressed as follows:

Rating under cyclic loading = Steady state rating $\times\ \lambda$

$$\text{where } \lambda = \sqrt{\frac{1.0 + \gamma}{\phi + \gamma}}$$

(For complete list of symbols see Appendix I.)

Steady state ratings may be computed by familiar methods [3–7]. In table IV *a* steady state and cyclic loading ratings based upon the 2 load cycles of equations 1 and 2 are shown. The earth base temperature was assumed to be 5 degrees centigrade in winter, and 20 degrees in summer. The maximum allowable temperature rise was $90-E$ where E is the voltage (kv) to neutral, which corresponds to the A.I.E.E. rule for temperature limit of 3-conductor shielded and single-conductor paper-insulated cables.

Belted cables require a lower rating than shielded cables for 2 reasons: the greater thermal resistivity

Table I—Ratio of Harmonic Conductor-to-Duct Temperature Rise to Steady Load Rise, With Time Lags, for Shielded Cables

S_i = 700 deg C per watt per cu cm, 14 kv cables
S_i = 650 deg C per watt per cu cm, 25 kv cables
K_s = 1,100* deg C per watt per sq cm

C_c = 0.107 watt per gram
C_i = 0.493 watt per gram
C_s = 0.035 watt per gram

ρ_c = 6.9 grams per cu cm
ρ_i = 1.16 grams per cu cm
ρ_s = 11.4 grams per cu cm

Rated Kv	Thickness of Insulation, Cm	Size of Conductor, B & S Gauge or Cir Mils	Sheath Dimensions, Cm: Diameter	Thickness	Ratio of Harmonic (1 to 4) Rise to Steady State Rise: 1	2	3	4	Harmonic (1 to 4) Time Lag in Hours: 1	2	3	4
14	0.556	4/0	5.23	0.318	0.980	0.905	0.835	0.765	0.745	0.700	0.650	0.605
		350,000	6.30	0.357	0.970	0.885	0.795	0.715	0.865	0.810	0.750	0.685
		500,000	7.04	0.357	0.965	0.865	0.765	0.680	0.975	0.900	0.825	0.745
25	0.874	4/0	6.75	0.318	0.965	0.880	0.800	0.720	0.805	0.760	0.705	0.650
		300,000	7.43	0.357	0.950	0.860	0.765	0.680	0.905	0.845	0.780	0.705
		500,000	8.27	0.357	0.935	0.830	0.730	0.630	1.060	0.975	0.880	0.785

* It has been customary to use 1,200 degrees centigrade per watt per square centimeter for the surface heat resistivity of the sheath. It is believed that 1,100 more nearly represents the condition at average loading, as the surface resistivity decreases with increase of heat flow.[2]

Table III—Ratio of Harmonic Earth-Base-to-Duct Temperature Rise to Steady Load Rise, With Time Lags

ς_e = 3.5 deg C per watt per cu ft $C_e \times \rho_e$ = 20 watts per cu ft

Number of Ducts	Steady State Duct Constant	Inside Radius, Ft	Ratio of Harmonic (1 to 4) Rise to Steady State Rise				Harmonic (1 to 4) Time Lag in Hours			
			1	2	3	4	1	2	3	4
4	0.93	0.33	0.340	0.250	0.220	0.190	2.36	1.26	0.86	0.66
6	0.82	0.46	0.330	0.240	0.210	0.185	2.42	1.29	0.89	0.68
9	0.75	0.59	0.310	0.220	0.190	0.170	2.52	1.32	0.91	0.69
12	0.72	0.65	0.295	0.205	0.175	0.155	2.56	1.34	0.92	0.70

of the cable, and the A.I.E.E. rule for which E in the expression $90-E$ refers to line voltage rather than the voltage to neutral. Consequently a correction factor may be applied to the steady state rating of a shielded cable to make it apply to the belted cable of the same size and voltage rating. This correction factor may be computed from the following formula:

$$\xi = \sqrt{\left(\frac{R_0}{R'_0}\right)\left(\frac{90 - E' - \theta' - A}{90 - E - \theta - A}\right)} \qquad (3)$$

where primes refer to belted cable constants. The value of this expression does not vary much with different sizes of cable of the same voltage rating, so an average value may be used. This factor may be applied with reasonable accuracy to the cyclic ratings, neglecting the effect of the greater thermal capacity of the belted cable compared with that of the shielded cable of the same size, and the greater dielectric loss of the shielded cable. These errors are both small and are partly compensating. Average values of this factor accompany the ratings for shielded cables given in table V.

It has been the experience on some power systems that if the temperature of the earth surrounding a duct structure be higher than a certain critical value, the earth will dry out, increasing thereby its resistivity to heat flow, and still further increasing its temperature until finally dangerous temperatures are reached in the cable insulation. Experience in the system of the Edison Electric Illuminating Company of Boston indicates that the average duct temperature should not exceed 50 degrees centigrade in order that this unstable condition may not be encountered. For other localities this limit will be higher or lower depending upon the composition of the soil and average annual rainfall.

It is a comparatively simple matter to compute ratings for cables such that the limiting duct temperature may be 50 degrees centigrade. These ratings are tabulated in table VI. The proper rating to assign a cable under a given duct concentration should be chosen from either table V or VI, whichever gives the lower value. The larger cables are limited more often by maximum duct temperatures than by maximum copper temperatures, especially in summer when the base temperature is high, or where the load factor is high as indicated by the ratings for steady load which generally are limited by duct temperatures.

Ratings for ambient temperatures other than those given in tables V and VI may be obtained by multiplying the summer rating (ambient 20 degrees centigrade) by the factor η, obtained by substituting in the following formula:

$$\eta = \sqrt{\frac{(90 - E)(1 - \gamma) - A}{(90 - E)(1 - \gamma) - 20}} \qquad (4)$$

This correction factor obviously will apply to both the steady and cyclic ratings.

Partially Loaded Conduits

Ratings given in tables V and VI were calculated for fully loaded conduits. Very little error will be involved if these ratings are assumed to apply to the number of cables in a conduit disregarding the number of empty ducts. For instance, the 9 duct rating will apply to a 12 duct conduit with 9 fully loaded cables. This neglects the effect of ventilation in the empty ducts and the effect of the larger cross section of conduit for heat flow and storage, all of which will increase slightly the rating of the cable.

It is believed that little ventilation is obtained from empty ducts at the usual manhole spacings of 100 feet or more, the ducts usually being horizontal and not forming convection currents to any appreci-

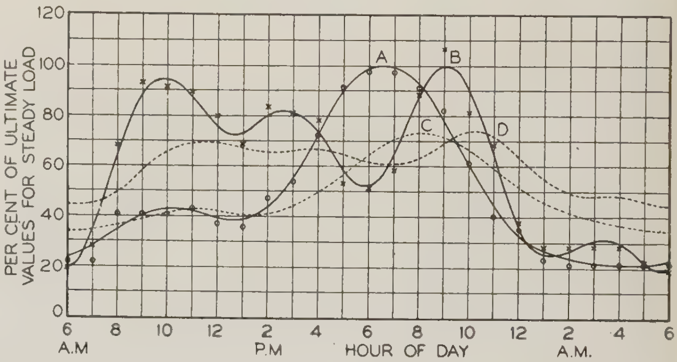


Fig. 1. Load cycles in terms of watts loss, and resultant temperature cycles of 4 400,000-circular mil 25-kv shielded cables in a 4 duct conduit

A—Winter load cycle, equation 1 in text plus dielectric loss

B—Summer load cycle, equation 2 in text plus dielectric loss

C—Temperature cycle of conductors with load cycle of curve A

D—Temperature cycle of conductors with load cycle of curve B

Plotted points (o, x) indicate actual losses

able extent. If it be desired to take the increased heat flow and storage capacity of the partially loaded conduits into account, the following method may be used, illustrated by 9 cables in a 12 duct conduit. The steady state duct constant of a 12 duct conduit is used. Equation 20 of reference 6 then may be modified as follows to obtain the steady state rating:

$$R_0 = \frac{0.00522SG}{n} + \frac{0.00411K}{D} + NH \qquad (5)$$

where H is the duct constant for the 12 duct conduit. The rating for cyclic loading may be calculated by multiplying by a modified value of ϕ, obtained by adding the cyclic temperature rise from duct to conductor, to $^9/_{12}$ of the cyclic temperature rise from earth to duct for 12 ducts. This method is predicated on the assumption that the loaded cables are on the outside of the conduit. If they be on the inside the ratings for the 9 fully loaded ducts should be used.

Soil Conditions

Ratings for almost any combination of partially loaded conduits, cables, or different soil conditions may be obtained by substituting the proper constants in the fundamental equations given in Appendix II.

Average soil conditions were assumed in the calculations in this paper. For dry sandy soils the thermal resistivity given in table III should be increased by as much as 50 per cent, and for wet clay soils decreased 25 per cent. The thermal capacity will not change much with different soil conditions.

Table IV—Ratio of Maximum Temperature Rise for Copper Loss to Rise for Steady Full Load Copper Loss; Shielded Cables

A—Temperature rise, duct to copper

Size of Conductor, Cir Mils	Rated Kv	Winter Load Cycle: Maximum	Winter Load Cycle: Time	Summer Load Cycle: Maximum	Summer Load Cycle: Time
500,000	14	0.952	7:31 p.m.	0.913	9:51 p.m.
500,000	25	0.932	7:35 p.m.	0.892	9:55 p.m.

B—Temperature rise, earth to duct; all ducts loaded with 400,000 circular mil 25-kv cables

Number of Ducts	Winter Load Cycle: Maximum	Winter Load Cycle: Time	Summer Load Cycle: Maximum	Summer Load Cycle: Time
4	0.597	9:12 p.m.	0.678	10:47 p.m.
6	0.590	9:13 p.m.	0.675	10:48 p.m.
9	0.585	9:14 p.m.	0.672	10:49 p.m.
12	0.575	9:15 p.m.	0.670	10:50 p.m.

C—Total per unit temperature rise, earth to conductor

Load Cycle	Size of Conductor, Cir Mils	Rated Kv	Values of ϕ for 4–12 Ducts: 4	6	9	12	Average Value of γ
Winter	500,000	14	0.718	0.693	0.669	0.648	0.025
Winter	500,000	25	0.708	0.686	0.659	0.642	0.041
Summer	500,000	14	0.756	0.743	0.727	0.715	0.039
Summer	500,000	25	0.750	0.737	0.722	0.712	0.067

Table V—Steady State and Cyclic Loading Ratings for 500,000 Circular Mil Shielded Cables

Based upon maximum copper temperature of 90 − E degrees centigrade

No. of Fully Loaded Ducts	Rated Kv	Summer Rating, Amp: Steady	Cyclic	ξ*	Winter Rating, Amp: Steady	Cyclic	ξ*
4	14	331	377	0.908	370	435	0.920
4	25	314	360	0.850	357	420	0.873
6	14	303	349	0.913	339	405	0.925
6	25	287	331	0.857	327	390	0.880
9	14	270	315	0.920	303	365	0.931
9	25	255	297	0.863	291	355	0.886
12	14	245	288	0.924	275	337	0.936
12	25	230	270	0.869	262	321	0.892

* Multiply shielded cable rating by ξ to obtain ratings for belted cables.

Table VI—Steady State and Cyclic Load Ratings for 500,000 Circular Mil Shielded and Belted Cables

Based upon average duct temperature of 50 degrees centigrade (ξ = 1.0)

No. of Fully Loaded Ducts	Rated Kv	Summer Rating, Amp: Steady	Cyclic	Winter Rating, Amp: Steady	Cyclic
4	14	280	368	355	526
4	25	276	360	345	507
6	14	245	322	305	452
6	25	240	312	301	442
9	14	210	276	260	386
9	25	205	267	257	378
12	14	185	243	230	341
12	25	180	234	226	332

Emergency Ratings

If an extremely liberal view be taken of the subject of emergency ratings, cables may be loaded to their normal ratings continuously; and if a transmission line in a group feeding a station be lost, the load may be carried by the remaining line or lines for a short time raising the temperature above the maximum allowable value on the theory that this would not occur often enough to injure the cable. However, if only 2 lines normally feed a station and one of them goes out of service at the time of peak load, this means that the remaining line will be loaded to 4 times its normal copper loss which, even though it occurred for only a short time, would injure the cable. If this liberal view be taken the greatest amount of load that a given cable may be required to carry in an emergency for the required length of time should be subjected to careful study in order that excessive temperatures may not be reached at any time. The excess temperature that a cable may withstand successfully is as yet uncertain, which introduces an added uncertainty to the point at which a cable may be loaded without injury.

The other and extreme conservative view is that a cable never should be allowed to exceed the maximum allowable temperature limit as specified by the A.I.E.E. Standards for cable temperatures, even for a short time. On this basis, if a cable has been loaded to its normal rating for an appreciable length of time

previous to application of any overload, the temperature of the cable necessarily will exceed its maximum allowable value shortly after the overload is applied and an emergency rating has therefore no meaning, being no higher in value than the normal rating.

To apply an emergency rating for any given length of time to a cable on this basis it must have been loaded to a value less than its normal rating for some specified time previous to the application of the overload. In figure 2 the solid and dash-dot lines give the per cent increase in rating for different sizes of cables that may be allowed for 2 hours when the cables previously have been loaded to various percentages of their normal ratings. These curves were obtained by substituting in the following expression, using the intersections of the curves in figure 3 with the ordinate 2.00 hours:

$$\mu = \sqrt{\left(\frac{1}{\beta} - 1\right)\left(\frac{1 - \alpha^2}{1 + \gamma}\right)} \times 100\% \qquad (6)$$

The curves in figure 3 were computed by making use of equation 11 in Appendix II.

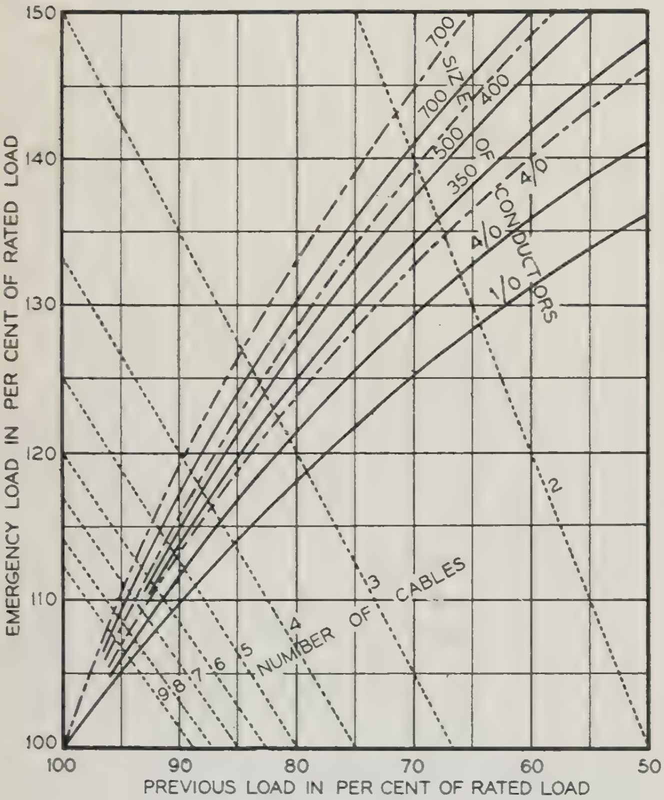


Fig. 2. Emergency overload ratings for various sizes of shielded 3-conductor cables installed in 9 duct conduit

Solid curves for 15 kv cables and dot-dash curves for 25 kv cables give allowable 2-hour emergency loads when cables previously have been loaded to various percentages of their normal ratings. Dotted curves give loads applied when one cable of a group goes out of service. Intersections of dotted curves with solid curves and dot-dash curves give maximum emergency ratings, reading on vertical scale, and maximum normal loads reading on horizontal scale that may be carried in order that emergency ratings will not be exceeded when one cable of a group goes out of service. Sizes of conductors are indicated in B. & S. gauge or thousands of circular mils

Since uncertainties are involved in the calculation of allowable overloads if cyclic loading be assumed, such as time of application of overload during the load cycle, it is believed that the suddenly applied steady load basis is the best way of attacking the problem. The results at least will approximate the condition where the overload is applied at or shortly after the peak of the load cycle, which will be the condition giving the highest copper temperature.

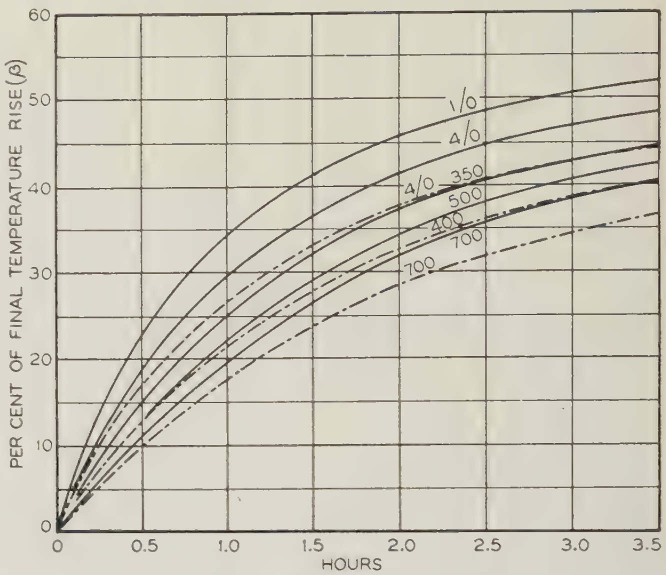


Fig. 3. Temperature rise curves with steady load suddenly applied to shielded 3-conductor cables of various sizes installed in a fully loaded 9 duct conduit

——— 15 kv cables —·—- 25 kv cables
Sizes of conductors are indicated in B. & S. gauge or thousands of circular mils

The load a cable has been carrying previous to the application of an overload determines the amount and duration of this overload. Most transmission systems are provided with spare capacity to take care of outages. Normally, therefore, the cables are operating at a load somewhat less than their ratings, the maximum percentage of full load to which they normally are loaded depending primarily upon the number of cables in the group. Usually a cable may be repaired within 24 hours of the time of breakdown so the load need be carried by the remaining cables through only one daily peak. When an outage occurs on one of the group the remaining cables, therefore, may be loaded to a certain percentage higher than their normal rating for a short time, say 2 or 3 hours.

The dotted lines of figure 2 give the percentages of normal rating that are applied to remaining cables of a group when one is out of service, for different initial loads and original numbers of cables in the group. The intersections of the dotted lines with the other curves in figure 2 give the maximum safe emergency rating that may be applied to the remaining cables in the group, reading on the vertical scale. The maximum percentage of rated load that may be allowed to be carried by the group under normal operating conditions also is given by these intersections,

reading on the horizontal scale. If these percentages of rated load be not exceeded during normal conditions, assuredly the maximum allowable temperature of the cables never will be exceeded with one cable of a group out of service.

There is, of course, some difference of opinion as to whether the liberal or the conservative view should be taken in loading cables in emergencies. Many believe that the A.I.E.E. temperature limit is conservative enough that it may be exceeded quite materially for a short time without injury to the cable. The author and other engineers of the company with which he is affiliated have taken the conservative view in assigning ratings to cables for foreseen emergencies. In unforeseen emergencies there would be no hesitation to exceed these ratings if it were necessary in order to maintain service.

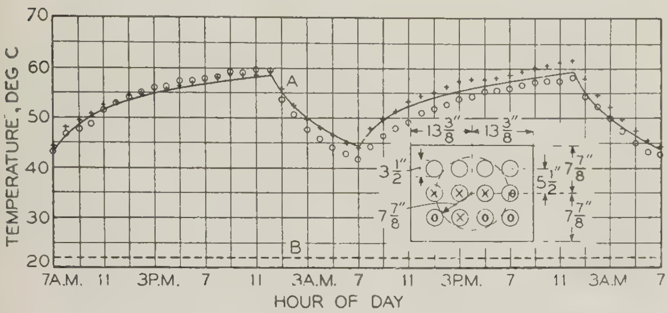


Fig. 4. Results of duct temperature tests on station tie lines

A—Calculated temperature rise
B—Temperature at a distance from duct structure beyond which load cycle had no appreciable effect on temperature cycle
Test points designated + and o taken in different manholes. Insert shows configuration of a 12 duct conduit with inside radius of equivalent cylindrical structure; x = ducts occupied by cables in test; o = ducts occupied by other cables. (See text for description of test)

Temperature Tests

Figure 4 shows the results of a test made in February 1932 on 4 generating station tie lines over which it was possible to maintain a constant load for any length of time desired. The theoretical curve, computed by the same method as was used to compute the curves in figure 3, may be seen to check reasonably well with the test points. Variations of these test points from a smooth curve are caused by other cables in the same duct bank carrying appreciable load only during the evening hours.

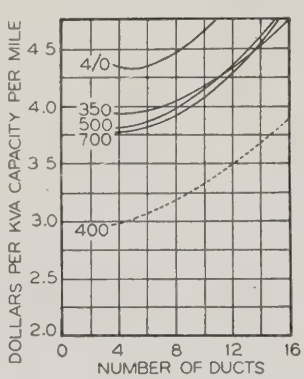
The high average temperature shown by this test was caused by the drying out of soil mentioned in a previous section of this paper, the cables having been loaded continuously at a load equal to the maximum of the test for several months previous. It has been found necessary to reduce the continuous ratings of the cables in these tie lines to a value considerably less than that calculated by the A.I.E.E. copper temperature limit, to prevent overheating during the summer months.

Economical Sizes for Duct Banks

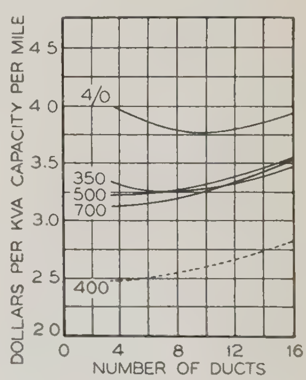
It may be noticed that the ratings of cables decrease quite rapidly as the number of fully loaded ducts increases. This leads to the conclusion that there is an economic limit beyond which the number of fully loaded ducts in a conduit should not be increased. This limit will be determined by 3 factors: the ratings of various sizes of cables in different numbers of fully loaded ducts; cost of conduit and cable; capitalized losses at the various ratings. The charts in figure 5 show cost of transmission per kilovolt-ampere per mile corrected for losses for different numbers of fully loaded ducts and various sizes of cables. These curves are based upon average conduit and cable costs on the Boston system and will vary in different localities. However, large variations in individual costs will not effect the relative values shown by the curves, and hence they will be representative for all systems. These curves do not include the cost of terminal apparatus, which will decrease the slopes of the curves but will not affect appreciably their minimum points. For the average lengths of transmission cable the cost of the terminal apparatus is a small part of the total cost, so the curves of figure 5 will not be greatly affected by it.

It can be seen from figure 5*B* that for transmission cables of the large sizes, the number concentrated in one conduit should be limited to not more than 6 and in some instances 4, using the winter cyclic rating as a basis. The winter load generally is so much higher than the summer load that it may be used as the limiting factor in determining ratings.

For distribution cables at lower voltages with lower costs per circuit foot, a similar calculation will show the lowest cost in 8 to 12 duct conduits. Sometimes congestion may dictate a 16 duct conduit, especially close to substations where end ventilation will compensate for the greater congestion. On long distances, however, it seems that efforts should be made toward keeping the concentration of ducts in any one conduit down to 12 or less, if possible, to obtain the maximum return from the investment.



A—Cost based upon summer cyclic rating



B—Cost based upon winter cyclic rating

Fig. 5. Cost of transmission for various load concentrations, 3-conductor shielded cables, ducts fully loaded

———15 kv cables ----25 kv cables
Sizes of conductors are indicated in B. & S. gauge or thousands of circular mils

Appendix I—List of Symbols

A = ambient temperature
C = specific heat
c = subscript referring to conductor
D = outside diameter of sheath
d = thickness of sheath
E = voltage to neutral
E' = line voltage
e = subscript referring to conduit and earth
G = geometric factor[6]
H = thermal resistance of conduit and earth (duct constant)
i = subscript referring to insulation
j = operator indicating 90 degree phase shift
K = sheath surface resistivity
N = number of loaded cables in conduit
n = number of conductors in cable
Q_0 = copper loss per unit length of conductor
Q_s = vector sheath loss per unit length of cable
Q_2 = vector heat flow at outside surface of sheath per unit length of cable
Q_2' = heat flow at outside surface of sheath at time t after application of a constant load
q = $\sqrt{\omega C \rho S}$
R_0 = total thermal resistivity, conductor to earth base
R_1 = inside radius of equivalent cylindrical duct structure
R_2 = outside radius of equivalent cylindrical duct structure
r_1 = radius of conductor
r_2 = outside radius of insulation
S = heat resistivity
s = subscript referring to sheath
T_0 = temperature of conductor at time t after application of a constant load
T_1 = vector temperature of conductors (same as inside surface of insulation)
T_2 = vector temperature of duct air
t = time in hours
α = ratio of initial load to rated load
β = fraction of final temperature rise at any time t after a constant load is applied to a cable carrying zero load
γ = dielectric loss expressed as a ratio of maximum load copper loss
δ = vector thermal impedance to heat flow[1]
η = factor for ambient temperature
θ = temperature rise of conductor at no load but with voltage applied (temperature of dielectric loss)
λ = ratio of rating for cyclic loading to constant load rating
μ = short time load expressed as a fraction of rated load
ξ = correction factor for belted cables
ρ = specific gravity
σ, τ, ψ = heat flow constants[1]
ϕ = maximum temperature rise expressed as a ratio of constant load rise
ω = angular velocity of heat flow vectors (0.262 for 24 hour period)

Appendix II—Heat Flow Equations

$$\delta = \frac{1}{2\pi} \times \left\{ \frac{1}{\dfrac{1}{S_i}\left[\dfrac{\tau}{\sigma} q_i r_1 - \dfrac{2\pi K}{S_i\sigma^2\{\pi D(1 + j\omega d C_s \rho_s K) - KQ_s\} + 2\pi K\psi q_i r_2}\right] + j\omega C_c \rho_c r_1^2} \right\} \quad \text{(single conductor)} \qquad (7A)$$

$$\delta = \frac{1}{2\pi} \times \left\{ \frac{1}{\dfrac{1}{S_i}\left[\dfrac{\tau}{\sigma} q_i r_1 - \dfrac{6\pi K}{S_i\sigma^2\{\pi D(1 + j\omega d C_s \rho_s K) - KQ_s\} + 6\pi K\psi q_i r_2}\right] + j\omega C_c \rho_c r_1^2} \right\} \quad \text{(3 conductor)} \qquad (7B)$$

$$T_1 = \delta Q_0 \qquad (8)$$

$$Q_2 = \frac{S_i\sigma}{2K}\left\{\left[\frac{2\tau}{S_i\sigma} q_i r_1 + j\omega C_c \rho_c r_1^2\right]\pi\delta - Q_0\right\} \text{(single conductor)} \qquad (9A)$$

$$Q_2 = \frac{S_i\sigma}{6K}\left\{\left[\frac{2\tau}{S_i\sigma} q_i r_1 + j\omega C_c \rho_c r_1^2\right]\pi\delta - Q_0\right\} \text{(3 conductor)} \qquad (9B)$$

$$\frac{T_2}{Q_2} = \frac{S_e\sigma_e}{2\pi q_e R_1 \tau_e} \qquad (10)$$

$$T_0 = \frac{2Q_0}{\pi}\int_0^\infty \delta_r \frac{1}{\omega} \sin \omega t \, d\omega, \text{ where } \delta_r = \text{real part of } \delta \qquad (11)$$

$$\frac{Q_2'}{Q_0} = \frac{2}{\pi}\int_0^\infty Q_{2r} \frac{1}{\omega} \sin \omega t \, d\omega, \text{ where } Q_{2r} = \text{real part of } Q_2 \qquad (12)$$

$$H = \frac{S_e}{2\pi} \ln \frac{R_2}{R_1} \qquad (13)$$

Equation 7 is an explicit function for the thermal impedance of the cable from conductor to air at sheath surface.

Equation 8 gives the temperature of the conductor in terms of the heat flow at the conductor.

Equation 9 is explicit function for the heat flow at the outside surface of the sheath in terms of the copper and sheath losses.

Equation 10 is an expression for the thermal impedance of an equivalent cylindrical duct structure representing the actual duct structure.

Equation 11 gives the temperature rise above ambient at any given time after a constant load is applied to a cable.

Equation 12 gives the heat flow at the surface of the cable sheath at any given time after a constant load is applied to the cable.

Equation 13 is the expression for thermal resistance of an equivalent cylindrical duct structure.

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Power Company Service to Arc Furnaces

Before an electric arc furnace, which is a rapidly fluctuating load during part of its operating cycle, is connected to an electric power distribution system, the electrical characteristics of the furnace must be known in order that the service may be provided in such a way that the resulting voltage fluctuations on the system will not exceed allowable limits. These and related factors are discussed in this paper which outlines a general procedure that may be followed in planning electric service for such loads.

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ASSOCIATE A.I.E.E.

The Detroit (Mich.) Edison Company

WHEN a power company considers serving an electric arc furnace, it is confronted with the problem of balancing 2 conflicting but essential interests. The good revenue producing possibilities make the load highly desirable, but to offset this there is always the chance that other customers may be disturbed by voltage fluctuations set up by the furnace. The expected power consumption and high load factor, however, usually warrant any special study and planning that may be necessary to provide service satisfactory to the prospective furnace user and at the same time protect other customers from any disturbing effects of the load on the distribution system. This paper outlines a general procedure that may be followed in planning service for such loads.

There are 4 important considerations which will be discussed in the following order:

1. A full and accurate knowledge of the electrical characteristics of the load is of prime importance.
2. Allowable limits of voltage fluctuation or "flicker" on the distribution system must be established.
3. Methods of furnace operation that either reduce the magnitude of current fluctuations or affect the time of occurrence may be considered.
4. Selection of the most economical method of serving a known fluctuating load without exceeding established flicker limits is the final objective.

A paper recommended for publication by the A.I.E.E. committees on (1) power transmission and distribution, and (2) electrochemistry and electrometallurgy, and scheduled for discussion at the A.I.E.E. winter convention, New York, N. Y., Jan. 28–31, 1936. Manuscript submitted Sept. 9, 1935; released for publication Sept. 18, 1935.

Electrical Characteristics of Arc Furnaces

It is highly essential that the operating characteristics of a prospective furnace be defined accurately. In the past, lack of definite knowledge of just how disturbing a prospective load might prove to other customers sometimes has caused power companies to be somewhat hesitant about providing service without either imposing prohibitive regulations or spending more than a reasonable amount to "play safe."

By systematic testing and study of existing installations, it has been possible to accumulate operating data on furnaces of various types and sizes which can be referred to whenever new jobs are proposed. Different installations of the same sized furnace may have slightly dissimilar electrical characteristics because of variations in the secondary bus layout or perhaps different voltage or reactance taps; but, in general, estimates for new jobs based upon similar existing installations prove to be reasonably accurate. The electrical characteristics, which must be known before intelligent service planning can be attempted, include the average load and power factor, the current fluctuations during different parts of the heating cycle, and the so-called starting currents.

There are 2 general types of arc furnaces: the single-phase indirect-arc type in which the arc plays between 2 electrodes and does not come into direct contact with the metal of the charge; and the 3-phase direct-arc type in which the arc plays from the 3 phase electrodes to the metal charge.

Single phase furnaces range in size from small laboratory models to a maximum size requiring transformer capacity of about 900 kva. Any of these single phase furnaces on regular production schedules operates at an average power factor ranging from 0.7 to 0.8, depending to some extent upon the arrangement and length of secondary bus between the transformer and the furnace. Oscillograph records show that the power factor at any given instant may be as low as 0.3 or as high as 0.9, but that the average power factor as determined by watt-hour and reactive kilovolt-ampere-hour meters generally will fall in the range between 0.7 and 0.8.

The 3 phase furnaces encountered on the system of The Detroit Edison Company range in size from about 200 kva to a maximum of 10,000 kva. The larger 3 phase furnaces operate at power factors ranging between 0.8 and 0.9, which are slightly higher than those of the smaller furnaces. One 10,000 kva furnace has had an average power factor of 0.86, a 2,100 kva furnace 0.84, and a 750 kva furnace 0.82.

The electrical load of a furnace can be regulated by the settings of the electrodes, but the stability of the arc and corresponding stability of load are dependent to a large extent upon the reactance in the circuit, the character of the furnace charge, and the temperature within the shell. During the preheating period of a cold furnace the arc is unsteady with resultant current fluctuations, but once the furnace becomes heated the arc is relatively stable and the current fluctuations are of a minor nature. Even when a furnace is operated only 8 hours per day,

the period of unstable arc operation is relatively small compared with the period of stable operation when the load is under direct control of the operator or under automatic electrode control. The upper limit of average load for any appreciable period is imposed by transformer heating and therefore under normal operating conditions does not exceed the rating of the transformer.

The largest current swing usually occurs when the arc is started, especially in the smaller furnaces. At times the arc may be started without causing a current surge of any appreciable magnitude, but usually there is an accompanying maximum current swing just as the arc is struck.

In general, the single phase and 3 phase furnaces have somewhat the same characteristics; however, the current fluctuations of the larger 3 phase furnaces do not represent as large a proportion of the normal full load current as is true for the smaller furnaces. For instance, both types of furnaces up to a size of about 2,000 kva may have a starting current of 2 to 3 times the rating of the furnace transformer; from 2,000 up to 10,000 kva the proportion decreases, and for the 10,000 kva furnace the starting current fluctuation in any single step does not exceed approximately full load at about 0.6 power factor. It is true that current drawn by a 10,000 kva furnace sometimes exceeds its normal full load value and occasionally may approach 3 times normal, but it does not reach such a value in a single swing or fluctuation. Overcurrent line relays set to pick up and start timing out at 3 times full load current give satisfactory protection and usually do not operate except in the event of actual trouble.

Experience does not indicate any particular advantage from the standpoint of voltage fluctuations of the small 3 phase furnace over the single phase furnaces in the sizes so far encountered on the Detroit Edison system. While a *continuous* single phase load of a given value will cause twice the voltage regulation of a *continuous balanced* 3 phase load of the

Table I—Current Fluctuations in Single Phase Furnaces

Furnace Rating			
Nominal Rating, Kw	Approximate Rating of Transformer, Kva	Metal Capacity With Average Scrap Iron	Maximum Swing, Kva
125	190	350 pounds	400 at 0.4 power factor
400	525	1/2 ton	1,000 at 0.4 power factor
600	875	1 1/2 tons	1,500 at 0.4 power factor

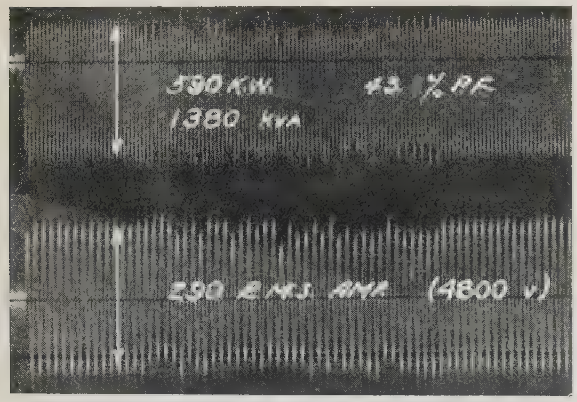


Fig. 1 (left). Star of operating cycle

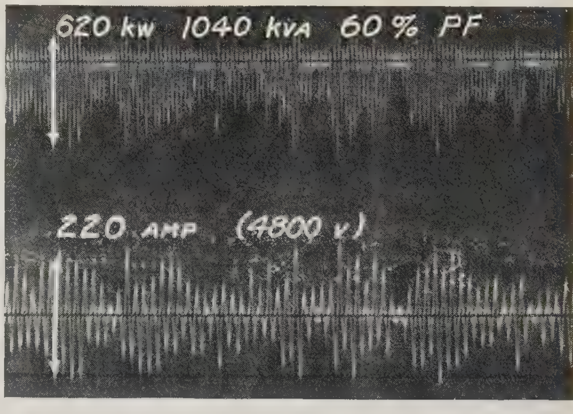


Fig. 2 (right). Preheating period

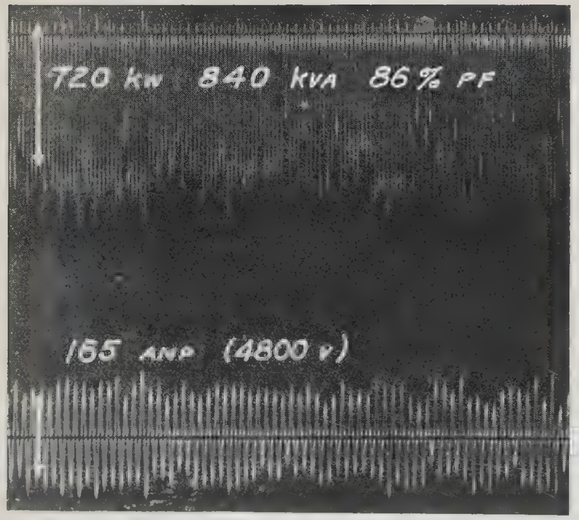


Fig. 3 (left). Melting down of cold charge

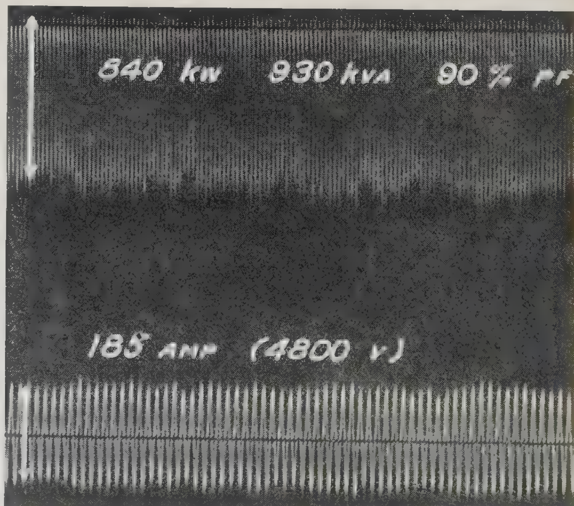
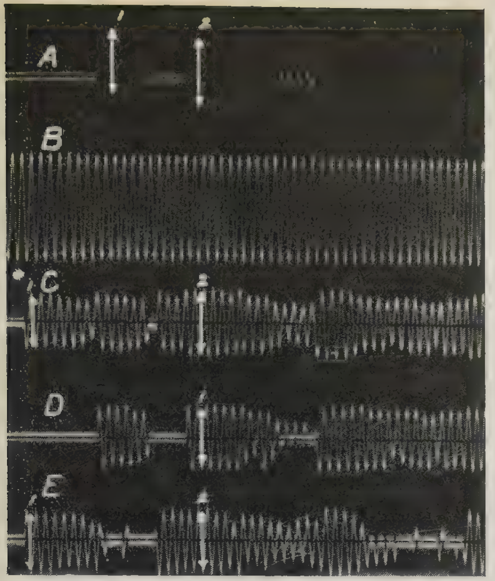


Fig. 4 (right). Normal operation with molten charge

Figs. 1 to 4. Line current and power input to a 600-kw 1 1/2-ton single-phase arc furnace for producing high grade cast iron

Upper traces show power input and power factor of load. Lower traces show line current in the 4,800 volt circuit supplying furnace transformer



1—3,200 kw, 5,900 kva, 0.54 power factor
2—3,700 kw, 6,200 kva, 0.60 power factor

23,500 volts

1—260 amperes
2—330 amperes

1—300 amperes

1—260 amperes
2—260 amperes

Fig. 5. Start of operating cycle

23,500 volts

1—330 amperes

2—230 amperes

3—280 amperes

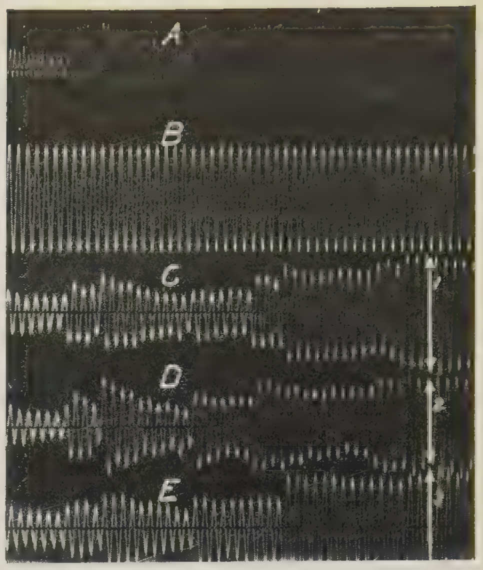


Fig. 6. Early part of melt with electrodes boring down through charge

Figs. 5 to 7. Line current and voltage and power input to a 10,000-kva 45-ton 3-phase arc furnace for producing high grade steel

A—Single phase power; current in phase *Y* and voltage across phase *YZ*
B—Voltage across phase *YZ*
C, *D*, and *E*—Line currents in phases *Z*, *Y*, and *X*, respectively

4,500 kw, 5,300 kva, 0.83 power factor

23,500 volts

205 amperes

205 amperes

190 amperes

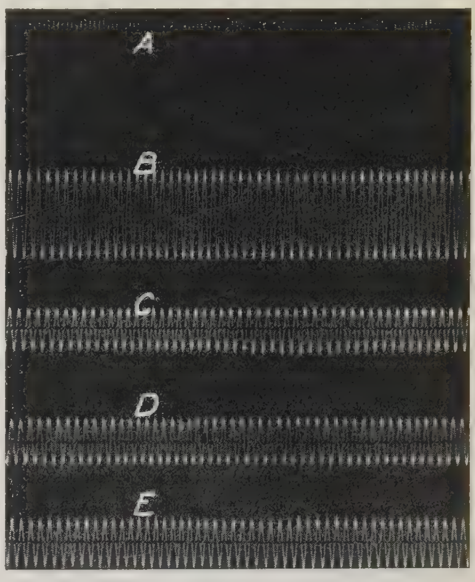


Fig. 7. Normal operation after pool of molten metal has been established at bottom of furnace

same value, this simple relationship does not apply to electric furnaces as built and used. In the 3 phase furnace, each phase arcs independently to the charge and, especially during the preheating periods, the phases act as 3 separate highly variable unbalanced single phase loads. It must be assumed that the so-called starting current may exist as a single phase current at any instant the same as in a single phase furnace.

Tables I and II show the maximum current swings that can be expected on several typical single phase and 3 phase furnaces. These values are the starting currents and are of a rapidly recurring nature only during the first 15 or 20 minutes of the preheating period.

Figures 1 to 4 show oscillograms of line current and power input to a 600-kw $1^1/_2$-ton single-phase furnace used for producing high grade cast iron in a jobbing foundry, for various periods during its operating cycle. It is interesting to note the progressive improvement in current stability from the highly erratic arc performance during preheating, through a much more stable period while melting down a cold charge of scrap, and finally to a period of comparatively smooth performance after the charge becomes molten.

Table II—Current Fluctuations in 3 Phase Furnaces

Furnace Rating: Rating of Transformer, Kva	Furnace Rating: Metal Capacity With Average Scrap Steel, Tons	Maximum Swing, Kva: 3 Phase	Maximum Swing, Kva: Single Phase
750	1	2,000	1,200 at 0.4 power factor
2,500	3	4,500	2,500 at 0.45 power factor
5,000	8	7,500	4,500 at 0.5 power factor
10,000	45	11,500	6,500 at 0.6 power factor

Figures 5 to 7 show oscillograms of line current and voltage and power input to a 10,000-kva 45-ton 3-phase furnace used for producing high grade electric steel, for various periods during its operating cycle. Figure 5 shows how the load at times is entirely a single phase one, especially at the start of a heat. The smooth performance indicated in figure 7 is typical of the major part of the cycle of operation.

Effect on Distribution System

Fluctuations in current drawn by an arc furnace through the impedance of that part of the power company's system which is ahead of the point of service, cause voltage variations or "flickers" which may

prove objectionable and disturbing on lighting circuits. After establishing the maximum value of current fluctuations that may be set up by the furnace under consideration, it is a relatively simple matter to calculate the voltage flickers that will result at different points on the distribution system.

The amount of fluctuation that can be allowed depends to a large extent on its frequency of occurrence. Tests show that flickers, or voltage changes caused by instantaneous changes in current, begin to be noticeable, under ordinary reading conditions with 40 and 60 watt lamps, at about 2 volts on 115 volt circuits. Therefore, this must be the maximum allowable on lighting circuits for rapidly recurring flickers; but for less and less frequent flickers the limit can be raised on the basis that, although the more pronounced flicker possibly may be noticed, it is not objectionable because of its infrequent occurrence.

Based upon this reasoning, tentative flicker limits, covering both magnitude and frequency of occurrence, have been adopted for use on the distribution system of The Detroit Edison Company. From a study of these limits, shown in table III, it may be noted that the magnitude of flicker allowable depends not only upon how often the flicker occurs, but also upon the class of customers affected. "Power line" customers buy essentially power, and consequently a greater flicker is tolerated than on distribution circuits where the predominating load is lighting.

Table III—Voltage Flicker Limits for Distribution System of The Detroit Edison Company

Location	Volts on 115 Volt Base		
	Infrequent*	Frequent†	Extremely Frequent††
A On a substation bus feeding only power lines	6	4	3
B On a power line primary circuit the entire output of which is not taken by one customer and read at the customer's premises	8	6	4
C On a power line the entire output of which is taken by one customer and read at the customer's premises		No limit	
D On a substation bus feeding distribution circuits (lighting) and on distribution circuit primary circuits	6	3	2

* Infrequent flicker shall include fluctuations occurring 6 times or less in 24 hours but not more than once between 6:00 p.m. and midnight. This provision is intended to cover apparatus such as motor generators, fans, pumps, etc., which normally runs continuously throughout the working day

† Frequent flicker shall include fluctuations occurring not oftener than 3 times per hour except that between 6:00 p.m. and midnight they shall not occur more than once per hour. This provision is intended to cover apparatus such as machine tools, electric furnaces, etc., which is started and stopped periodically throughout the working day

†† Extremely frequent flicker shall include all fluctuations occurring more frequently than the above. This provision is intended to cover such apparatus as flashing signs, welders, gravel pit hoists, and certain electric furnaces, which are stopped and started frequently and repeatedly or rapidly loaded and unloaded during normal use

Applying these flicker limits to the specific problem of furnace operation, it is reasonable to allow a 2 volt limit for lighting customers affected during the preheating period of the furnace when the flickers are of a rapidly recurring nature. For less frequent fluctuations, as caused by starting the furnace during normal operation, a more liberal limit in the order of 3 to 6 volts is generally permissible. Local conditions and experiences of the local power company are a big factor in the determination of allowable flicker.

Solving the Problem of Voltage Flicker

The logical procedure to follow in determining just how a given furnace shall be served with electric power is to select the simplest method of service and calculate the amount of disturbance to other customers. If the flicker be excessive, the furnace supply must be tapped onto the distribution system nearer the source, that is, where the impedance is lower and the flicker correspondingly less. Using this method a point finally is reached where the flicker is within allowable limits. The cost of providing service then can be estimated, and by balancing this cost against the expected revenue a decision can be made as to whether or not it is commercially and economically feasible to provide the service.

To be balanced against system changes, there are possible changes in the furnace installation itself that may be made to limit the current fluctuations. For single phase furnaces, this has been accomplished by utilizing reactor starting; although The Detroit Edison Company has had no experience with similar reactor starting for small 3 phase furnaces, it is logical to expect that like results can be accomplished. Large 3 phase furnaces generally have

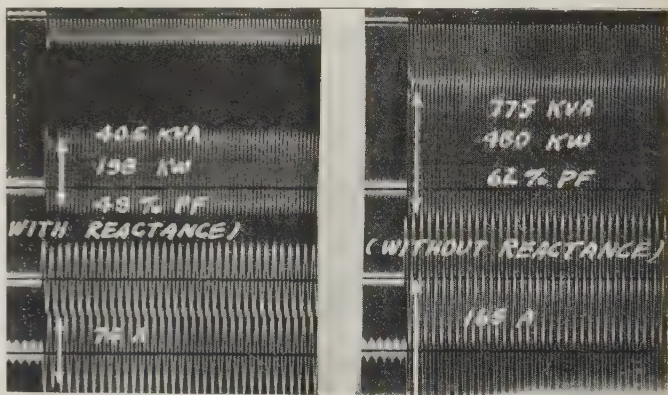


Fig. 8. Start of 300-kw single-phase arc furnace with and without starting reactance in primary circuit

Lower traces show line current in 4,800 volt circuit supplying furnace transformer. Second trace from top shows the power input and power factor of load

such inherently high impedance in the secondary bus and electrodes that any further temporary addition of reactance for starting is of doubtful value. However, similar results are accomplished by operating at reduced voltages during the start of a melting down period as the electrodes bore down through the charge. When a molten pool has been established at the bottom of the furnace and the electrodes are no longer in direct contact with the steel, the operating voltage is raised about 25 per cent higher for faster melting.

The usual single phase furnace transformer is equipped with a combination of taps to provide reactance ranging from 10 to 40 per cent. By adding another block of 40 or 50 per cent reactance and an additional primary bushing, a starting reactor is provided which, with the necessary short-circuiting breaker and relays, adds from 10 to 15 per cent to the installation cost. By having this reactance in the circuit whenever the arc is struck, the current surge is reduced to the following values:

Nominal Rating of Furnace, Kw	Maximum Swings, Kva	
	Without Reactor Starting	With Reactor Starting
400	1,000	550
600	1,500	900

By using reactor starting it is sometimes possible economically to serve a furnace for which otherwise the cost of service would be prohibitive, thereby obviating the necessity for using a smaller furnace. By leaving the reactor in the circuit during the preheating period of the furnace cycle, the worst of the fluctuations can be reduced to permissible values. Tests show that the heating time remains about the same with or without the reactor in the circuit. Figure 8 shows oscillograms of starts of a 300 kw furnace equipped with a starting reactor which reduces the current surge to about 50 per cent of the value obtained on a start with the reactor short-circuited.

As the current fluctuations during the preheating period usually cause the most serious disturbance, an alternative operating procedure is the simple expedient of governing the furnace operating schedule in such a way that the furnace always is preheated during a period of the day when there are few lighting customers to be affected, such as between midnight and 6 o'clock in the morning. During the daylight hours, flicker will be confined to the occasional starting of the arc; and since the lighting load is small during the daylight hours, this might not prove objectionable. On such a basis the last heat would have to be started before the late afternoon lighting load comes on.

If a schedule such as the foregoing one is not practicable and service must be available at any time, then the supply must be taken from the power system at a point far enough back toward the substation or generating source so that the resulting flicker at that point is not objectionable. For instance, a given circuit feeding both power and lighting customers might be of such length and impedance as to be unsatisfactory for serving a prospective furnace load at the end of the line. In this event, it would be necessary to build a separate line for the furnace back to a point on the existing line where the impedance is small enough not to cause an objectionable flicker. Sometimes it will be necessary to carry the furnace service all the way back to a substation, that is, to provide a separate line for the furnace. If the voltage disturbance on the substation bus be in excess of accepted standards, a separate transformer and bus section must be set aside to serve the furnace.

In an industrial center of any magnitude there is often sufficient furnace and welder load to make it economical to set aside one or more bus sections per

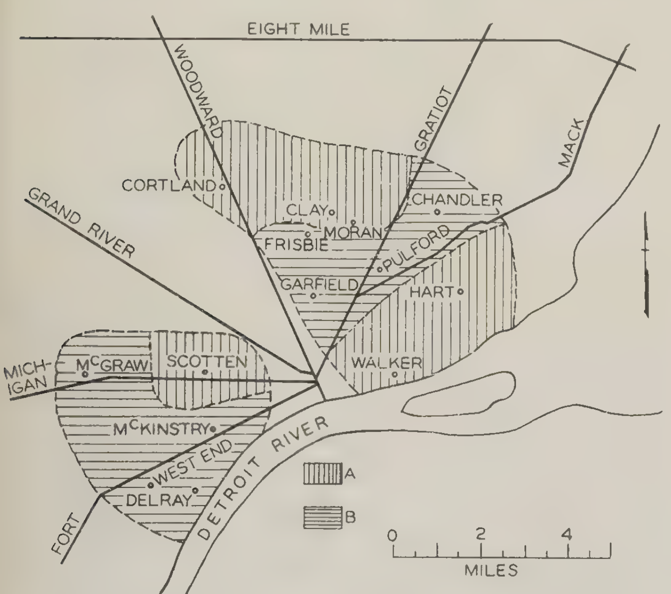


Fig. 9. Map of industrial area of Detroit, showing substations capable of serving highly fluctuating loads

A—Areas in which fluctuating loads are served from special busses

B—Areas in which special busses can be established

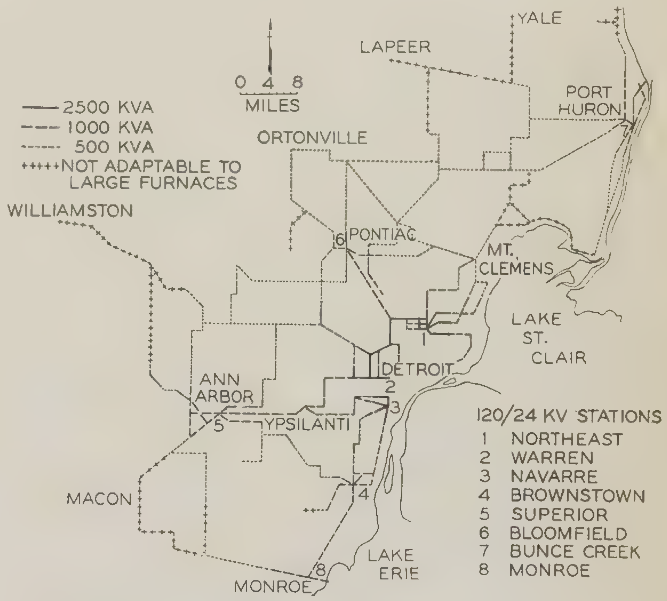


Fig. 10. Map of 24 kv transmission lines in suburban territory of The Detroit Edison Company showing maximum single phase load swings allowable without exceeding a 2 volt flicker limit on the distribution circuit

substation for these so-called "bad" loads. By grouping such loads and keeping all lighting and "good" power loads off this bus, the need for keeping the voltage fluctuations within standard limits often can be avoided, and there still will be a reasonable load for the bus section set aside.

There has been sufficient fluctuating load, including both furnaces and welders, in the central industrial area of Detroit to warrant setting aside special busses in 6 out of 14 substations serving this area. The total peak load on these special busses amounts to more than 30,000 kva, including about 2,000 kva of railway motor generator sets. The railway equipment not only helps to load the bus sections, but also aids in smoothing out the voltage flickers. Figure 9 shows the industrial load area in which it is possible to provide special bus service and also the area already served by the existing special busses in 6 of the substations.

In suburban areas the substations usually are equipped with small transformers, and it is generally useless to consider serving a furnace of any appreciable size from existing substations. Here, a simple tap to a near-by transmission line is often the solution, and the only voltage disturbances imposed on other customers are those set up on the 24 kv system. Figure 10 shows a map of the suburban area served by The Detroit Edison Company, showing the 24 kv transmission lines and the allowable load fluctuations for any point on the system without exceeding a 2 volt flicker. Where a customer is situated near a transmission line, reasonable service can be provided by extending the 24 kv line direct to the customer's property and establishing a small single-phase or 3-phase step-down transformer bank for serving only the furnace.

There are bound to be a few installations that simply cannot be served economically even when using reactor starting or when operating on a restricted schedule; for these few exceptional cases, the only remaining solution is service through a motor-generator provided by the customer. The motor-generator need not be an expensive flywheel set, such as is used in steel mill service, but can be an ordinary induction or synchronous set. A synchronous set has additional value to the customer in improving the power factor of the plant, and sometimes the investment can be justified partially on that basis. It is true that a flywheel set or even an induction set without a flywheel is more effective than the synchronous set in shielding the distribution system from current surges, but usually the benefit from the synchronous set is found to be sufficient. The current surges at the furnace are at low power factor and do not represent enough variation in energy to justify a flywheel.

In conclusion, there are 2 points that should be stressed: First, electric furnaces are large power users and therefore the power company can well afford to spend some money to take them on; and second, once the electrical characteristics and operating schedules are accurately defined and well understood, excessive margins of safety can be eliminated, thereby keeping the cost of providing adequate service at a minimum.

Flashing of Railway Motors Caused by Brush Jumping

Theories and concepts developed in connection with circuit rupturing by switches and circuit breakers are used to analyze the factors involved in the flashing of railway motors. The investigation is limited to cases where flashing is caused by brush jumping, which occurs principally with railway motors. Voltage conditions as applying at the commutator circumference and as affecting the sustenance of an arc are studied for varying designs of main poles with different ratios of armature ampere turns to field ampere turns. The effect of load current and speed is subsequently taken into account. Actual test results are compared with the theories outlined. Finally, the modifying effect of certain secondary phenomena is discussed.

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FLASHING at the brushes of commutating machines has been a common difficulty for many years, particularly in railway motors. However, an analysis of the factors entering into flashing has proved very difficult and little progress has been made. A great deal of experimental work has been carried on, but usually nothing but a very rough qualitative interpretation of the results obtained was possible on account of the great many variables entering into the problem. In the absence of analytical methods, designers have used certain empirical rules relating to such matters, as, for instance, the ratio of armature ampere turns to field ampere turns, but naturally the results obtained with such rules have been irregular and unreliable in practice. With the extensive studies made during recent years in connection with arc rupturing of switches and circuit breakers and certain concepts that have been established through these studies, the possibility of analyzing flashing characteristics of commutating machinery successfully has been improved; an attempt will be made in this paper

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to indicate a method of attacking this problem. In order to make progress in clarifying this rather involved problem, certain simplified assumptions will first be made and subsequently other factors will be taken into account. Furthermore, the present study will be limited to certain kinds of flashing and certain types of motors.

Referring particularly to railway motors, it is known that there are 2 major causes of flashing. One of these is the interruption of the power supply caused by trolley bouncing, third-rail gaps, etc., and subsequent sudden re-establishment of the power supply. A second cause is the jumping of brushes. While it is difficult to state which of the 2 major causes mentioned is encountered most in practice, it is believed that the jumping of brushes has frequently been the reason for flashing troubles; at any rate, there is much evidence that it has been a rather common cause of flashing in railway work. This statement is supported by the fact that in a great many cases where flashing occurred, increasing the brush pressure has yielded marked improvement, either by preventing the brush from leaving the commutator or by reducing the time interval during which the brush is not in contact with the commutator. Flashing caused by brush jumping therefore will be made the subject of the present analysis.

Causes of Brush Jumping and Flashover

There seems to be some doubt as to the cause of brush jumping, but it is quite possible that the mechanical vibrations of the motor are responsible in some instances. In other cases roughness of the commutator may be the principal cause of brush jumping. While variations in mechanical conditions may affect the nature and the duration of the brush jumping and consequently the results obtained, the electrical theories subsequently given will apply regardless of what the cause of the brush jumping may be.

Reference may be made to figure 1. Figure 1*A* shows the brush at the instant it has left the commutator, and the arc *S* which has formed between the brush and the commutator segment *a*. Experimental investigations have indicated that there is little tendency for this arc to cause flashing if the brush is negative. On the other hand, the very worst flashing conditions are obtained when a positive brush is lifted and the direction of rotation is such that segment *a* approaches directly the other brushholder of a 4 pole motor having only 2 brushholders. This is to be expected, because with a positive brush a hot-cathode spot forms on the segment *a* and therefore the arc terminal tends to remain at the hot-cathode spot of segment *a* while it moves along as indicated in figure 1*B*. Figure 1*C* shows the condition just after the brush has returned to the commutator. The arc *S*, however, may still continue to exist if the voltage between the brush b_1 and segment *a* is of sufficient magnitude to sustain it with the condition of ionization prevailing at the time. It therefore seems expedient to consider more in detail both the ionization condition and the voltage conditions as they may exist at the time when the brush recontacts the commutator and during the time following.

Ionization Affected by Armature Current

The arc and the amount of ionization present in it, when the brush leaves the commutator, are primarily governed by the armature current flowing at the time. This current, previous to the formation of the arc, depends upon the load, the speed, and the control position. Equilibrium in the motor circuit for such given condition exists if the impressed electromotive force minus the counter-electromotive force equals the ohmic drop. When the brush lifts, the circuit resistance is increased by that of the arc, and since the arc resistance may be an appreciable percentage of the previously existing ohmic resistance, there is a tendency for the current to decrease. With decreasing current, the flux of a series motor tends to decrease. This tendency of the flux to decrease with the current is counteracted by the damping effects in the motor frame, which prevent any sudden and appreciable field changes. Nevertheless, certain portions of the main flux do not go through the solid frame and therefore a certain decrease in flux will take place with decreasing current. As soon as the flux decreases, however, the counter-electromotive force decreases as well, which, in turn, increases the voltage available for the ohmic drop. Therefore, a new condition of equilibrium will be established with a current but slightly decreased. Even this somewhat smaller current may not be reached in the short time intervals under consideration, because any decrease in current will be temporarily opposed by the self-inductance of the armature and field coils, as well as by any other inductance in the system, as may be present with third rail installations and the like. Especially with heavy currents flowing, it is therefore not to be expected that during the very short interval which the brush is away from the commutator any appreciable decrease in current will take place, which also means that no appreciable decrease in ionization will occur before the brush returns to the commutator. With smaller currents, causing less ionization in the first place, a decrease of the current and the consequent deionization of the arc during the time the brush is raised, may be of practical importance. If this deionization is sufficient to cause an interruption of the current before the brush returns to the commuta-

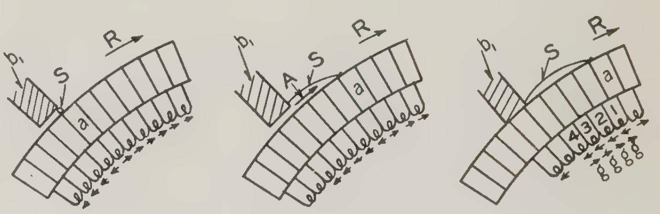


Fig. 1. Formation of arc on d-c machines during and after jumping of brushes

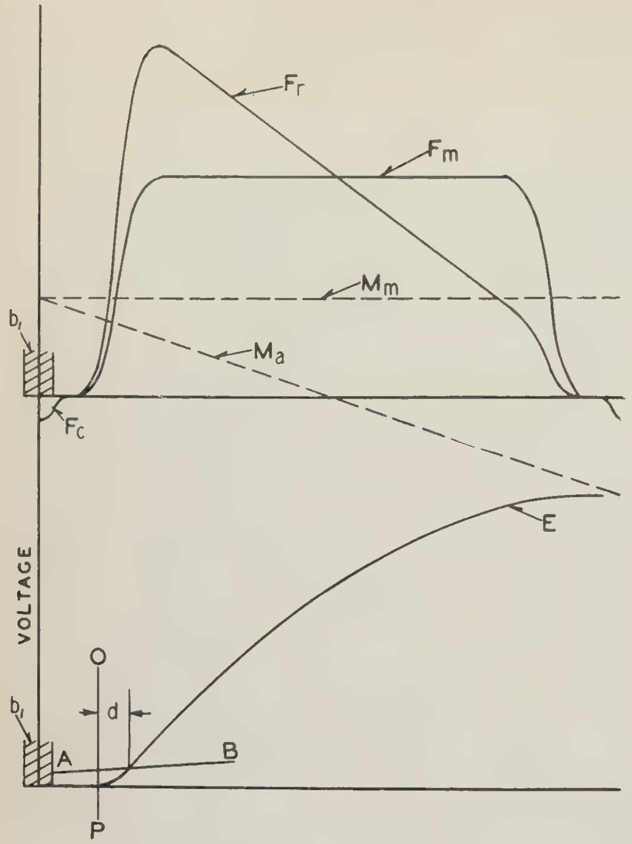
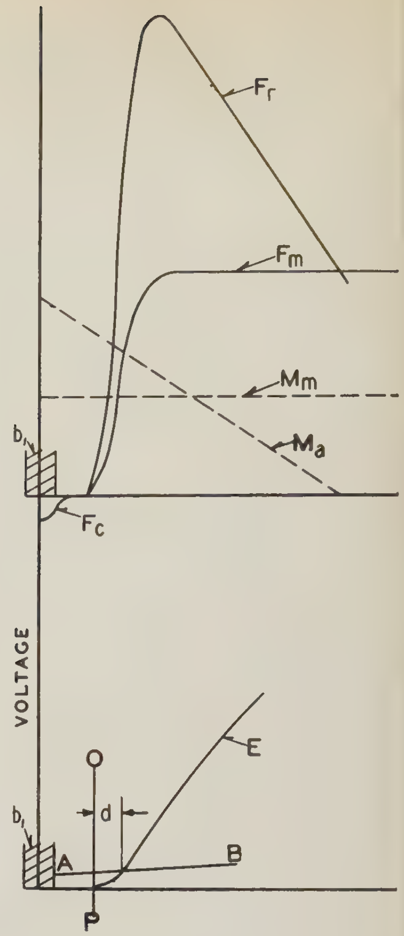


Fig. 2 (left). D-c machine with narrow poles and low armature ampere turns

Fig. 3 (right). D-c machine with narrow poles and high armature ampere turns

Upper curves—Field form

Lower curves—Voltage between brush and various points on commutator



tor, there is no longer any problem in connection with flashing, and therefore no need to consider such cases further.

Although the phenomena described as applying during the time that the brush is away from the commutator will have some influence upon the flash characteristics, especially in the case of smaller currents, the designer has very little latitude in appreciably affecting such conditions. As already pointed out, the load current is definitely fixed by the work to be performed at the time, and with the conventional motor design the other factors discussed, such as self-inductance, resistance of the circuit, and the damping effect of the frame, cannot be altered economically over wide ranges for a given rating. However, these factors vary appreciably with the rating of the motor. The inductance and resistance, for instance, decrease as a rule with the rating of the motor, so that a motor with a large rating may have less tendency to sustain certain small currents while the brush is away from the commutator than a motor of a lower rating would have with the same current (in amperes).

Effect of Induced Voltage

If conditions are such that the arc going from the brush to the hot-cathode spot of segment *a* is still in existence when the brush returns to the commutator, with a definite ionization present the subsequent behavior of this arc will be governed largely by the voltage induced at that time and during subsequent periods between the brush and segment *a*. In order to study these voltage conditions, a field form F_m, without armature reaction, a commutating field form F_c and a resultant field form F_r have been shown in figure 2*A* with the field and armature magnetomotive forces indicated by the lines M_m and M_a, respectively. The voltage between any point on the circumference of the commutator and the edge of the brush b_1 can be determined by graphical integration of the resultant field F_r up to such a point. Thus the voltage curve E in figure 2*B* has been obtained. It may be assumed that, at the time the hot-cathode spot has reached the line OP, the brush has returned to the commutator. It is known that the voltage necessary to sustain an arc is from 25 to 80 volts, depending upon the current in the arc and its length. The line AB has been drawn to indicate the voltage necessary to sustain the arc for different positions of the segment with the hot-cathode spot. This line shows a slight increase with the length of the arc. It will be seen that over the distance d the induced voltage available to sustain the arc is lower than the line AB, which means that a certain amount of deionization of the arc takes place while the cathode spot travels over this distance d. If for the present a definite speed is assumed, the distance d also represents the time elapsed. If the distance and time indicated by d is sufficient for bringing about complete deionization, there will of course be no flashing. However, even though deionization occurs very rapidly, it nevertheless requires a finite time, and moreover there are other factors such as the self-induction of the armature coils, 1, 2, 3, and 4, which

tend to maintain the arc, as will be discussed more in detail later on. If as a consequence of this the deionization is only partly accomplished the arc may be fully re-established after the time d when the voltage again increases appreciably. The chances that this will occur and bring about motor flashing depend essentially upon the ionization remaining at the end of the time d and the subsequent rate of increase in the voltage, as indicated by the steepness of the voltage curve E.

Effect of Ratio of Ampere Turns

In order to study the influence of the ratio of armature ampere turns to field ampere turns upon these conditions, figure 3 has been worked out by using the same main field F_m but doubling the armature ampere turns M_a. Figure 3B shows the resultant voltage curve E. It will be seen that the distance and time d are decreased to about 80 per cent of the value shown in figure 2B and that the rate of rise of the voltage is also greater. Nevertheless, it is quite conceivable that with the assumptions made, the time d is still sufficient for complete deionization and if such is the case the greater rate of rise of voltage is of no practical importance; in other words, it is quite conceivable that under the conditions assumed, a very appreciable increase in the ratio of ampere turns will have no effect of practical importance upon the flashing characteristics.

An entirely different effect may be obtained, however, in a case as shown in figures 4 and 5. In this case a wider pole arc has been assumed, which results in an appreciably reduced distance and time d as shown in figure 4B, with the smaller ratio of armature ampere turns to field ampere turns. Nevertheless, it is conceivable that the deionization obtained during the time d and the subsequent small rate of rise of voltage may be insufficient to sustain the arc. However, if in a case like this the ratio of armature ampere turns to field ampere turns is materially increased, as shown in figure 5, conditions change materially. It will be noted in figure 5B that the time d available for deionization is zero because, at the time the brush returns to the commutator, the voltage available for the arc is equal to that necessary to sustain it and, furthermore, from then on there is a high rate of rise in the voltage. Flashing is almost certain to occur, with the assumptions made.

Figures 6 and 7 assume conditions somewhat similar to those in figures 2 and 3 except that the rise in the field curve starts nearer to the brush and is more gradual, as might be obtained with changed pole tips or by skewing the slots with poles similar to those in figures 2 and 3. Inspection of these figures indicates that conditions are less favorable than with figures 2 and 3. The deionizing time has been reduced to 80 per cent with the small ratio of ampere turns (compare figures 2 and 6), and to 66 per cent with the large ratio of ampere turns (compare figures 3 and 7).

Figures 8 and 9 show conditions that may be obtained with a pole having a tapered air gap. Here the conditions are more favorable than in figures 2 and 3. The deionizing time has been increased to 155 per cent with the small ratio of ampere turns

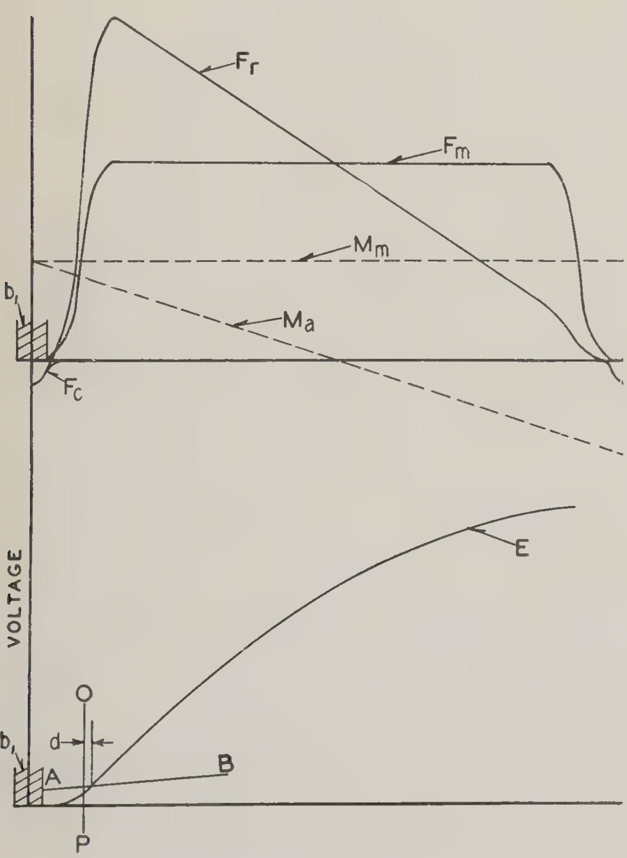


Fig. 4 (left). D-c machine with wide poles and low armature ampere turns

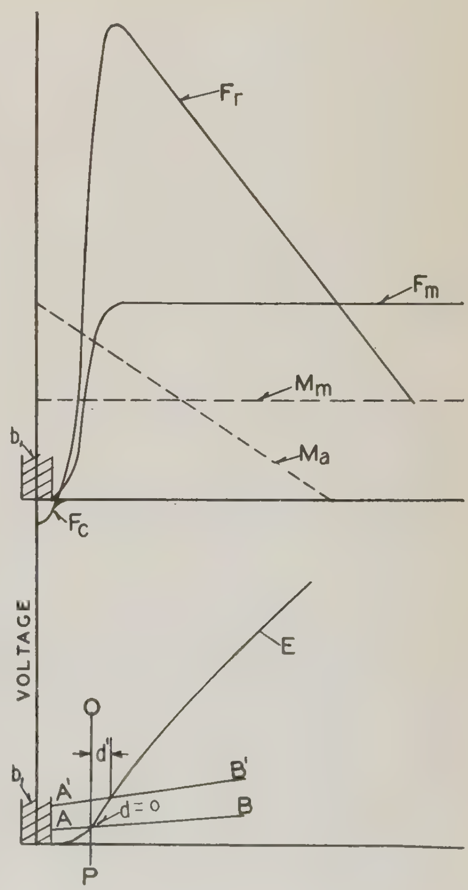


Fig. 5 (right). D-c machine with wide poles and high armature ampere turns

Upper curves—Field form

Lower curves—Voltage between brush and various points on commutator

(compare figures 2 and 8), and to 125 per cent with the large ratio of ampere turns (compare figures 3 and 9). As a matter of fact, the deionizing time with the large ratio of ampere turns of figure 9 is the same as that with the smaller ratio used for figure 2. The rate of rise in voltage also has been reduced with figures 8 and 9 as compared with figures 2 and 3, respectively.

Effect of Current and Speed

Comparisons so far have been made under the assumption of a definite current and a corresponding voltage line *AB;* it has also been assumed that the speed is the same in all cases. If the same study were made for a smaller current but still with the same speed, it would be necessary to simply enter a line *A'B'*, as indicated in figure 5*B*. This line is higher in accordance with the well-known arc characteristics indicated in figure 10. It will be noted that with the smaller current and everything else being assumed the same, there is a deionizing time *d'* so that flashing might be avoided, while with the larger current and a corresponding line *AB* it would most likely occur, because the deionizing time *d* is zero.

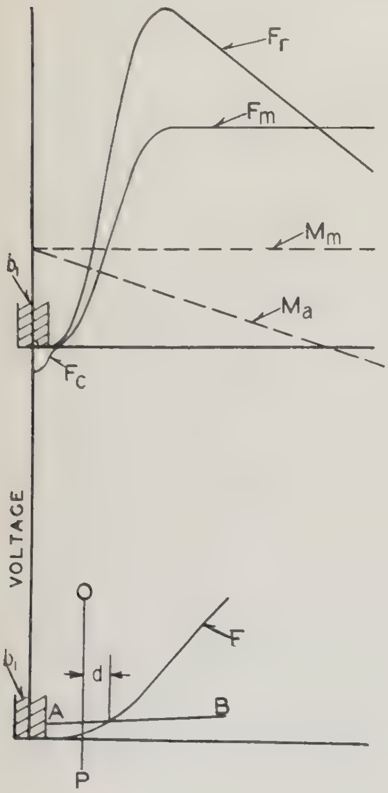


Fig. 6 (left). D-c machine with narrow skewed poles and low armature ampere turns

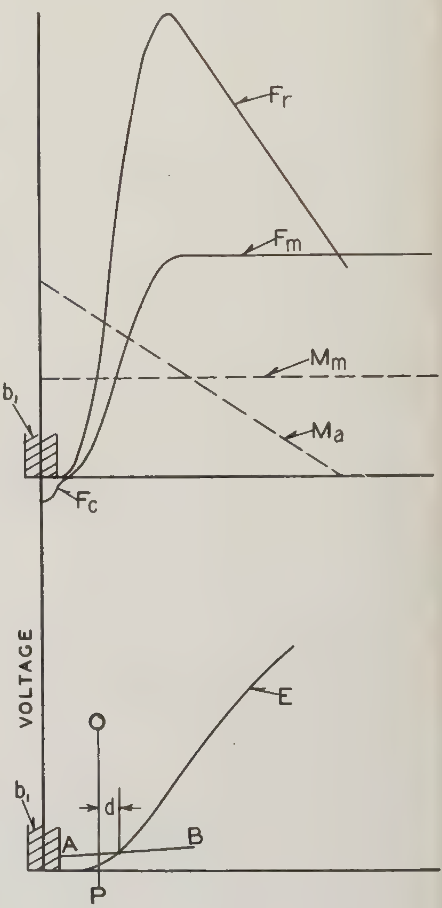


Fig. 7 (right). D-c machine with narrow skewed poles and high armature ampere turns

Upper curves—Field form

Lower curves—Voltage between brush and various points on commutator

Although the current flowing at the time of the brush jumping may have an appreciable influence upon flashing, the speed of the motor is of even greater importance. So far it has been assumed that the speed is the same for all cases considered, making the distance *d* equivalent to the time. However, if in figure 7 for instance, a 50 per cent increase in speed is assumed with other conditions unaltered, the voltage curve *E*, reproduced in figure 11 on a larger scale from figure 7, changes to curve *E'* for the higher speed if time is used as a basis. In this case it is further assumed that the time during which the brush is off the commutator has not been changed. This, in turn, means that the hot-cathode spot has traveled 50 per cent farther, when the brush returns to the commutator and has reached a position for which the voltage between the spot and the brush is *e*. It will be seen that while there is a chance of avoiding flashing with the voltage curve *E* and the deionization time *d*, such chance is practically zero with the 50 per cent increase in speed, as indicated by the fact that the voltage curve *E'* is equal to the values needed for sustaining the arc at the time the brush returns to the commutator, and the fact that the subsequent rate of rise in voltage is increased.

In actual practice, especially over the operating range of a series motor, all of the conditions previously discussed, such as load current, speed, and the effect of the armature ampere turns upon the resultant field form undergo certain changes which, as indicated in the beginning of the paper, make it somewhat difficult to interpret the facts as they actually are. However, this will be greatly facilitated by the foregoing discussion of the individual factors.

Study of a Previous Experiment

Figure 12 reproduces some experimental data obtained some time ago by W. A. Brecht, but not pre-

viously published, on a small railway motor, by means of a mechanical arrangement permitting adjustment of the time during which the brush is off the commutator. The curves were obtained by gradually increasing the time for any load condition and recording the time at which flashover occurred.

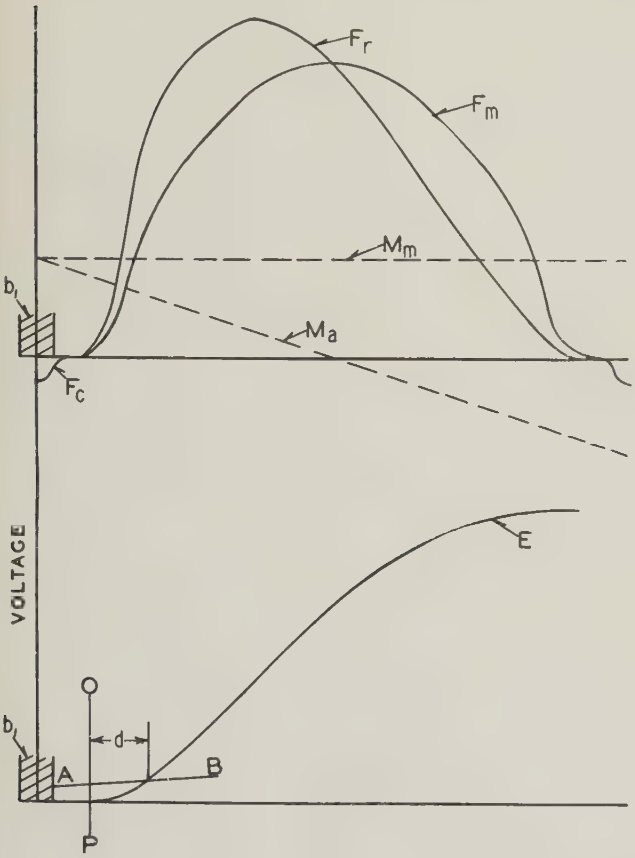
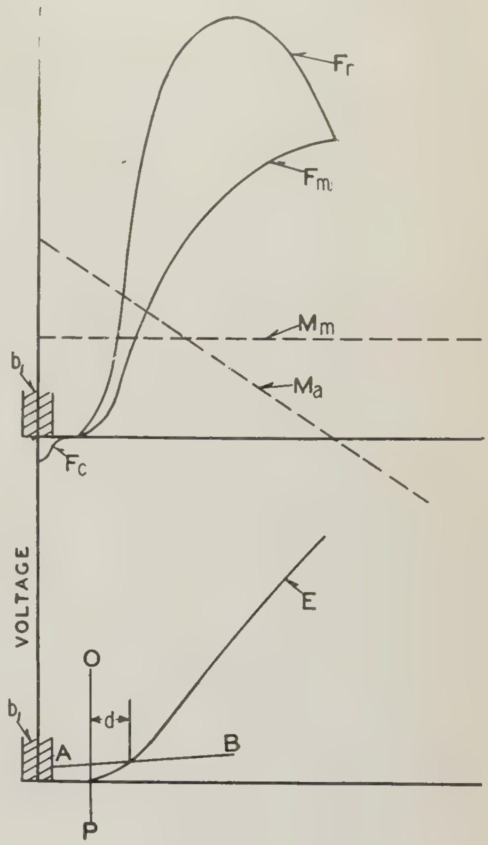


Fig. 8 (left). D-c machine with poles having tapered gap, and low armature ampere turns

Fig. 9 (right). D-c machine with poles having tapered gap, and high armature ampere turns

Upper curves—Field form

Lower curves—Voltage between brush and various points on commutator



Curve A applies to a motor operated as a series motor; curve B, to operation with a constant field excitation of 60 amperes; and curve C to operation with constant field excitation of 30 amperes. It will be noted that curve B, with a constant field excitation of 60 amperes, is rather flat for the range over which tests were made. In this case, increased armature current results in an increased ratio of armature ampere turns to field ampere turns which in turn tends to crowd the flux into the interpolar zone and to thus make field conditions less favorable. This effect is, however, not appreciable with this particular motor on account of the saturation of the pole tips. Whatever unfavorable effect along this line existed was probably compensated for by the favorable effect of a slight decrease in speed. With such conditions, the flatness of the curve seems to indicate that, starting with a certain value, increasing the current above such value does not affect the arc conditions materially, which is quite in accordance with the tendency of normal arc characteristic curves to flatten out above certain current values, such as shown in figure 10.

Curve C, as will be seen, is appreciably below curve B, the major part of this difference undoubtedly being due to the very much higher speed obtained with the 30 ampere field excitation. In addition to this, however, conditions are unfavorably affected because the field distortion caused by the heavy armature currents crowds the field into the interpolar zone and thus makes field conditions less favorable. It will also be noted that curve C has a drooping characteristic even for the higher currents, which is probably due to the fact that the armature ampere turns can have greater influence with the lower saturation obtained in the pole tips with the 30 ampere field. The rather steep part of the curve for the lower current values is, of course, caused even more by the decreased distortion with decreased ratio of armature ampere turns to field ampere turns, since there is no saturation at all in the pole tips with the lower load currents. In addition, it is likely that the reduced ionization and the consequent higher voltage necessary for maintaining the arc is partly responsible for the steepness of the lower branch of curve C. The effect of speed is, of course, of somewhat secondary importance with the reasonably constant speed with the constant excitation of the fields.

Coming now to curve A applying to the series operated motor, it is evident that with 60 ampere loads, meaning in this case also 60 ampere excitation, this curve should closely approach curve B; similarly curve A should closely approach curve C at the 30 ampere load, which will be noted to be the case. Below 30 amperes curve A should be expected to be, and it is, somewhat less favorable than curve C on account of the greater field distortion, as well as

on account of the higher speed obtaining with series motor operation over this range. It will be seen that, considering the nature of these tests and certain limitations in their accuracy, the practical results obtained agree very closely with the theory previously outlined, and the latter, in turn, explains rather satisfactorily the peculiar curve obtained under series motor operation, which indicates most unfavorable conditions at normal rating in this particular motor of older design.

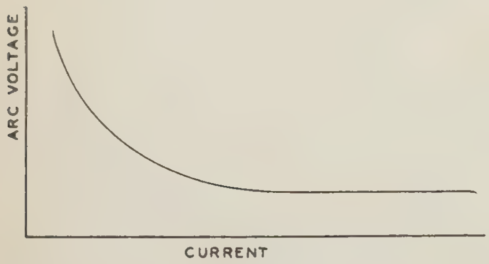


Fig. 10. Typical arc characteristic

Other Phenomena—Voltages Induced During Current Reversal

So far, quite a number of phenomena of secondary importance have been neglected in order to simplify the consideration of the basic conditions. The importance of these phenomena varies, but they may under certain conditions be of practical consequence and it therefore seems necessary to outline some of them briefly.

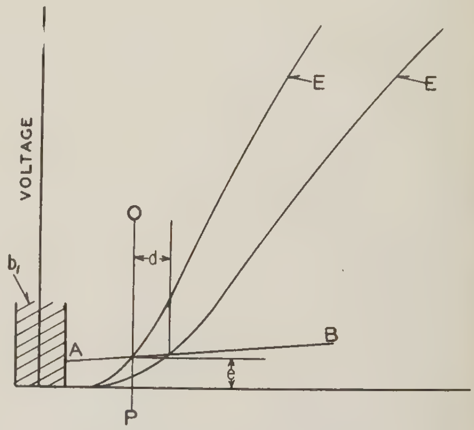
By reference to figure 1*B*, it will be seen that, while the current is conducted into the commutator by the arc, the current direction is, as indicated by the arrows below the coils, with the direction of rotation as indicated by the arrow *R*. When the brush returns to the commutator (figure 1*C*) the current will tend to flow in a normal direction in coils 1, 2, 3, and 4. This means that the current in these coils has to be reversed, as indicated by the opposite arrows *g* near these coils. This reversal will induce a self-inductive voltage in these coils, which adds to the voltage induced by rotation, as covered in the previous discussion. In other words, this self-inductive voltage will assist in maintaining the arc. The amplitude of this voltage increases with the size of the current which has to be reversed. This, in turn, means that the effect is less favorable with the heavier loads. Furthermore, the greater the number of turns in the armature coils, the larger the self-inductance will be, which is merely another reason for keeping the number of conductors in the armature as small as other practical considerations permit. While the effect of this self-inductive voltage is probably not negligible in a great many cases, the variations possible within practical design ranges are somewhat limited, so that, in practice, the effect of such voltage probably does not vary over wide ranges for a given rating.

As already indicated, the armature current tends to decrease while the brush is off the commutator, and consequently the field excitation will also decrease and the main field will tend to diminish. After the brush returns to the commutator, opposite changes will take place. Any such change will also induce certain voltages in the armature coils 1, 2, 3, and 4 with which the main flux interlinks; however, as already pointed out, such changes will be small on account of the various reasons previously indicated. Therefore, the influence of these voltages may not be appreciable.

Effect of Slots and Teeth, Interpole Flux, Armature Flux, and Hot-Cathode Spots

The figures showing field forms and voltages give rather smooth curves because the effects of slots and teeth and the commutating oscillations which may take place under the brushes have been neglected.

Fig. 11. Rate of rise of voltage between brush and cathode spot on commutator segment for 2 different motor speeds



In actual practice these influences usually result in rather irregular curves with many higher harmonics. The higher harmonics in the voltage curve, if they cause sudden rises at the critical moment of arc rupturing, may be instrumental in sustaining the arc; on the other hand, such harmonics, if they happen to result in sudden decreases at the critical moment, may assist in extinguishing the arc.

In the figures previously given, the interpole flux F_c has been assumed to be the same in all cases. Since this interpole flux is negative in the interpolar zone, it assists in reducing the voltage over the critical range. It is therefore evident that by increasing the interpole flux, conditions may be favorably affected. A wide interpole extending beyond the interpolar zone, for instance, may be of practical value. Unfortunately, however, commutation considerations definitely limit practical possibilities along this line.

In developing the field forms of the figures given, the armature flux has been assumed to be constant. As a matter of fact, there is some shift in the distribution of the armature ampere turns while the current flows as shown in figure 1*B*. This is equivalent to a shift in the brush, which, in turn, has the same effect as a chorded winding. The result is a decrease of the magnetomotive force in the commutating zone and this permits the commutating flux to increase and results in somewhat more favor-

able conditions, as pointed out in the previous paragraph.

In the considerations up to this point it has been assumed that there is a single arc, with the hot-cathode spot remaining on segment a. With larger currents, it is not entirely inconceivable that while this arc is lengthening, a new cathode spot will form at the segment near the brush.* If this occurs, it is of course equivalent to the brush temporarily returning to the commutator at that point. Considerations, therefore, are much the same as outlined before, the difference merely being that such a phenomenon will reduce the time during which the original arc carries the entire load current, which in turn favors the elimination of flashing.

Artificial Means of Reducing Flashing

A great many attempts have been made in the past to reduce flashing by artificial means, such as air currents or magnetic-blow effects, but on the whole, such attempts have not been very successful. The reason for this apparently is that during the critical point in the arc formation, the arc is short and the air currents readily obtainable have little effect upon it. As the arc becomes longer the air currents have the effect of changing its direction and possibly blowing it to other parts of the motor, but, in general, little has been accomplished thereby in the way of entirely preventing flashing. The use of magnetic-blow effects for the same purpose seems to have had similar effect. Practical observation seems to indicate that the air currents A shown in figure 1B caused by the rotation of the commutator and being carried along near its surface tend to carry the arc along and favor an arc formation as indicated in figure 1B.

Numerous attempts have been made to influence flashover conditions favorably by mechanical arrangements of parts, and certain minor improvements have been accomplished. Certain tests, for instance, have indicated that with other conditions unchanged a somewhat larger commutator and wider spacing has a noticeable, but not an appreciable, favorable effect. Certain improvements have also been made in mechanical arrangements for diverting the arc and the consequent damage from the most vulnerable parts of the motor. However, as a rule, the possibilities of eliminating flashing by mechanical arrangements seem to be definitely limited by the very much restricted space conditions in railway work.

Effect of High Speeds, and Number of Brushes per Arm

In the previous theoretical study of the voltage conditions, the time during which the brush is off the commutator has been assumed to be a definite value. This, of course, is not the case in actual practice, as it is very probable that this period increases with higher speeds. This, together with the other unfavorable effects of high speeds previously discussed, explains the fact that most flashing occurs with the higher speeds of the motor. The only redeeming fact is that the currents are usually somewhat smaller at the higher speeds, which partly counteracts the unfavorable effects.

In the previous considerations, it has been assumed that there is only one brush per arm. If there are 2 or more brushes side by side, there is, of course, the possibility that one brush will remain on the commutator while the other brush jumps, which in turn should materially reduce flashing. Unfortunately, however, high commutator bars, causing brush jumping, may at times extend over the entire commutator width, and any other mechanical shocks causing brush jumping may act upon both brushes alike. In some such cases no marked difference has been noted in practice between the results obtained with 1 and 2 brushes per arm, but in general it should be expected that 2 brushes per arm will reduce the chances for flashing caused by brush jumping. If 4 brush arms per motor are used and only 1 set of a given polarity leaves the commutator, the current can be transferred to the other brush of the same polarity; this transfer, however, means the overcoming of the self-induction of the one armature coil connected between the segments of the same polarity. Depending upon this self-inductive voltage, the voltage available for the arc between the raised brush and segment a may or may not be sufficient to sustain it. However, even if the arc is sustained until the brush b_1 returns to the commutator, at least some of the current will have been transferred to the other brush of the same polarity. It is therefore to be expected that the presence of the second set of brushes of equal polarity will be favorable to the elimination of flashing conditions as affected by brush jumping, even though it should not be expected to eliminate flashing entirely in all cases. A complete analysis of this more involved case is beyond the scope of this paper.

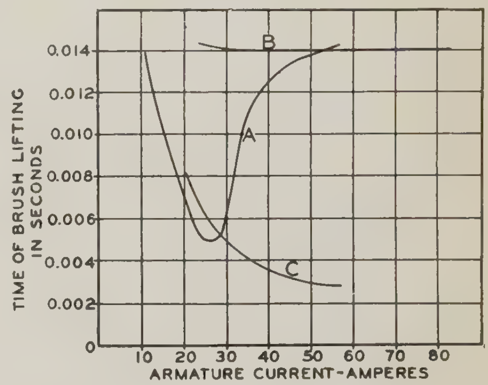


Fig. 12. Curves indicating the time of brush lifting required to cause flashing

A—Field in series with armature

B—Separately excited field, 60 amperes

C—Separately excited field, 30 amperes

Many of the phenomena have by necessity been touched upon only briefly as it is obviously impossible to cover this inexhaustible subject in full in the limited space available here, particularly because of the many different conditions encountered in practice. It is believed, however, that the fundamentals given will assist in clearing this rather intricate problem.

*An analysis of the test results shown in figure 12 seems to indicate that this actually took place with some of the larger current values, but further investigations will be necessary to confirm it.

Applications of a Photoelectric Cell

A photoelectric cell of the dry disk type, consisting of a light-sensitive disk and not requiring any vacuum or liquid was developed a few years ago, and has proved to be a very useful device. Characteristics of one type of dry disk photoelectric cell are described in this paper, and a number of typical applications of the cell are given.

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PHOTOELECTRICITY is not new to science. There are records of the Becquerel effect of 1839, the Fritts dry disk cell of 1855, and the Willoughby Smith discovery of the resistor type of selenium cell in 1873; however, the application of photoelectricity in the commercial world has progressed rather slowly and has taken a definite stride forward only in the last decade. While the actual commercialization has just recently begun, it was not lack of vision on the part of our early engineers which prevented its quicker adoption but rather lack of suitable associated apparatus such as sensitive microammeters and relays, and the fact that the older cells were relatively insensitive.

With the advent of inexpensive electronic amplifiers, the photoelectric tube in combination with an amplifying system, was found to offer a solution to many industrial control problems, but early in the last decade it became evident to research engineers that the photoelectric tube, with its associated amplifying tubes, was not the complete answer to most photometric measurement and control problems.

Accordingly, intensive research work was carried on, first with the wet type of copper oxide cell, then with the same cell in its dry disk form and finally as an outgrowth of these investigations the first commercially practical dry disk type of photoelectric cell, which is described in this paper, was offered to the industrial world late in 1931. Due to its high current output (about twice that of the photoelectric tube), the simple and sturdy construction, and the ease of adapting it to existing measuring and control devices, it has found a ready and fertile field.

The dry disk photoelectric cell consists essentially of a metal disk on the surface of which light-sensitive material is deposited. (See figure 1.) Contact fingers resting against the face and back of the disk collect the current generated, which is then carried directly to the measuring or control instrument. The light-sensitive disk is usually mounted in a bakelite or metal case, the case is not evacuated or filled with gas, as is the photoelectric tube, and no liquid is used. The action of the light striking the sensitive surface is entirely electronic, that is, the radiant energy is directly transformed into electrical energy and there is no physical or chemical change in the sensitized material; thus a practically unlimited life is to be expected.

In the photoelectric tube, or emissive type of photoelectric cell, electrons are ejected into space by the impinging light; but a polarizing potential of a relatively high order, that is, 90 volts or more, is required to collect them. In the dry disk photoelectric cell there is no space between electrodes but instead they are bonded together electrically at the barrier layer thus allowing the direct flow of current without the use of an external aiding voltage.

The total current generated in the cell is a function of the illumination and is proportional to it. This fact has been verified in experimental circuits, from low to the very highest values of illumination. This current divides between an internal conducting path in the cell and the external path, or load, in accordance with the ordinary laws of parallel circuits.

Frequency Response and Internal Resistance of Cell

A consideration of the equivalent cell circuit (see figure 2) which has been proved experimentally, will indicate the reason for some of the unusual and interesting characteristics of this cell.

It is interesting to know that this cell, in common with other types of electronic photoelectric cells, is a

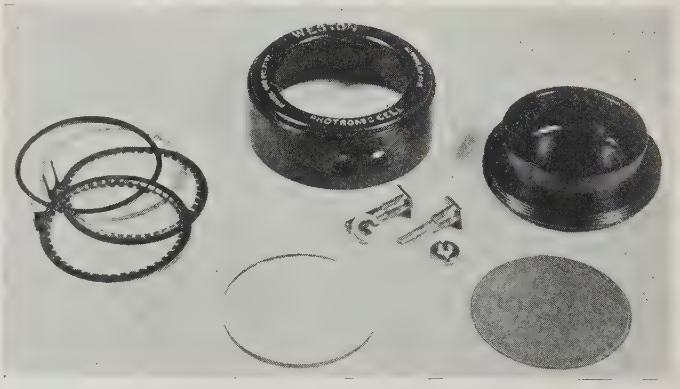

Fig. 1. View of component parts of the dry disk photoelectric cell

rectifier (*R*, figure 2), that is, it offers a resistance to the flow of current in one direction many times that of the opposite direction. This characteristic is often utilized. The capacitance *C* between the conducting surfaces of the cell is of the order of 0.5 microfarad. This causes a drop in output with an increase in

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frequency of the modulated light (see figure 3) and makes the cell unsuited for use at audio frequencies, such as are used in talking pictures. By the use of special filters and coupling transformers, a practically flat response curve may be obtained up to 5,000 cycles but the power level is reduced and additional amplification would be required.

There are many applications, however, where linear response is not required at all frequencies, such as, for instance, in the measurement of projectile velocity. The U.S. War Department has built a device using dry disk photoelectric cells to detect the passage of a rifle bullet at speeds up to 3,000 feet per second with an accuracy of plus or minus one foot per second.

The internal resistance of the cell is relatively high, varying from 6,000 to 7,000 ohms for a dark cell to 1,000 to 2,000 ohms under intense illumination. (See figure 4.) This allows the use of fairly high resistance meters in the external circuit and accounts for the fact that lead resistance is negligible, and, therefore, the cell may be mounted any desired distance from the meter or relay. This characteristic

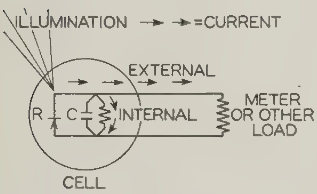


Fig. 2. Equivalent circuit of the dry disk photoelectric cell

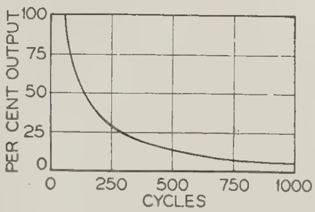


Fig. 3. Frequency response of the cell

was taken advantage of by the engineers for the Holland tunnel under the Hudson River, who mounted a cell in the middle of the tunnel and had it record smoke density at that point, on a recorder in the New York control room, over 3,000 feet distant.

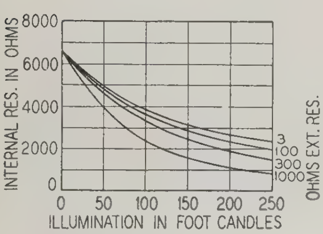


Fig. 4. Internal resistance of cell under varying conditions of illumination and external resistance

Tungsten filament source at 3,000 degrees Kelvin

There are many other installations where leads pass from 300 to 500 feet through water, earth, or air, from the cell to its associated meter or relay.

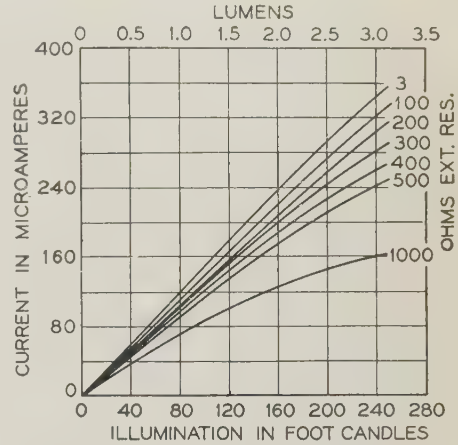
Current Output and Spectral Response

For low external resistances, the current output of the cell varies directly with the illumination. (See figure 5.) This has permitted the making of direct reading foot candle meters having a uniformly divided scale. Such meters are made with ranges from as low as 1 foot candle up to 20,000 foot candles.

For ranges of 50 foot candles and higher one cell supplies sufficient current to operate a rugged microammeter. For lower ranges of illumination 2 or more cells are connected in parallel, and the current increase is approximately in proportion to the number of cells. (See figure 6.) In the instance of meters made for checking very low values of illumination such as for street lighting measurements as many as 16 cells in parallel are sometimes used. This may

Fig. 5. Current output and effect of external resistance

Tungsten filament source at 3,000 degrees Kelvin



sound cumbersome but actually it is not, as the cells all fit nicely into the standard instrument cover. The dry disk photoelectric cell, especially when more than one is mounted in a single housing, is often referred to as a light target, or light collector. In figure 7 a light collector is shown containing 6 cells, another contains 3 cells in a weatherproof case used for mounting at the tops of poles or towers; for instance, on lighted airways the 3-cell light collector is mounted on top of the beacon tower, sometimes 150 feet above the ground, and the relays are mounted at the base of the tower.

The spectral response of the cell is fortunately such that it overlaps that of the human eye in both the violet and red ends of the spectrum. (See figure 8.) Thus by the use of a colored glass filter it is possible to cut out the overlapping response and make the cell practically identical in response with the human eye. As will be seen on the curve, the cell also responds well to invisible radiation and by simply using a quartz window instead of glass it was possible to calibrate meters for use in reading ultra-violet intensities directly. It has also been found practical, when a suitable metal or composition filter is used, to determine X ray intensities.

It may readily be seen, therefore, that where sensitivity to any part of the spectrum from the X ray through the infra-red is required, it is merely necessary to use a filter which will cut off most of the undesired radiations. As an additional example, in the steel industry temperatures in open hearth furnaces are being measured by means of a dry disk photoelectric cell and are being directly recorded. This would be impossible by any ordinary pyrometric method of temperature measurement. Billet temperatures are also being measured by a cell as the billet moves along through the rolls.

The Illumination Meter and the Exposure Meter

The usefulness of the dry disk photoelectric cell has been enhanced by the fact that only extremely simple circuits need be used to obtain the full advantages of direct measurement and control. For the measurement of illumination, color, opacity, etc., it is necessary only to connect the cell directly to the terminals of a suitably calibrated microammeter, or milliammeter.

One of the first measuring instruments to be devised incorporating this cell, was the illumination meter shown in figure 9. Prior to its introduction, only such relatively inadequate comparison methods, as used in the grease-spot photometer, were available in portable instruments for the measurement of illumination, and they were good only in normal white light.

This illumination meter consists of a microammeter, calibrated in foot candles, and connected, by a 5 foot cord, to a light target containing 2 cells connected in parallel.

Another useful application of this cell, for the measurement of light values, is in the photographic exposure meter shown in figure 10. All prior, and commonly used, exposure meters relied on the ability of the human eye to distinguish between various degrees of brightness, since it is the brightness of the object at which the camera is directed that determines the required exposure time. The human eye, however, is quite capable of reading newsprint in moonlight and in sunlight, a range of about a million to one, but is incapable of determining minor variations, over this great range, with sufficient exactitude to suit the relatively narrow requirements of light intensity of the usual photographic emulsions. Having the relation between brightness and the density of exposure resulting, it was a relatively simple matter to make a calculator, or translator, that would interpret brightness measurements in terms of exposure time.

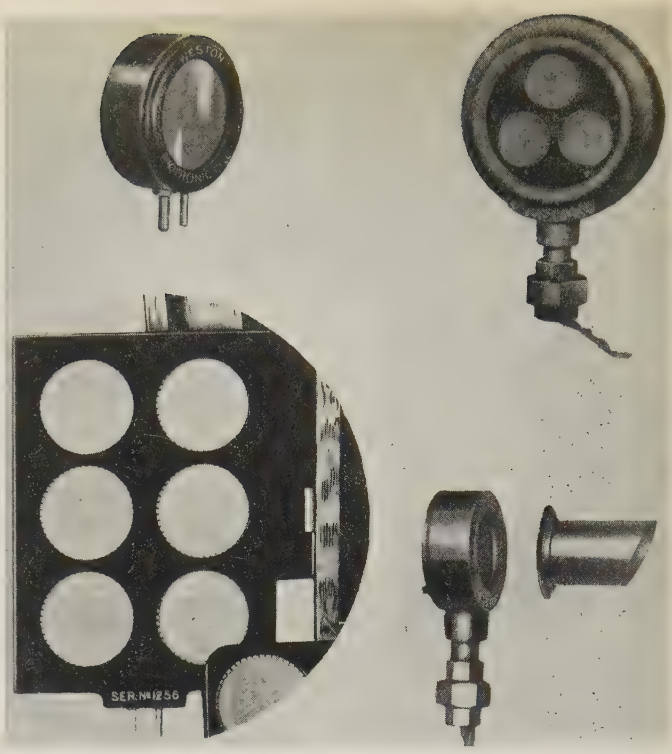


Fig. 7. Different types of cell housings

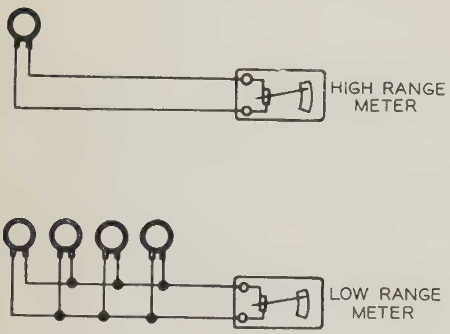


Fig. 6. Parallel connection to obtain higher output

Door Opening Circuit and Automatic Weighing Circuits

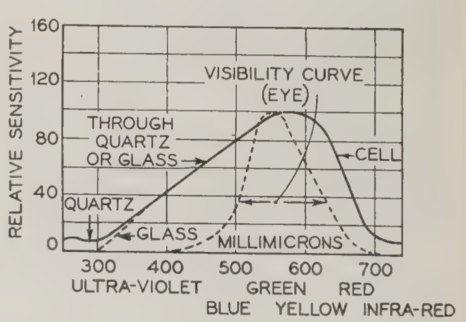
The dry disk photoelectric cell readily adapts itself to most industrial control problems and as the technique of applying it as a mechanical tool is becoming well established, it is finding a very definite place in this field. Industrial control may usually be divided into 3 groups:

1. Complete cut-off of the light beam.
2. Change in light intensity.
3. Change in color.

Complete cut-off of the light beam is usually associated with the counting of opaque objects moving through a light beam. This is a simple type of control and is used successfully in nearly every industry which packs or manufactures small articles. But of greater interest to technicians are some of the unusual applications of this versatile cell.

Photoelectrically controlled doors, for instance, operate by the complete cut-off of a light beam as a person or vehicle passes through. It has been found necessary at all recent installations of photoelectric cell controlled doors, such as those in the Pennsylvania Railroad Station in New York City, to pass a diagonal beam of light through the open doorway so that if anything should remain in the doorway, the

Fig. 8. Spectral response of dry disk photoelectric cell



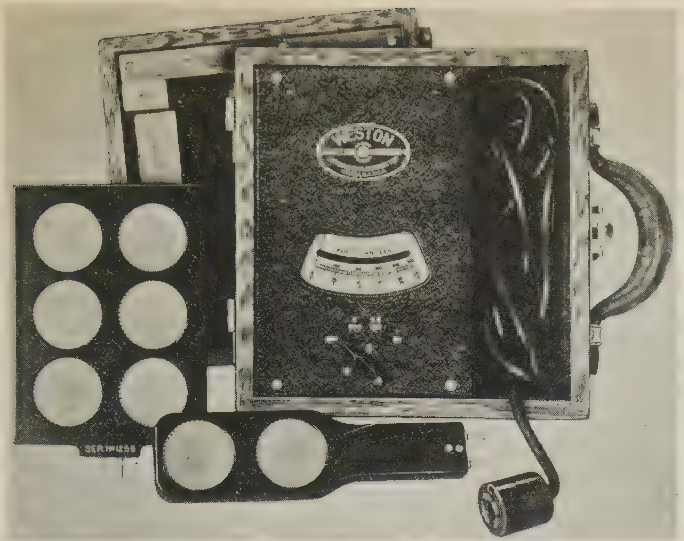


Fig. 9. Illumination meter

holes in the dial face. Behind each hole is an electric light and over each hole a photoelectric cell. The cells are connected in parallel to one relay. Operation is as follows (in the case of automatic coal weighing): The truckman turns a rotary switch, marked in tons, to the number of tons desired. The switch is arranged to light only the lamp under the proper cell so that the target or the pointer will pass through the light beam when the proper weight is reached. The other cells are dark, but since their dark resistance is so high, they have practically no effect upon the circuit. The one cell which is lighted energizes the relay and causes the coal to start flowing; this continues until the correct weight is reached at which instant the tail of the pointer darkens the lighted cell and causes the relay to operate and stops the flow of coal.

These control circuits are extremely simple when one considers the difficult and complicated task per-

doors will not close. As shown in figure 11, there are 3 cells connected in series, cells 1 and 2 being operated from either approach, and cell 3 being used to hold the door open when anyone remains in the doorway. When the door closes cell 3 is short-circuited by the switch *S*. The dark resistance of the cell is 5 to 6 times that of the cell when illuminated; therefore, when the door is closed and cells 1 and 2 are connected in series, blocking either light beam will reduce the output current practically to zero and thus operate the relay. Likewise when the door is open and all 3 cells are in series, blocking the diagonal beam or either of the other beams, will cause the relay to close and the door to remain open.

Another application of complete cut-off control

Fig. 11. Door opening circuit

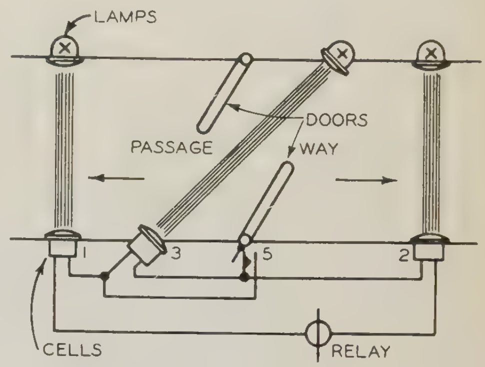
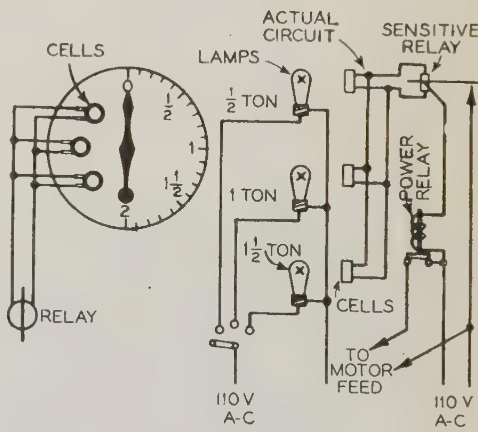


Fig. 12. Automatic weighing



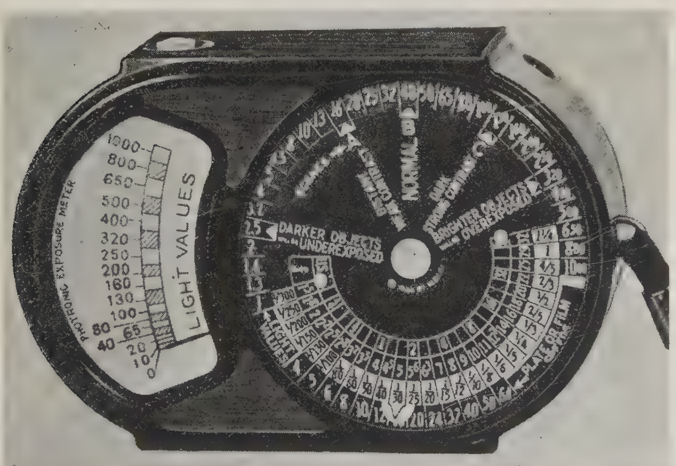


Fig. 10. Exposure meter

is in the use of dial scales for automatic weighing of coal, coke, liquids, etc. Again, one industrial control relay is connected to the cell circuit but in this application the cells are connected in parallel. (See figure 12.) On the tail of the scale pointer a target *T* is mounted, and arranged to pass over 2 or more

formed. These controls were made possible only by understanding and taking advantage of the characteristics of this cell.

Other Control Circuits

Control by light intensity change is illustrated well by a smoke alarm. (See figure 13.) A light beam is directed across the inside of a stack, striking a cell which is connected to a relay. The device is so adjusted that when a predetermined amount of smoke is present in the stack, an alarm rings in the boiler room, thus warning the attendant. The relay

is mounted in the boiler room and the cell is mounted any distance away and at any desired height on the stack.

A simple type of control is used at a sewage disposal plant where a cell and light source are placed in the liquid in the last tank. (See figure 14.) As the sludge becomes sufficiently dense at the light

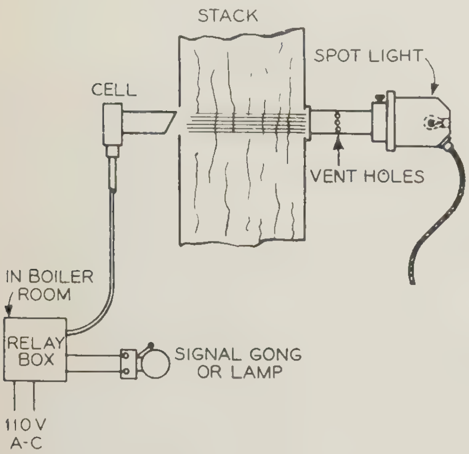


Fig. 13. Smoke alarm

beam, the cell output decreases and operates the relay which starts a pump so that the sludge is removed and placed back into the system. By using this equipment it is possible to operate the plant at considerable overload without danger of overflow of the sludge into the river which carries off the liquid. In this installation, the relay box is mounted indoors approximately 200 feet away from the cell which with its light source is mounted 5 feet below the surface of the sludge tank.

The street light control relay is another instance of control on slowly changing light intensity. (See figure 15.) The relay box may be mounted in a manhole under ground or at any other point near to the power mains, while the cell may be mounted on top of a pole or roof or wherever convenient. This same relay is used for lighting radio towers, and airway and marine beacons, and in each installation the cell is mounted wherever convenient without regard to the length of leads to the relay box. This relay may be set, by adjusting an index, to turn the lights on and off at any desired value within its range, and any range may be supplied to meet the desired conditions.

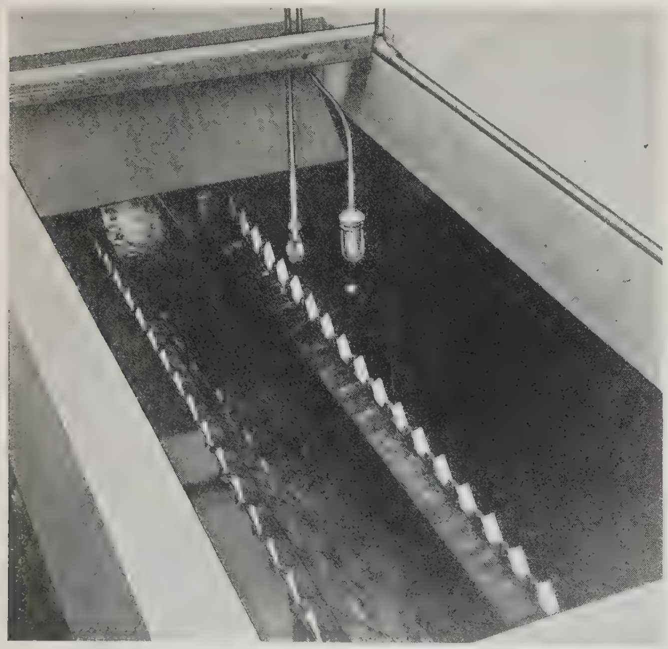


Fig. 14. Sludge level control

Color change may be made to operate these relays since the cell produces a different current output for any change of color. In automatic coffee roaster control, the green beans produce by reflected light an output several times higher than the finished brown beans. It is, therefore, an easy matter to adjust the relay to dump the roast or operate a signal when the proper shade of brown is reached.

While this paper covers only a few of the high spots of recent advances in the application of the dry disk

Fig. 15. Street light control

photoelectric cell, it will indicate that many new fields of photoelectric measurement and control have been opened. The simplicity of the circuits involved and the use of standard instruments and relays operating on well-known principles, largely eliminate the hesitancy of the industrial engineer toward adaptation of photoelectric methods to his control problems.

Photoelectric measurement and control is young but it will surely take its place in the commercial world along with the lamp, the battery, and the motor; and simple devices, such as are made possible by the correct technical application of the dry disk type of photoelectric cell, will hasten its acceptance.

"Angle Switching" of Synchronous Motors

Commercial applications of "angle switching" of synchronous motors are given especial attention in this paper. "Angle switching" refers to a method of starting synchronous motors whereby the time of application of the excitation voltage with respect to the relative angular position of the field poles and the impressed voltage is controlled. The advantages of this method of starting are pointed out, the equipment is described briefly, and a number of practical applications in which increased starting torque, reduced starting current, or other features were obtained are described.

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As the speed of a synchronous motor during the starting period approaches synchronism, d-c excitation is applied and the motor pulls into step. If the time of application of the excitation voltage to the synchronous motor is controlled with respect to the relative angular position of the field poles and the impressed voltage, the method of starting is referred to herein as "angle switching." Several papers have been presented before the Institute[1] on the subject of the effects of this type of control. As previously pointed out, one of the chief advantages of such control is to increase the torque against which the motor will consistently pull into step with a given value of starting current. The present paper points out certain commercial applications of "angle switching" and gives some experimental results obtained along these lines.

The equipment used for commercial installations consists essentially of a means of checking the position of the rotor against the space position of a particular phase winding at a predetermined point on the wave of voltage impressed across that phase. This angle switching equipment is used in conjunction with synchronous motor starting control of standard type. The control functions in an ordinary manner up to the point of applying the excitation voltage, but instead of applying the excitation at this point, the angle switching equipment is energized. This device, in turn, applies the excitation voltage to the motor the first time the rotor and the impressed voltage pass through the predetermined relative angular relation.

Let a in figure 1 represent a portion of the voltage wave impressed across one phase of the motor. For convenience, a check is made each time the voltage has its maximum value in an arbitrarily chosen positive direction, as at the point indicated by the arrow. Let b in figure 1 represent a section through a full pitch coil in the phase across which the voltage shown at a is impressed, and let the positive direction of the wave shown be such as to tend to make current flow in the coil in the direction shown by the arrows. For a machine operating in synchronism with the direction of rotation shown, and at true no load, a "north" pole will be directly under the coil side shown each time the voltage wave has its maximum positive value. For various steady loads the pole will be a certain distance ahead of or behind the position shown at the time of maximum voltage, depending upon whether the machine is loaded as a generator or a motor, respectively. Positions up to 180 degrees ahead of the no load position are said to be in the "generator region" and positions through 180 degrees behind are said to be in the "motor region."

The Saturating Transformer and Inductor Generator

To determine the time at which the voltage impressed upon the reference phase of the motor is maximum, a small saturating transformer is connected across this phase through a potential transformer. This saturating transformer gives a sharp peaked wave in time phase with the maximum value of the voltage wave. The rotor position is checked by means of a small generator. This generator is of the inductor type without rings, brushes, or other

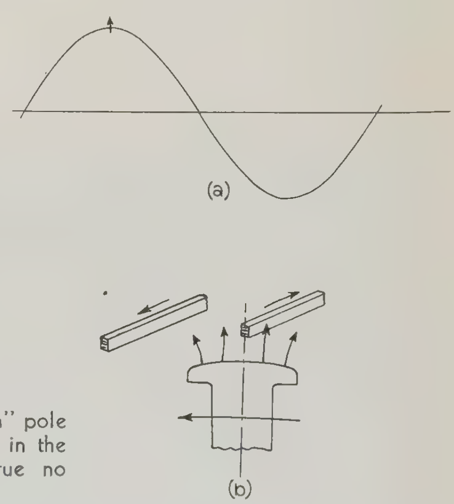

Fig. 1. Principles involved in angle switching

a—Voltage wave impressed on reference phase of the motor. Rotor position is checked at point indicated by arrow

b—Position of a "north" pole with respect to a coil in the reference phase at true no load

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1. For references, see list at end of paper.

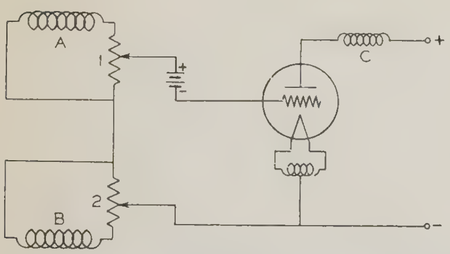
wearing parts. The rotor is mounted on the shaft or spider of the main motor and the stator is mounted on one of the bearing pedestals or the brush rigging support. The magnetic circuit is so arranged that during one revolution a sharp voltage impulse is generated for each pair of poles on the motor. Provision is made for circumferential adjustment of the stator to obtain the required angular position of the generated voltage peaks. The power required from the generator and from the saturating transformer is practically negligible.

The outputs of the saturating transformer and the inductor generator are connected in series between the grid and the cathode of an electronic tube of the gas or mercury vapor filled type as shown in figure 2. *A* and *B* are respectively the secondary of the saturating transformer and the voltage coil of the inductor generator. The bias on the grid is such that a coincidence of the voltage peaks from the 2 sources is necessary to render the tube conducting. When the motor is operating on its starting windings below synchronous speed the impulses from the generator still denote the position of the rotor of the motor in space but its position is continually changing with respect to the constant frequency voltage impressed across the reference phase. However, since the frequency of the impulses generated by the saturating transformer is different from the frequency of the voltage produced by the inductor generator, there will soon be a coincidence of the voltage peaks from these 2 sources. As soon as the tube becomes conducting the current through the operating coil *C* of a high speed relay causes the relay to close and apply the excitation to the motor.

Figure 3 shows the 2 voltage peaks and the resultant wave as impressed between the grid and cathode of the tube. This film was taken with the 2 voltage waves at the same frequency and approximately 60 degrees apart. Figure 4 is a view of the tube and its panel. The saturating transformer is mounted on the back. The dials are for the adjustment of the 2 potentiometers. It is evident that the tube need be energized during only a part of the starting period. This gives it a relatively long life.

Results Obtained With Angle Switching

Pull-in tests have been taken on motors from 200 to 900 horsepower and with different values of excitation, load inertia, and field circuit time constants. These tests show that for a given motor with a given starting current the maximum torque against which it pulls into step consistently with a fixed moment of inertia can be increased more than 50 per cent in some cases by the use of angle switching. The results of such a test on a 290 horsepower, 1,200 rpm, 60 cycle, 80 per cent power factor motor are shown in figure 5. These curves are plotted between the maximum torque against which the motor will synchronize consistently and the angle at which the excitation



A—Secondary of saturating transformer
B—Voltage coil of inductor generator
C—Operating coil of high speed relay
1 and 2—Potentiometers

Fig. 2. Schematic diagram of tube circuit

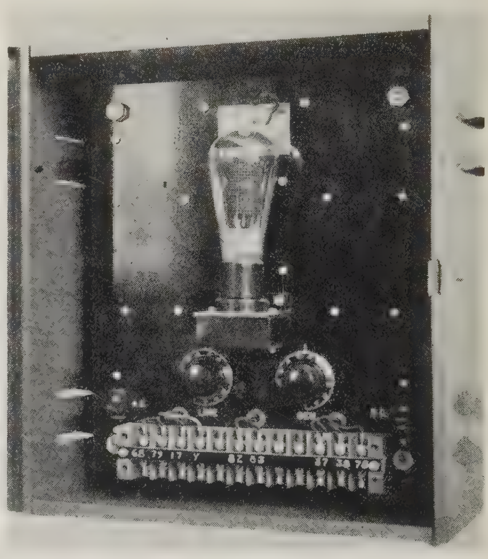


Fig. 4. Electronic tube and its panel. The saturating transformer is mounted on the rear

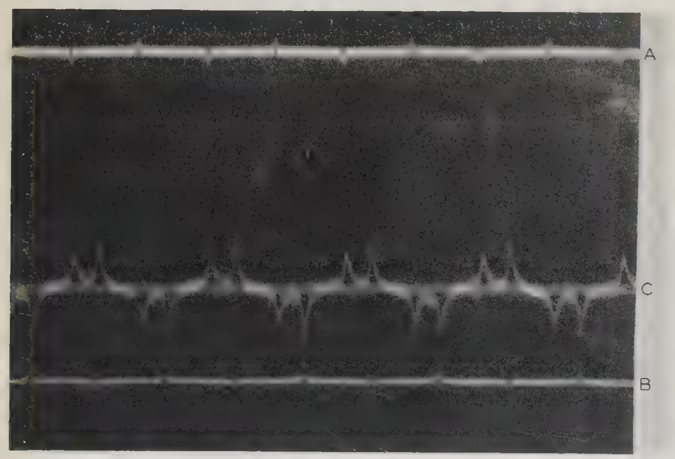


Fig. 3. Voltage peaks and resultant load

A—Voltage output of saturating transformer
B—Voltage output of inductor generator
C—Total voltage between grid and cathode, not including bias

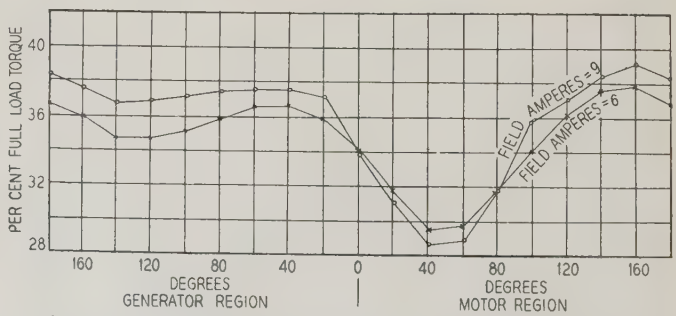


Fig. 5. Tests of pull-in torque against angle for a 290 horsepower, 1,200 rpm, 60 cycle, 80 per cent power factor motor

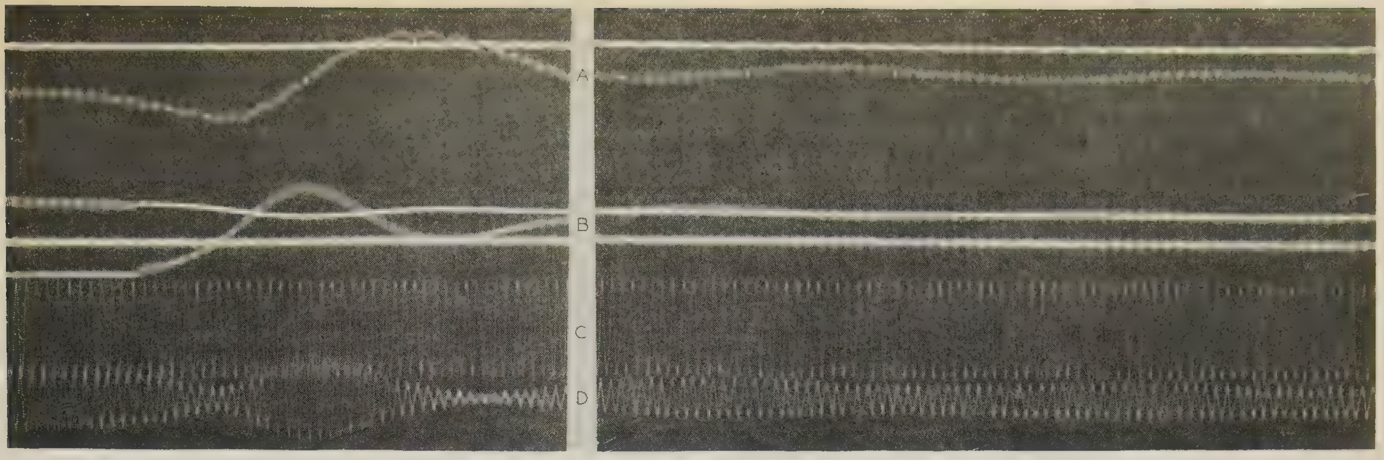


Fig. 6. Pull in with field energized at 40 degree generating position

A—Speed *B*—Field current *C*—Line voltage *D*—Line current

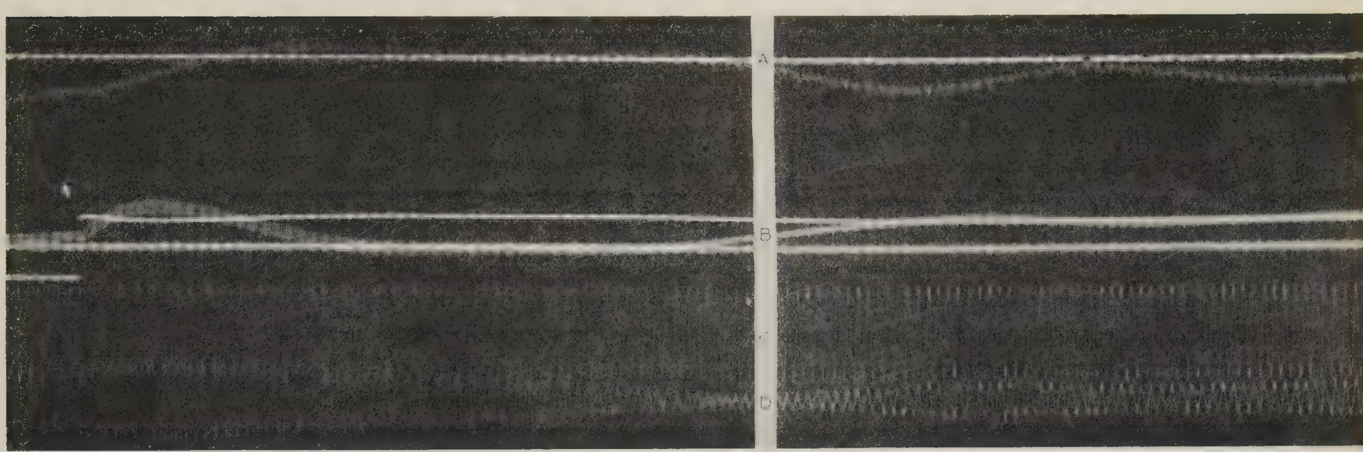


Fig. 7. Pull in with field energized at 100 degree motoring position

A—Speed *B*—Field current *C*—Line voltage *D*—Line current

voltage was applied. The 2 curves submitted are for the same value of total moment of inertia, and 2 different values of field current. These particular curves were taken at half voltage.

The greatest practical value of this means of increasing the torque against which a motor is certain to synchronize appears to be that it permits a decrease in the starting current for a given pull-in requirement. Figure 5 shows that this particular motor with the particular value of moment of inertia, and the higher value of excitation would pull in every time against only 29 per cent torque with uncontrolled field switching. At the voltage at which these tests were taken the motor had 65 per cent starting torque and 170 per cent starting kilovolt-amperes. Assume that 29 per cent pull-in torque is sufficient for the particular load. The speed-torque characteristic for this motor shows that the slip corresponding to 29 per cent torque on the starting windings is 2.5 per cent. However, figure 5 shows that the motor will synchronize consistently from 3.5 per cent slip with angle switching, as 3.5 per cent is the slip corresponding to 38 per cent torque on the starting windings. Therefore, this motor with angle switching will synchronize against 29 per cent torque with a higher impedance damper winding which will bring it to only 3.5 per cent slip with this value of torque. Because of the higher impedance damper winding the starting current will be reduced from its present value of 170 per cent to approximately 125 per cent.

An example of the effects of synchronizing against the same load at different angles is shown in figures 6 and 7. The set up for taking these films consisted of the 290 horsepower motor, referred to above, connected to a 200-kw d-c generator. This generator was loaded on a resistance bank and compounded in such a manner that the torque was substantially constant over a considerable speed range. The motor was controlled through an angle switching device similar to the one described above. The different character of the pull-in phenomena is evident from the films. Figure 6 shows the motor being synchronized with the field applied at a 40-degree generating position. Figure 7 shows synchronizing from the less favorable angle—from the torque standpoint—of 100 degrees motoring. When synchronized from 40 degrees generating, the motor at once pulled into step and oscillated over a rather wide angle before settling down to its final angular position. When

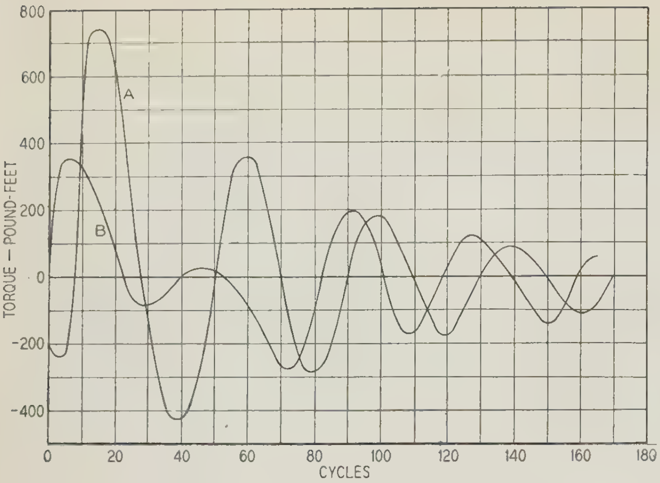


Fig. 8. Torque producing oscillation of motor rotor and load during pull in

A—Field applied at 40 degree motoring position
B—Field applied at 100 degree generating position

synchronized from 100 degrees generating position, the fluctuations in speed are of considerably less magnitude even though the motor does not synchronize at the first opportunity.

Another aspect of this synchronizing phenomena is shown in figure 8 which gives the calculated torque which produced mechanical oscillations based upon the speed variations shown on the films and the known moment of inertia of the system. It is seen from curve *A*, which is for the 40 degree generating position, that the oscillating torque is from 750 pound-feet positive to 460 pound-feet negative. For the 100 degree motoring position shown at *B*, the oscillation is from between 360 pound-feet positive to 275 pound-feet negative, or only 53 per cent as much total pulsation. The constant load torque was 280 pound-feet. Since the total torque is the sum of the inertia torque and the load torque, there was no reversal of mechanical torque in the mechanical system during the entire synchronizing process.

An example of an application of this principle in which angle switching has been applied is in the case of the cement mill of the National Portland Cement Company, where the requirements of the tube mill drives were such that the torques during the synchronizing period had to be controlled within certain limits. These mills are driven by 800 and 900 horsepower, 720 rpm motors. One of the requirements was that the torque delivered through the gears, which coupled the motor to the load, should not reverse during the synchronizing period. There was also a limit on the maximum value of positive torque. By tests under load conditions the angle switching equipment was adjusted so that the desired torques were obtained.

Another point which may be of interest in some applications is the comparison of the line current drawn by synchronizing from the 2 angles. The currents are shown in figure 9, which is drawn from the oscillograms shown in figures 6 and 7. Curve *A* shows the current for switching at 40 degrees generating, which is the more favorable angle from the torque standpoint. The maximum root mean square value is 87 amperes and the average value of the effective current over the first 2 seconds is 40 amperes. For the more unfavorable angle of 100 degrees motoring, the maximum effective value is 96 amperes and the average value of the effective current is 50 amperes for the first 2 seconds.

A Modification of Angle Switching

Another practical problem to which a modification of the angle switching equipment has been applied is that of synchronizing 2 motors so that they will operate with a predetermined angular relation between the rotors of the 2 machines. A particular case was in the air conditioning of the new Kansas City Auditorium in which it was specified that the motors driving 2 identical refrigerant compressors must be synchronized in such a manner that the cranks of the 2 compressors would always be displaced by a definite angle relative to each other. The purpose of this requirement was to distribute the current pulsations caused by the torque variations in the 2 compressors. The motors were rated at 400 horsepower, 225 rpm.

To accomplish this an inductor generator was mounted on each machine. With one motor started in a normal manner and operating in synchronism the impulse generator gives an impulse each revolution when the rotor is in a particular position. The inductor generator on the second machine is adjusted so that it produces an impulse each revolution at a position displaced by the required amount from the position at which the rotor on the first machine produces its impulse. These generators differ slightly from those previously described in that the former give an impulse for every pair of poles. Since the first motor is operating in synchronism and the second on its starting windings, it is evident that the peaks from the 2 generators will sooner or later coincide.

The tube control circuit is quite similar to that shown in figure 2. As shown in the figure, potentiometer 2 is energized from the inductor generator of the machine which is to be synchronized. But potentiometer 1, instead of being energized from the line is energized from the inductor generator which is

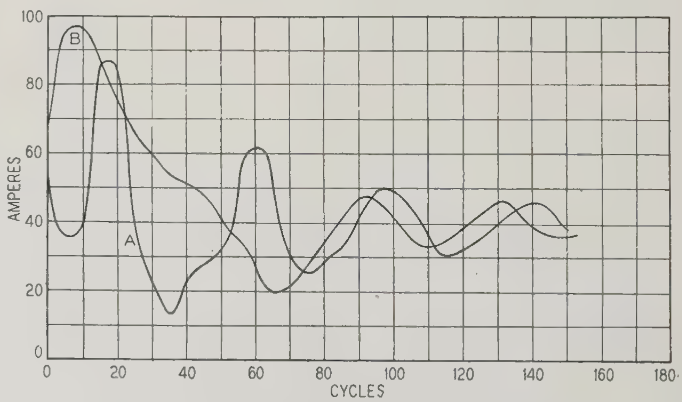


Fig. 9. Current variation during pull in

A—Field applied at 40 degree motoring position
B—Field applied at 100 degree generating position

mounted on the shaft of the machine which has already been synchronized. When the tube becomes conducting the excitation is applied to collector rings of the second machine. Obviously the same result could be approximated by some sort of electrical or mechanical contactors. But no such devices appear to have the speed, accuracy, and freedom from wearing parts found in the angle switching equipment described herein.

Since the first machine is synchronized and operating at some rather definite average angular displacement, it is entirely possible to adjust the impulse generator on the second motor so as to take advantage of the increase in pull-in torque which can be obtained through angle switching.

The angle switching equipments which have been built have been used in conjunction with the standard, definite-time type of control. There may be certain types of loads where it is desirable to incorporate a means of checking the speed at which the tube is energized. The inductor generator affords a convenient means of doing this. But unless the load conditions vary widely from one start to the next, such a feature does not appear to be necessary.

In addition to increasing the torque against which a motor is certain to synchronize, it is seen that angle switching may be used to advantage for other purposes as outlined in this paper. The installed equipment to date, using electronic tube type of equipment, consists of 10 motors totalling 5,200 horsepower. Such control of the "pulling-into-step" process is now a special feature. But with a growing understanding of its advantages, some form of angle switching equipment will probably become more or less standard for many applications of synchronous motors.

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2. The Pulling-Into-Step of a Salient Pole Synchronous Machine, H. E. Edgerton and Paul Fourmarier. A.I.E.E. Trans., v. 50, June 1931, p. 769–81. (A good set of references to previous articles is given in this paper.)

Electric Furnaces With Carbon Radiator

A type of electric furnace for application to metallurgy and to operations at very high temperatures has been developed in which the heat producing element is a carbon electrode raised to a very high temperature and conveying heat to the charge by radiation. Furnaces of this type, which have incorporated in them a number of novel features, have been devised and perfected in France in the electrothermic laboratory of the company with which the author is connected.

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THE ESSENTIAL feature of the electric furnaces described herein is a carbon radiator mounted within the furnace chamber and raised to a very high temperature by the current flowing through it. This radiator can be reduced to a simple graphite rod, which allows high current densities (200 to 500 amperes per square centimeter) to be used, and which radiates freely to the charge and to the walls; hence the furnace, which has been frequently called a "radiation furnace," can be compared to a large incandescent lamp, the filament of which is the resistor.

The general properties of the furnaces with an internal radiator can be derived from their physical characteristics:

1. From the electrical viewpoint, these properties are those of a resistance furnace: constant load for each value of the supply voltage, and absence of any inductance.

2. From the thermal viewpoint, the use of graphite resistors makes it possible to attain very high temperatures (3,000 degrees centigrade in special furnaces) or, in the case of normal operation, to build very flexible furnaces, the power rating of which may be quite considerable.

Construction

The radiator type furnaces embody, besides the conventional elements, several interesting features which are made important by the necessity of bringing large currents to a radiator of small diameter, of eliminating eddy current losses, and also of avoiding the inductance caused by any closed loops in the feeders.

Before examining these features, a small single-phase 100-kw furnace for melting steel, which has a capacity of 100 kilograms (220.5 pounds) will be described briefly. Figure 1 represents a vertical cross section of that furnace. The chamber consists of a sheet metal cylinder which is terminated at each end by truncated cones. The lining com-

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Translated from the French by L. A. Huguemont, of the General Electric Co.

prises a heat insulating layer and a refractory lining baked on the job. Two circular tracks are secured to the central furnace cylinder and these rest on rollers. Thus equipped, it can rotate or oscillate for churning and for pouring the molten metal. Both the spout and the charging door are on the cylindrical part of the furnace wall.

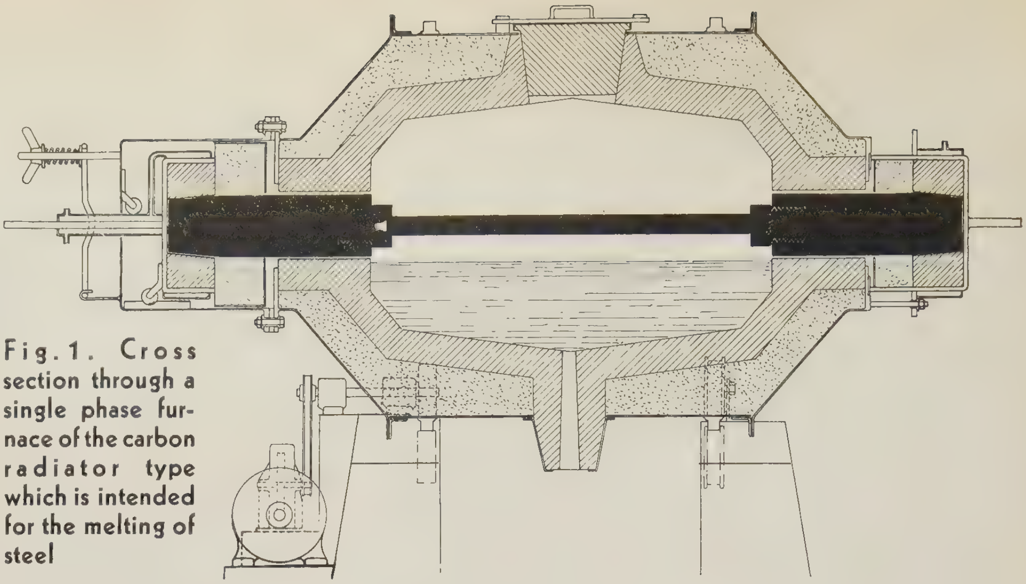

Fig. 1. Cross section through a single phase furnace of the carbon radiator type which is intended for the melting of steel

The radiator is a graphite rod coaxial with the furnace; one of its ends is screwed to a cylindrical graphite element while the other end, which is conical, rests in a conical recess in another cylindrical graphite element into which it is forced by a spring outside the furnace. Each one of the end parts is screwed into an extension consisting of amorphous carbon which in turn is fitted into a metal collar through which water circulates. It is through these collars that the current is conducted to the heating element. The furnace is of the closed type; on one side it is closed by one of the carbon extensions that rests against an insulating disk which in turn is supported by the furnace plating. On the other side it is closed by a sheet metal box which contains the second carbon extension, the electric connection of which slides in a tubular guide built on the bottom of the box. The carbon extension can move freely within the box, and it is forced toward the interior of the furnace by a spring which also presses the resistor element against its electric contact, thus preventing loose contacts such as may be brought about by the motion of the furnace and by expansion and contraction.

Fig. 2. A 200 kilogram (441 pound) furnace which is installed in an iron foundry

In order to withdraw the electrode from the furnace, the movable carbon extension is disconnected and removed from the furnace by means of the carriage provided for that purpose. In this manner the resistance element can be changed or placed aside to protect it from being broken while the furnace is being charged. In order to prevent the oxidation of the graphite parts that have been removed while still hot, it is necessary only to cover them with a hood or to lower them into a suitably arranged well.

In general, the furnace is subjected to voltages

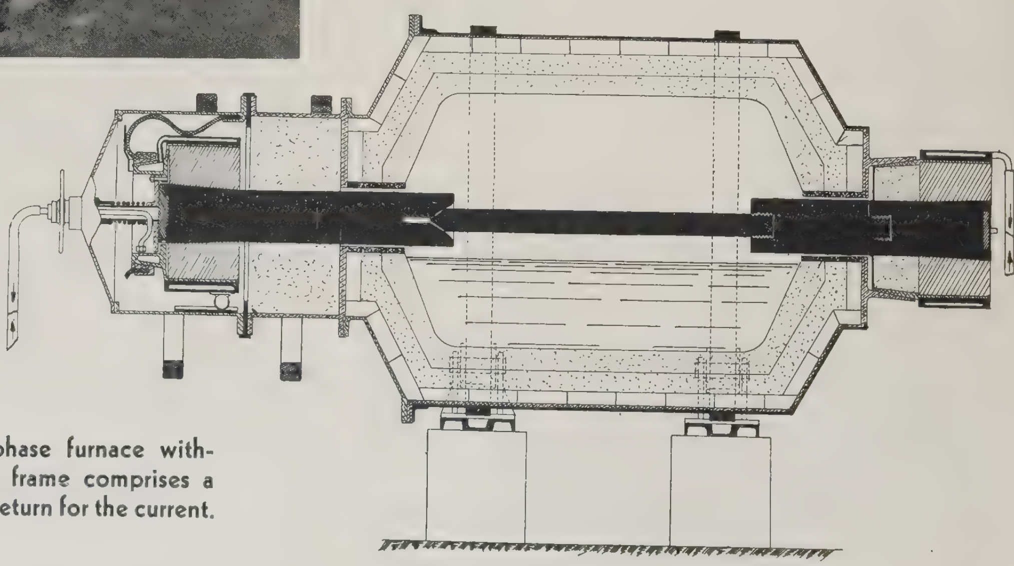

Fig. 3. High-capacity single-phase furnace without magnetic losses. The steel frame comprises a copper lining which serves as a return for the current. The feeder system has no loop

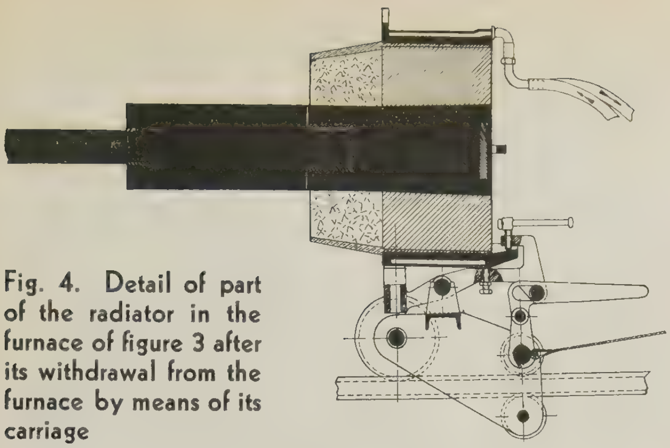

Fig. 4. Detail of part of the radiator in the furnace of figure 3 after its withdrawal from the furnace by means of its carriage

which may be varied with respect to a nominal value. The range of the regulation depends upon the wear which is allowed for a given resistor.

The small 100-kw furnace of figure 1 has given the following results:

For steel which is cast at 1,650 degrees centigrade, the power consumption amounts to 0.8 kilowatt-hour per kilogram.

For cast iron and bronze, the corresponding figures are 0.5 and 0.25 kilowatt-hour per kilogram, respectively.

The furnace is provided with a graphite rod of 35 millimeters diameter which weighs 1.320 kilograms (2.91 pounds); after 7 operations, the diameter of that rod or resistor is reduced to 20 millimeters throughout its entire length. Should it have to be replaced then, this would imply a consumption, including the drop, of 1.650 kilograms (3.64 pounds) of graphite per thousand kilowatt-hours; this is much lower than can be attained with an arc furnace. These low consumptions, which have never been attained in such a small furnace before, become still lower in large furnaces.

Figure 2 represents one of the first furnaces with an internal radiator that were installed in France. Its design is nearly in conformity with the layout of figure 1. It is used for the casting of cast iron piston segments.

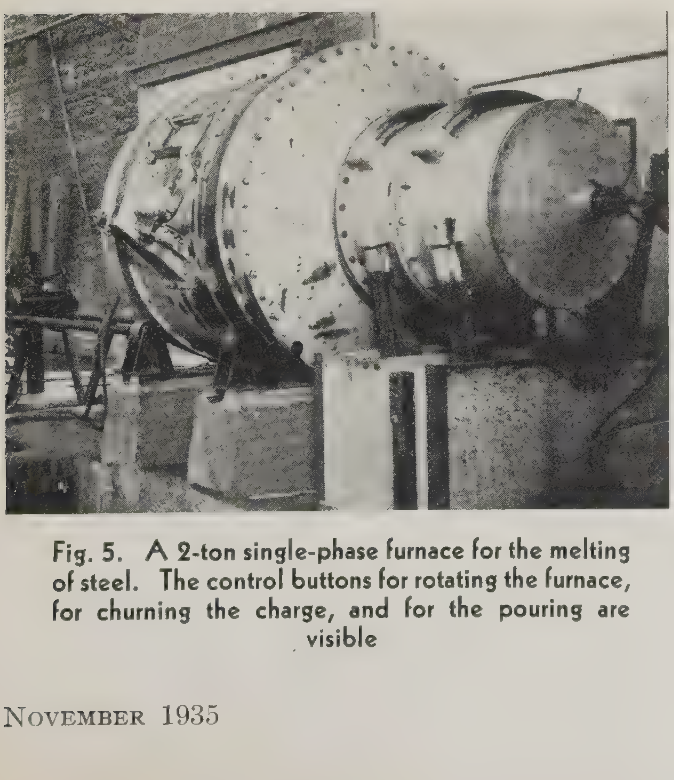

Fig. 5. A 2-ton single-phase furnace for the melting of steel. The control buttons for rotating the furnace, for churning the charge, and for the pouring are visible

Elimination of the Magnetic Losses and of the Inductance of the Loop

The construction which has been described herein is satisfactory only in the case of low-capacity single-phase furnaces which bring into play only currents below 5,000 amperes. Beyond that, the eddy current losses in the magnetic parts of the frame become considerable, and at the same time the power factor drops as a result of the inductance of the loop formed by the supply conductors.

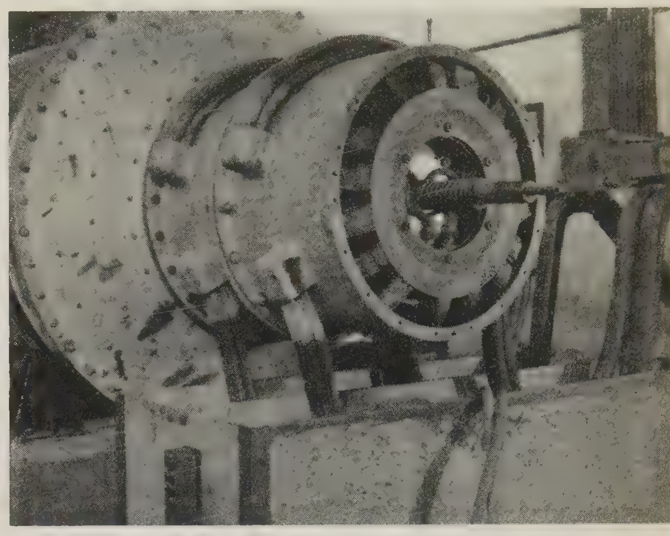

Fig. 6. Detail of the furnace of figure 5, showing external and internal connections

One can evidently avoid the losses by avoiding any magnetic metal in the construction of the frame. However, the difficulty can be overcome better by annulling the field in the magnetic parts and by bringing about the return of the current which has traversed the resistor, by means of a copper lining on the metallic housing of the furnace so that the 2 incoming conductors terminate at the same end of the furnace. In this manner the magnetic losses and the loop formed by the supply conductors are eliminated.

Figure 3 represents a cross section of a detail of that type of construction, which has also the advantage of facilitating the handling of the radiator, which is made an integral part of the carbon extension at which no conductor whatever terminates.

Figure 4 represents, on its carriage, the radiator and its connecting members withdrawn from the furnace. Attention is called to the bronze piece which, at its cylindrical part, is clamped by a ring to the carbon extension, and the conical part of which fits into the conical recess at the end of the furnace which is electrically connected to the copper lining of the frame. By means of this construction it was possible to lower the power consumption, for the

Fig. 7. The radiator of the furnace of figure 5 is represented withdrawn from the furnace for the charging, and tilted into its pit

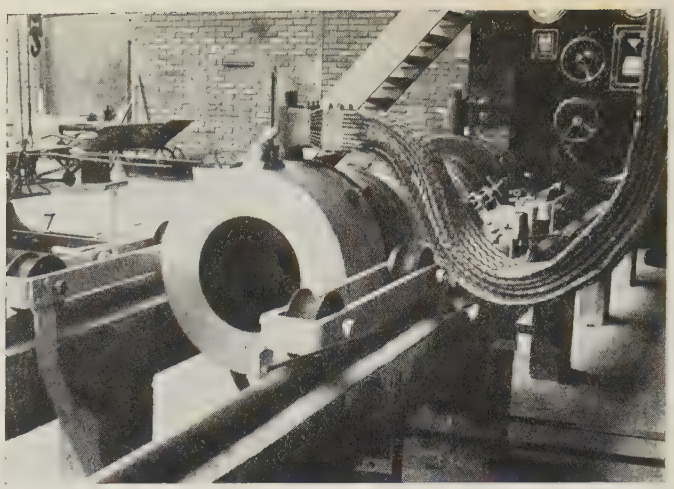

Fig. 9. A 300 kw centrifugal furnace for calcination and fusion at very high temperature

casting of steel in furnaces of 500 kilograms capacity, to 0.68 kilowatt-hours per kilogram.

Figures 5, 6, and 7 represent various views of a single phase furnace of 2-tons capacity which is installed in a steel mill. The current rating for this furnace is 15,000 amperes; the power factor is 98 per cent. It is obvious that the furnaces may comprise several radiators and may be connected directly to a 3 phase system.

Centrifugal Furnaces With Internal Radiator for Operation at a Very High Temperature

An interesting application of carbon radiator furnaces on an industrial scale is found in fusion and calcination processes at high temperature. At the temperatures which come into consideration here, no refractory linings are available, and it is necessary to have recourse to special artifices.

When it is a question of melting or calcining pulverized or granular products, centrifugal furnaces may be built. In these furnaces the enclosure is a steel cylinder which rotates at a fairly high speed so that the pulverous material which it contains assumes the shape of a tube. The internal radiator is pushed into that tube by means of its carriage, but it does not participate in the rotation of the enclosure. Figure 8 represents schematically furnaces of that type. The part of the charge which is to be molten or fritted has been represented by surfaces shaded by short lines. It will be noted that there is no contact between the radiator and the charge, and that a slight air circulation makes it possible to avoid any reduction or carburization of the product which is being treated.

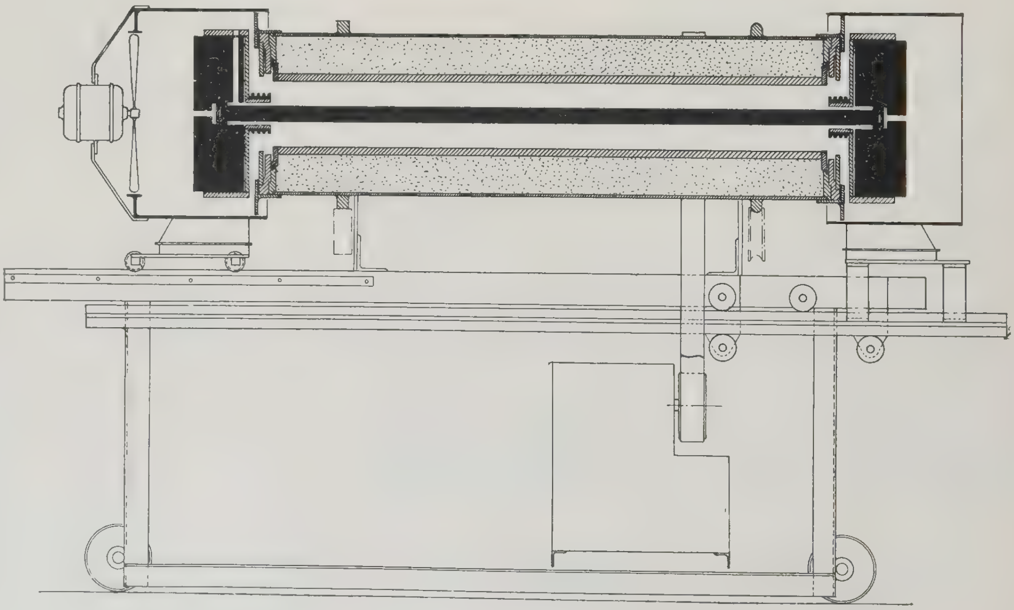

Fig. 8. Schematic cross section of a centrifugal furnace with graphite radiator

In the 300 kw furnace which is shown in figure 9, thorium oxide, which is used as refractory material, has been successfully calcined without carburization, at 2,700 degrees centigrade.

It is evident that with furnaces of this type one can adopt the very high power ratings which the principle of the internal radiator furnaces permits. With a special industrial furnace, which has been used since 1932 in the manufacture of fused silica tubes, one can obtain ingots of 20 kilograms within not more than 6.5 minutes, starting from a cold charge on which a power of 270 kw is applied.

Advantages of the Carbon Radiator Furnace

These furnaces are the simplest and least expensive among the industrial electric furnaces. Be-

cause of limited space here, it is possible to dwell but briefly on their metallurgical advantages, which are likewise remarkable and which can be summarized as follows:

Flexibility, uniformity of temperature, a definite atmosphere, the possibility of operating in a vacuum, easy regulation of the carburization, facility of reduction processes, and last, but not least, the constancy of the analysis of the metal prior and after fusion.

This furnace has met with the instant approval of French metallurgists, as soon as it was placed on the market in 1934.

Power companies are greatly interested in its promotion since they hope that its low cost will make for its ready adoption by metallurgists who so far have refrained from installing furnaces of another system.

Resolution of Surges
Into Multivelocity Components

Based upon a theory of traveling waves previously expounded by the author, this paper presents a method of resolving oscillograms of traveling waves on electric power lines into their multivelocity components. The effects of a ground wire on multivelocity waves also are pointed out.

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THE multivelocity theory of traveling waves has proved essential for explaining several peculiar attenuation and distortion effects observed in the behavior of both natural and artificial surges on transmission lines.[1] The theory also has found a practical application in analyzing the counterpoise.[2,3] While these previous applications were confined either to qualitative predictions, or to calculations based upon known ground planes and line constants, no method has been given for obtaining the multivelocity components directly from cathode ray oscillograms without knowing the line constants. It is the object of the present paper to show under what conditions this can be done and, incidentally, to point out the effect of a ground wire on multivelocity waves.

In general, there will be as many multivelocity components on a multiconductor system as there are separate line conductors;[1] but for certain conditions of symmetry one or more of these velocities may become redundant. Thus for a 3 phase line with horizontal configuration there will be only 2 independent velocities. For a double 3 phase circuit completely transposed there will be 2, not 6, velocities. Most lines are transposed, or may be so regarded for the purpose of this paper without introducing much error. It is shown in appendix I that on a completely transposed n-conductor system with corona on one of the conductors (the one carrying the main surge), there are only 3 velocities, and all waves induced on adjacent conductors are alike. The multivelocity equations are

$$e_1 = f_1(x - v_1t) + f_2(x - v_2t) + f_3(x - v_3t)$$
$$e_2 = a_1f_1(x - v_1t) + a_2f_2(x - v_2t) + a_3f_3(x - v_3t)$$

in which f_1, f_2, and f_3 are the 3 wave functions, v_1, v_2, and v_3 their respective velocities, and a_1, a_2, and a_3 coefficients depending upon the circuit constants. The inductances and capacitances in turn depend upon the diameter of the corona and the average depth of the ground plane, neither of which in general can be determined. At the point of inception of the surge, all the wave components have identically the same shape, as proved in appendix I; but very soon thereafter the waves become dissimilar because of the different distortions experienced by the components as a result of different corona and ground currents. Thus the problem presented is to find 3 different wave shapes traveling at different velocities, and 3 amplification factors, when all that is given are the 2 oscillograms of the main and induced surges, e_1 and e_2, respectively. Analytically this cannot be done; but as a matter of fact it is possible by cut-and-try sketching to draw fairly good approximations of the wave components, because the points on the oscillograms where the slower components arrive usually are well defined.[1] Of course, even this possibility is defeated if the surge has not traveled far enough for the component waves to separate out so that their presence can be distinguished on the oscillograms.

In the event that the voltage of the surge is below the corona limit and the conductors are completely transposed, a most fortunate simplification takes place. As shown in appendix I, only 2 velocities are present and the a coefficients are evaluated to fixed values independent of the circuit constants; so there results for an n-conductor system:

$$e_1 = f_1(x - v_1t) + f_2(x - v_2t)$$
$$e_2 = \frac{-1}{(n-1)} f_1(x - v_1t) + f_2(x - v_2t)$$

or resolved

$$f_1 = \frac{n-1}{n}(e_1 - e_2) = \text{fast wave}$$
$$f_2 = \frac{e_1 + (n-1)e_2}{n} = \text{slow wave}$$

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1. For all numbered references see list at end of paper.

For a 3 phase line these relationships become:

$$f_1 = \frac{2}{3}(e_1 - e_2) = \text{fast wave}$$

$$f_2 = \frac{1}{3}(e_1 + 2e_2) = e_1 - f_1 = \text{slow wave}$$

Thus by plotting the oscillograms of e_1 and e_2 together and to the same scale, their difference can be found quickly by a pair of dividers and $^2/_3$ of this difference is the fast wave. The difference between e_1 and this fast wave is the slow wave. Figure 1 shows oscillograms of main and induced surges taken 9.3 and 22 miles from the origin. At the bottom of the figure, these oscillograms have been replotted and resolved into their multivelocity components by the foregoing "$^2/_3$" rule.

The fast wave so found may be seen to fit the front of the main surge quite well, as it should because the slow wave has not yet arrived. The front of the slow component is longer than that of the fast component, and in the 12.7 miles of travel represented by the 2 pairs of oscillograms, the slow component suffers more attenuation. This is because the slow component has an image current in the earth, and therefore experiences the flattening of the front and elongation of the tail characteristic of waves with earth return.[1]

From figure 1 it is also possible to find the velocities of the waves. It is known from both theory and test with traveling waves on transmission lines that the velocity of the fast wave is the velocity of light (or at least very nearly it). Let

Δt = separation of waves at point x on the line
$\Delta t'$ = separation of waves at point x' on the line
t = time of passage of the fast wave

Then

$$(x' - x) = v_1 t = v_2 (t + \Delta t' - \Delta t)$$

$$v_2 = \frac{x' - x}{t + \Delta t' - \Delta t} = \frac{x' - x}{(x' - x)/v_1 + \Delta t' - \Delta t}$$

$$= \frac{(x' - x)\, v_1}{(x' - x) + v_1 (\Delta t' - \Delta t)}$$

In figure 1 the 2 sets of oscillograms were taken 12.7 miles apart, and the separation of the wave components are $\Delta t' = 15$ at tower B and $\Delta t = 7$ at tower A. Therefore

$$v_2 = \frac{12.7 \times 5280 \times 985}{12.7 \times 5280 + 985\,(15 - 7)} = 880 \text{ feet per microsecond}$$

Thus the slow wave travels at approximately 89 per cent the velocity of light.

It should be remarked that if the main and induced surge oscillograms are not properly matched with respect to their starting points, spurious oscillations and residues will appear in the derived multivelocity components and the front of the fast wave component will not coincide with the front of the main surge. This point is illustrated in figure 2. The top sketch shows a perfect match of the main surge e_1 and the induced surge e_2. The component waves derived from the "$^2/_3$" rule are smooth and definite. The middle sketch shows the same e_1 and e_2, but mismatched by $-\epsilon$. The derived f_1 component no longer coincides on its front with the main surge, while the f_2 component now shows a long undulating toe; several major errors have crept into the analysis: The f_1 component is too high and has too long a front, the f_2 component is too low and has a spurious toe, and the indicated wave separation Δt is too much. The bottom sketch shows the same e_1 and e_2, but mismatched by $+\epsilon$. The derived f_1 component now contains a ripple and has a reduced crest, and its front does not coincide with that of the main surge; f_2 is led by a reversed polarity loop. Since the exact start of a surge is usually indefinite on the oscillograms by a microsecond or more, matching by trial so that the spurious residues vanish and the front of the derived fast wave com-

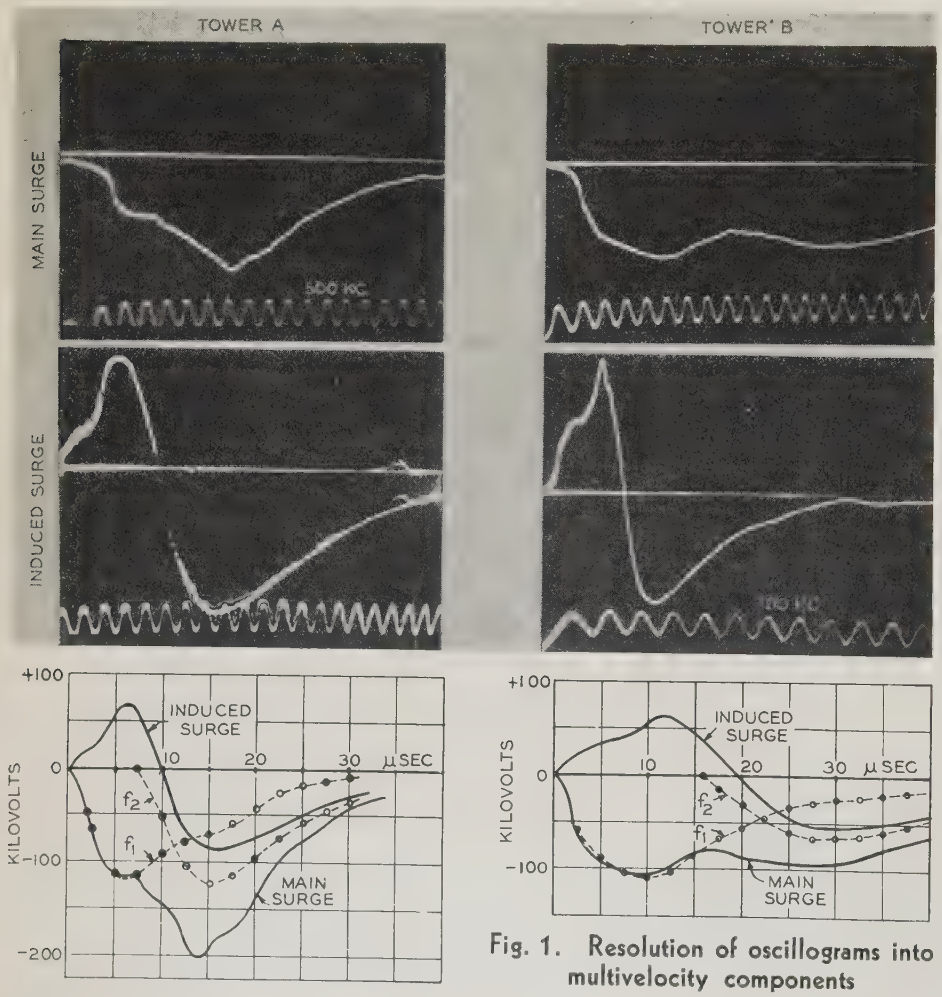

Fig. 1. Resolution of oscillograms into multivelocity components

ponent coincides with the front of the main surge up to the point where the slow wave component appears, seems to be the best criterion for proper matching of the oscillograms.

Effect of Ground Wires

In appendix II it is shown that a ground wire does not add to the number of multivelocity components, but it does change the relative velocities of those components. The proximity of a ground wire lowers the potential at which corona can appear on an adjacent conductor, and the corona envelope is of larger diameter for a given potential; but the greater the diameter of the corona envelope the slower the slow wave,[1] and consequently the more the separation of the components for a given distance of travel. For a surge the voltage of which is below the corona level, however, the slow wave moves faster if a ground wire is present, because the effective inductances are decreased; thus the separation of the waves is not as much in a given distance of travel. From these considerations it would be concluded that the attenuation of a given surge is initially more, but finally less, with a ground wire than without, and this is exactly the situation from field tests.[4] The point is illustrated in figure 3 for both positive and negative surges. It may be noticed that for the negative surge the transition from more to less attenuation occurs at a higher potential than for the positive surge. This is in line with the fact that corona appears at a lower positive than negative voltage. However, the differences of attenuation with and without ground wires are not great.

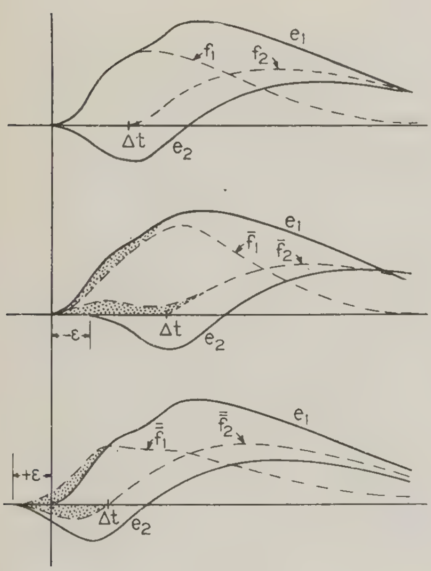

Fig. 2. Spurious oscillations and residues in the multivelocity components resulting from displacement between main and induced surges

Simplified Theory Adequate for Practical Purposes

Multivelocity waves owe their origin to corona and to ground currents in a resistive earth. The effect of a resistive earth on the propagation of steady state currents over a system of parallel wires has been treated rigorously by J. R. Carson in several excellent articles,[5,6] and for such steady state solutions the "modes of propagation" are the counterpart of what are called here the "multivelocity components" of a surge. When the propagation of a single surge over a system of parallel wires is considered, as contrasted to periodic currents, a rigorous mathematical solution based upon Maxwell's equations not only becomes too formidable, but even if it could be achieved the chances are that it would be too complicated for ordinary use. In this paper and its forerunners[1,2,3] only simple multiconductor traveling wave theory is used, and the different velocities appear as a consequence of the corresponding inductance and capacitance coefficients being based upon different ground levels and conductor diameters. That this simplified theory is quite adequate and sufficiently exact for purposes of this paper is now supported by 3 groups of evidence:

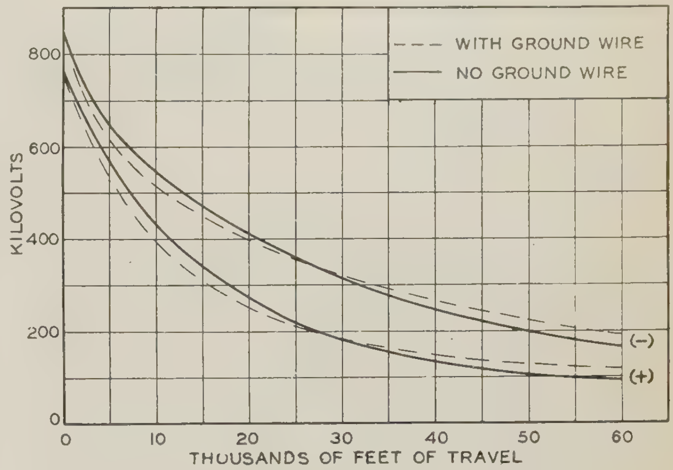

Fig. 3. Effect of ground wire on attenuation of traveling waves

1. A great variety of peculiar and otherwise unexplainable distortion and attenuation effects observed on cathode ray oscillograms of surges were shown in the first of these papers[1] to be described qualitatively by the theory.

2. From experimentally determined circuit constants it was shown in the third of these papers[3] that multivelocity waves calculated with these constants checked the cathode ray oscillograms for all the reflections calculated.

3. In the present paper it is shown that the multivelocity theory offers a simple rule for resolving the oscillograms into multivelocity components and that the components so derived obviously fit the oscillograms.

Appendix I—Resolution of Oscillograms of Surges Into Multivelocity Components

The object of this appendix is to derive the relationships existing between multivelocity wave components on a multiconductor transmission system, and to formulate the rules for resolving oscillograms of the main and induced surges at a given point into their multivelocity components. The methods of tensor analysis[7] are found most convenient for this purpose.

In general[1] there will be as many multivelocity components as there are separate line conductors, although for certain conditions of symmetry, one or more velocities may become redundant. As shown in appendix II, ground wires do not add to the *number* of

velocities present, but their presence does change the relative magnitudes of these velocities. Therefore, a system comprising n line conductors is specified with respect to its multivelocity components by n simultaneous equations,[1] which can be written in tensor notation as

$$e_r = a_r^s \left[f_s (x - v_s t) + F_s (x + v_s t) \right] \qquad (1)$$

$$i^r = Y^{rs} \left[f_s (x - v_s t) - F_s (x + v_s t) \right] \qquad (2)$$

where, in accordance with that notation, r may be any number from 1 to n, and the dummy index s implies summation with respect thereto. The a_r^s coefficients involve all the surge impedances and admittances of the system and are to be determined from the general differential equations. Now the general differential equations of an n conductor system are[1]

$$0 = \left(I_r^s \frac{\partial^2}{\partial t^2} - \delta_r^s \frac{\partial^2}{\partial x^2} \right) e_s \qquad (3)$$

where $I_r^s = L_{rw} K^{sw}$, and δ_s^r is the Kronecker delta ($\delta_r^s = 1$ for $s = r$, $\delta_r^s = 0$ for $s \neq r$).

Substituting any wave component of equation 1 in equation 3 there results (for the forward waves):

$$\left(I_r^s \frac{\partial^2}{\partial t^2} - \delta_r^s \frac{\partial^2}{\partial x^2} \right) a_s^Q f_Q (x - v_Q t) = 0 \qquad (4)$$

Equating coefficients of waves with like velocities, there is obtained for the system of equations defining a_r^s and v_Q

$$(I_r^s - v_Q^{-2} \delta_r^s) a_s^Q = c_r^s a_s^Q = 0 \qquad (5)$$

where in equations 4 and 5 the summation convention with respect to q is dropped by writing it as a capital index Q. The velocities are given by

$$|c_r^s| = 0 \qquad (6)$$

This makes it possible to find all the a_r^s coefficients in terms of any number n of them taken arbitrarily, and it may be assumed that

$$a_1^r = 1 \qquad (7)$$

Equations 4 and 5, however, imply that the inductance and capacitance coefficients L_{rs} and K^{rs} are known so as to calculate the $I_r^s = (L_{rw} K^{sw})$. Suppose, however, that only the values of e_r are known as cathode ray oscillograms at different points along the line. Then since equation 1 contains $(n^2 - n)$ unknown coefficients a_r^s, n unknown functions f_s, and n unknown velocities v_s, it is clear that the measurement of all values of e_r at a given point on the circuit is not sufficient to evaluate all these unknowns; nor would measurements of the currents obviate the difficulty. Perhaps by taking oscillograms at enough points along the line, the unknowns could be evaluated; but practically this would involve attenuation and distortion effects not included in these equations, and of course would necessitate using many oscillographs. There is, however, a condition that can be taken advantage of to secure a reasonable solution: In order for appreciable distortion to develop as a result of the separation of multivelocity components, the waves must travel so far that the line can be regarded as completely transposed. For a completely transposed line

$$\left. \begin{aligned} L_{rr} &= L \\ L_{rs} &= L' \qquad r \neq s \end{aligned} \right\} \qquad (8)$$

The corresponding relationships between the capacitance coefficients may be unsymmetrical because of corona. Suppose that the surge is impressed on conductor 1 (a direct stroke to that conductor). Then the corona will be confined principally to that conductor, and, consequently, for a completely transposed line:

$$\left. \begin{aligned} K^{rr} &= K^{11} \text{ if } r = 1 \\ K^{rr} &= K \text{ if } r \neq 1 \\ K^{rs} &= K' \text{ if } r \neq S \end{aligned} \right\} \qquad (9)$$

Therefore

$$\begin{aligned} I_r^s = L_{rw} K^{sw} &= L_{rr} K^{sr} + (L_{rw} K^{sw})_{w \neq r} \\ &= (L - L') K^{sr} + L' \sum_{w=1}^{n} K^{sw} \\ &= K^{ss} L' + (n-1) K'L' + (L - L') K^{sr} \end{aligned} \qquad (10)$$

Hereby

$$\left. \begin{aligned} I_1^1 &= L\,K^{11} + (n-1) L'K' \\ I_r^r &= L\,K + (n-1) L'K' &&= I,\ r \neq 1 \\ I_r^1 &= L'\,K^{11} + L\,K' + (n-2) L'K' &&= I'',\ r \neq 1 \\ I_r^s &= L'\,K + L\,K' + (n-2) L'K' &&= I',\ r \neq s,\ s \neq 1 \end{aligned} \right\} \qquad (11)$$

Solving equation 5 subject to equation 7,

$$a_s^Q = \frac{-A_s^r}{|b_r^s|} I_r^1 \qquad s \neq 1, r \neq 1 \qquad (12)$$

where $|b_r^s|$ is the determinant $|c_r^s|$ with the first row and first column corresponding to $r = 1$ and $s = 1$, respectively, deleted, and A_s^r is the cofactor of b_r^s. However, by equation 11 all values of I_r^1 are equal, and the determinant $|c_r^s|$ has all elements along the diagonal equal to $b_r^r = b$ while all its other elements are equal to $b_r^s = b'$. Consequently,

$$a_s^Q = \frac{-I''}{I + (n-2) I' - v_Q^{-2}} = a_Q = \text{constant for } s \neq 1 \qquad (13)$$

Now by equation 11 the elements of equation 6 are of 4 types:

$$\left. \begin{aligned} &c_1^1 \\ &c_r^1 = c'' && r \neq 1 \\ &c_r^s = c' && s \neq 1, r \neq s \\ &c_r^r = c && r \neq 1 \end{aligned} \right\} \qquad (14)$$

and therefore equation 6 becomes the bordered determinant

$$|c_r^s| = \begin{vmatrix} c_1^1 c' \ldots\ldots\ldots c' \\ c'' c \ldots\ldots\ldots c' \\ \ldots\ldots\ldots\ldots \\ c'' c' \ldots\ldots\ldots c \end{vmatrix}$$

$$= (c - c')^{n-2} \{ c_1^1 [c + (n-2) c'] - (n-1) c' c'' \} = 0 \qquad (15)$$

Substituting $c_r^s = (I_r^s - \delta_r^s v^{-2})$ there results

$$(I - I' - v^{-2})^{n-2} \{ (v^{-2})^2 - [I_1^1 + I + (n-2) I'] v^{-2} + [I_1^1 I + (n-2) I_1^1 I' - (n-1) I' I''] \} = 0 \qquad (16)$$

This equation has 3 nonredundant roots for v^{-2}, that is, only 3 velocities are present. By virtue of equations 13 and 16 the conclusion is reached that on the completely transposed transmission line of any number of conductors and with corona on one conductor only: (1) all waves induced on adjacent conductors are alike, and (2) only 3 velocities are present. Consequently equation 1 may be written

$$\left. \begin{aligned} e_1 &= f_1 (x - v_1 t) + f_2 (x - v_2 t) + f_3 (x - v_3 t) \\ e_r &= a_1 f_1 (x - v_1 t) + a_2 f_2 (x - v_2 t) + a_3 f_3 (x - v_3 t),\ r \neq 1 \end{aligned} \right\} \qquad (17)$$

where f_1, f_2, f_3 may differ in magnitude, shape, and sign.

At the point of inception of the surge all the wave components have the same shape. To prove this, let $E(t)$ be impressed on conductor 1 at the point where $x = x_0$. The current in all other conductors at that point must be zero. Then by equations 1 and 2,

$$E(t) = a_1^s f_s (x_0 - v_s t) \qquad (18)$$

$$0 = Y^{rs} f_s (x_0 - v_s t) \qquad r \neq 1 \qquad (19)$$

Equation 19 may be solved for f_s in terms of f_1, and these results inserted in equation 18 yield

$$E(t) = a f_1 (x_0 - v_1 t) \qquad (20)$$

where a involves a_1^s and Y^{rs}; and thus all components, for both forward and backward waves, have identically the same shape, and therefore at the point of inception

$$e_r = a_r^s f (x - v_s t) \qquad (21)$$

The waves of different velocity experience very different attenuations and distortions from corona and ground current losses. Consequently, although alike at the point of origin, they soon become dissimilar. For this reason equations 17 must be retained as the general description.

Equations 17 cannot be simplified further, as there are no fixed relationships independent of the line constants among the *a coefficients*. These equations contain 9 unknowns: 3 *velocities* (v_1, v_2, v_3),

3 wave *shapes* (f_1, f_2, f_3), and 3 *coefficients* (a_1, a_2, a_3). There are, however, only 2 given data, the oscillograms of e_1 and e_2. Nevertheless, from an inspection of these oscillograms it is often possible to identify the components by the cut-and-try method.

Now in the special case that the surges are at a voltage below the corona level, $K^{11} = K$ and by equation 11, $I_1^1 = I$ and $I'' = I'$. Hence by equation 16 the velocities are

$$\left.\begin{aligned} v_1^{-2} &= (I - I') \\ v_2^{-2} &= I + (n-1)I' \end{aligned}\right\} \quad (22)$$

and equation 13 then gives

$$\left.\begin{aligned} a_1 &= \frac{-I'}{I + (n-2)I' - (I - I')} = \frac{-1}{(n-1)} \\ a_2 &= \frac{-I'}{I + (n-2)I' - I - (n-1)I'} = 1 \end{aligned}\right\} \quad (23)$$

Thus equations 17 for a completely transposed transmission line with surge voltage below the corona level become

$$\left.\begin{aligned} e_1 &= f_1 + f_2 \\ e_r &= -\frac{1}{(n-1)} f_1 + f_2 \qquad r \neq 1 \end{aligned}\right\} \quad (24)$$

Solving these equations,

$$\left.\begin{aligned} f_1 &= \frac{n-1}{n}(e_1 - e_2) = \text{fast wave} \\ f_2 &= \frac{e_1 + (n-1)e_2}{n} = \text{slow wave} \end{aligned}\right\} \quad (25)$$

These equations give the multivelocity components directly from the oscillograms of the main surge e_1 and the induced surge e_2 of a completely transposed line when the surge voltages are below the corona level.

Appendix II—Effect of Ground Wire on Multivelocity Waves

To determine the effect of a ground wire on multivelocity waves it is sufficient to consider the ground wire as perfectly grounded throughout its length, because differences in velocity are not very great and the wave separation that can take place in a few spans is quite insignificant.

It may be recalled[8] that the conventional differential equations of a multiconductor line are set up on the assumption that there is a conductance between wires and from wires to ground; but for perfectly grounded wires the conductances to ground are infinite. When these limiting values are inserted in the general differential equations, it is found that the order of the equations is reduced by twice the number of ground wires. If the only conductances present are those infinite conductances to ground of the ground wires, then the differential equation still is satisfied by pure waves, but the number of possible independent velocities is equal to the number of ungrounded wires. Therefore, while ground wires do not add to the *number* of velocities, they do change the numerical *values* of these velocities, because the inductance and capacitance coefficients of the ground wires become involved in the equation.

The difficulties attendant on inserting infinities in the differential equation can be circumvented by considering the number of conductors in the system equal to the number of line wires, and introducing equivalent inductances to take care of the ground wires. In general, if there are m line conductors numbered from 1 to m inclusive and $n - m$ ground wires, numbered from $m + 1$ to n inclusive, the flux linking each conductor, per unit length, is

$$\phi_r = \sum_{s=1}^{n} L_{rs} i^s = \sum_{s=1}^{m} L_{rs} i^s + \sum_{m+1}^{n} L_{rs} i^s, \quad (r = 1 \text{ to } r = n) \quad (26)$$

However, for the ground wires $\phi_r = 0$, and therefore

$$\sum_{m+1}^{n} L_{rs} i^s = -\sum_{1}^{m} L_{rs} i^s, \qquad (r = m+1 \text{ to } r = n) \quad (27)$$

From this latter set of $n - m$ simultaneous equations the currents i_{m+1} to i_n inclusive are determined in terms of the currents i_1 to i_m, and then equation 1 reduce to

$$\phi_r = \sum_{s=1}^{m} L'_{rs} i^s \qquad (r = 1 \text{ to } r = m) \quad (28)$$

where L'_{rs} is the equivalent inductance for use in the traveling wave equations.

For example, if there are 3 line conductors and 2 ground wires, by equation 27,

$$\begin{aligned} L_{44} i^4 + L_{45} i^5 &= -(L_{41} i^1 + L_{42} i^2 + L_{43} i^3) \\ L_{54} i^4 + L_{55} i^5 &= -(L_{51} i^1 + L_{52} i^2 + L_{53} i^3) \end{aligned}$$

from which

$$i^4 = \frac{(L_{45}L_{51} - L_{55}L_{41})i^1 + (L_{45}L_{52} - L_{55}L_{42})i^2 + (L_{45}L_{53} - L_{55}L_{43})i^3}{L_{44}L_{55} - L_{45}^2}$$

$$i^5 = \frac{(L_{45}L_{41} - L_{44}L_{51})i^1 + (L_{45}L_{42} - L_{44}L_{52})i^2 + (L_{45}L_{43} - L_{44}L_{53})i^3}{L_{44}L_{55} - L_{45}^2}$$

Equation 26 then gives

$$\begin{aligned} \phi_r &= L_{r1}i^1 + L_{r2}i^2 + L_{r3}i^3 + L_{r4}i^4 + L_{r5}i^5 \\ &= \left[L_{r1} + \frac{L_{r4}(L_{45}L_{51} - L_{55}L_{41}) + L_{r5}(L_{45}L_{41} - L_{44}L_{51})}{L_{44}L_{55} - L_{45}^2}\right] i^1 \\ &+ \left[L_{r2} + \frac{L_{r4}(L_{45}L_{52} - L_{55}L_{42}) + L_{r5}(L_{45}L_{42} - L_{44}L_{52})}{L_{44}L_{55} - L_{45}^2}\right] i^2 \\ &+ \left[L_{r3} + \frac{L_{r4}(L_{45}L_{53} - L_{55}L_{43}) + L_{r5}(L_{45}L_{43} - L_{44}L_{53})}{L_{44}L_{55} - L_{45}^2}\right] i^3 \\ &= L'_{r1}i^1 + L'_{r2}i^2 + L'_{r3}i^3 \end{aligned}$$

which is the same form as equation 28.

The capacitance coefficients K are calculated in the usual way, taking all conductors into account, ground wires as well as line wires.

The traveling wave equations then are set up as though the ground wires were absent, but using the equivalent inductances as given by equation 28 and the capacitance coefficients associated with each line wire.

The effect of a ground wire for surge voltages below the corona level is to *increase* the velocity of the slow component. For this reason attenuation is less with a ground wire (the 2 waves do not separate as rapidly), once the surge voltage gets below the corona level. While the surge voltage is high, however, corona forms on the conductor, and the proximity of the ground wire intensifies this corona, thereby increasing the effective diameter of the conductor and with it the capacitance; and so the velocity of the slow component is less than it would be if the ground wire were not present.

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Power and Energy, Positive and Negative

An attempt to clarify the definitions of power, power factor, and related quantities has led to an intensive study which has included a large number of actual test measurements of the quantities involved. Measurements of positive and negative power, and the use of a "power ratio" based upon these 2 quantities are proposed, particularly for circuits in which the voltages and currents are nonsinusoidal or unbalanced. Positive and negative energy and power can be measured approximately with apparatus commercially available.

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THE development of electrical engineering has called for a continuous readjustment of the concepts that have previously been in use. Up to the time of the introduction of alternating current, the law of Ohm (I equals E/R) was looked upon as unalterable, and as dependable as the laws of the Medes and Persians. This law, upon the introduction of alternating current, had to be modified, i. e. (I equals E/Z). Early a-c theory, for the sake of simplicity, was delimited by the assumption of sinusoidal waves. Various conclusions that were true on the basis of sinusoids were no longer valid when harmonics were present. Upon the introduction of polyphase alternating current, the assumption of a balanced load was the most attractive method of gaining simplicity. Eventually Fortescue's method of symmetrical components came into use for handling unbalanced polyphase circuits.

One of the concepts which in its definition has not kept pace with the development of the art, is power factor. In the good old days of direct current, power was the product of current and voltage. This constituted a law as fundamental as the law of Ohm. To deny this was an act of heresy. In fact, a past-president of the British Institution of Electrical Engineers, prior to 1884 (see page 423 of April 1922, I.E.E. *Transactions*) wrathfully said to an engineer who had experimental proof that in alternating current practice the power was not always equal to the product of the voltmeter and ammeter readings, "You're a liar." Investigation soon proved that the past-president was wrong and the concept of power factor was born. In all these 50 years the definition of power factor has been a difficult problem. The concept, which is exceedingly simple and useful in sinusoidal cases, has become more and more involved as engineers have tried to modify it to cover nonsinusoidal cases and unbalanced polyphase systems. Even to this day engineers have not got over the easily demonstrable fallacy of multiplying voltmeter readings by ammeter readings in nonsinusoidal cases. This product is absolutely meaningless. Definitions of power factor based upon this product are equally meaningless. A desperate effort to clarify and rationalize the situation so far as the definitions of power and related quantities are concerned appeared in a recent paper by H. L. Curtis and F. B. Silsbee.[1] This paper was liberally discussed with no eventual solution in sight for clarifying the definition of power factor for universal application.

The authors of the present paper have been struggling with the same difficulties for a number of years. Finally, in view of the fact that the situation as to definition and measurement of power factor and re-

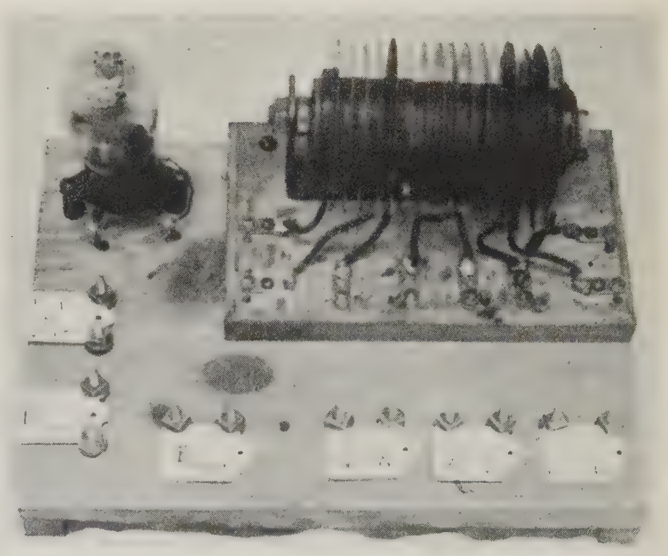


Fig. 1. Apparatus used for measuring positive and negative power and energy

W_1 W_1—Binding posts for connecting the current terminals of the positive wattmeter or watt-hour meter

W_2 W_2—Binding posts for connecting the current terminals of the negative wattmeter or watt-hour meter

LL—Binding posts for connecting the alternating current before rectification

E_{in}—Binding posts for the alternating voltage before rectification

E_{out}—Rectified potential supply for instruments

A paper recommended for publication by the A.I.E.E. committee on instruments and measurements, and scheduled for discussion at the A.I.E.E. winter convention, New York, N. Y., Jan. 28–31, 1936. Manuscript submitted Apr. 21, 1935; released for publication Oct. 8, 1935.

1. For all numbered references, see list at end of paper.

active factor seems to have reached an impasse, some new method of approach was sought for. This line of attack which has been under way since May 1934 involved an intensive study of positive and negative power, and of positive and negative energy. After this paper was partly written, it was found that certain others had been working along a similar line, investigating the properties of positive and negative power. Notable among these were Messrs. Knowlton, Pratt, and Karapetoff. Since some of this material has already been presented,[2,3,4] it will be used as a starting point, principally that part appearing in the latter half of reference 2. Positive and negative power are entities which are measurable by apparatus to be described later. Whenever the voltage and the current are positive, the power will be positive. If either of them is negative and the other positive, the power will be negative. If both are negative, the power is positive. Integration of the positive power alternations in respect to time gives positive energy. The statement of this definition and the method of measurement of these quantities are perfectly general and not limited to sinusoidal conditions.

For the purpose of charging the customer of electricity, an obvious method would be to itemize his positive energy and negative energy, bill him at so much per kilowatt-hour of positive energy and rebate him at so much per kilowatt-hour of negative energy. This would represent a simple method which avoids entirely the use of the power factor term. In contrast with this proposal the present practice is to meter the net energy (positive − negative) and apply a penalty rate based upon the readings of a power factor meter. In either case 2 meters are required.

The object of this paper is to make a thorough study of positive and negative power and of positive and negative energy. The first part of the paper will be used to describe the apparatus in its final form without detailing the various preliminary developmental steps. The next portion will present the various groups of experimental results, which will be divided into 2 parts: (I) energy data; and (II) power data. Finally, some interpretations of the results and conclusions will be presented.

Apparatus

Figure 1 represents a photograph of the apparatus. Figure 2 represents a wiring diagram, including the apparatus of figure 1 and the accompanying instruments. The objectives in assembling this apparatus were: (1) supply of positive potential and current to one instrument, which will be designated as the positive instrument; and (2) supply of positive potential and negative current to the second instrument which will be designated as the negative instrument. It is necessary ordinarily to base measurements upon only one positive alternation of potential since the second negative alternation of potential repeats the first. In those rare and exceptional cases where the preceding statement is not true, a reversing switch should be included in the potential circuit, so that 2 positive and 2 negative readings may be obtained. The instruments used were electrodynamometer type wattmeters for power measurements and d-c type watt-hour meters for energy measurements. For rectifying the potential a type 80 vacuum tube was used. For rectifying the current a 4-part copper-oxide rectifier unit, arranged in the familiar bridge circuit, is used. A wide range of current values may be measured by interposing a current

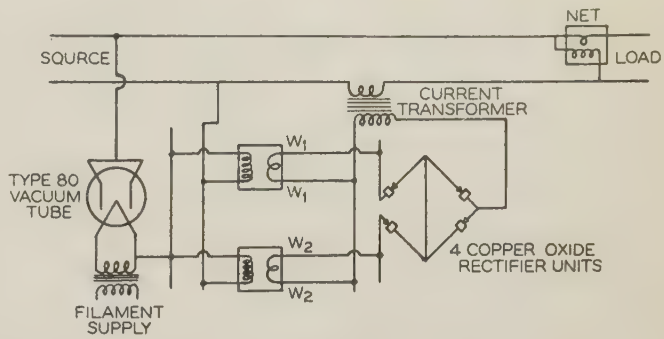


Fig. 2. Wiring diagram of apparatus for measuring positive and negative power and energy, with the necessary meters

transformer between the line and the copper oxide rectifier unit. The diagram of figure 2 is applicable to single phase tests. By use of a 2 element wattmeter or watt-hour meter and 2 such pieces of apparatus as shown in figure 1, measurements may be made on 3 phase circuits. In the present experiments a special metering panel was used to make 1 wattmeter serve in the familiar 2 wattmeter method.

Experimental Results on Positive and Negative Energy in Single Phase Cases

In these tests since only one d-c watt-hour meter was available, the watt-hour meter is placed across W_1W_1 (figures 1 and 2) for positive energy measurements and across W_2W_2 for negative energy measurements. A dynamometer type wattmeter was connected to W_2W_2 during the positive energy measurements and to W_1W_1 during the negative energy measurements. The watt-hour meter, a 10 ampere 110/220 volt d-c meter, was first calibrated on an ordinary d-c supply. Next the watt-hour meter potential circuit was corrected for the drop in the type 80 tube by shunting a decade resistance box across the resistance unit in the armature circuit, the resistance being set at 1,400 ohms. After this adjustment, a test was made to determine the positive and negative energy supplied to a load of 2 carbon lamps in parallel.

Results of the first test were as follows:

Positive energy by the watt-hour meter; 6 revolutions in 108 seconds; $6 \times {}^1/_2 \times 2$ equals 6 watt-hours. (In this product, "$^1/_2$" is the meter constant and "2" is used because the meter is in action only every other alternation.)

Negative energy by the watt-hour meter; 0 revolutions in 108 seconds; zero watt-hours.

Net energy by watt-hour meter; 6 watt-hours.

Net energy by a wattmeter connected to the load in the ordinary way; reading 200 watts; $200 \times 1/3{,}600 \times 108$ equals 6 watt-hours.

A second test was made on a condensive load of about 15 per cent power factor. The load consisted of 40 2 microfarad paper condensers in parallel with a resistance load.

Positive energy by watt-hour meter; 8 revolutions in 150 seconds; $8 \times 1/2 \times 2$ equals 8 watt-hours.

Negative energy by watt-hour meter; 5 revolutions in 150 seconds; $5 \times 1/2 \times 2$ equals 5 watt-hours.

Net energy by watt-hour meter; $8 - 5$ equals 3 watt-hours.

Net energy by wattmeter; reading 67 watts; $67 \times 1/3{,}600 \times 150$ equals 2.79 watt-hours.

A third test was made on a condensive load of about 26 per cent power factor.

Positive energy; 13 revolutions in 412 seconds; $13 \times 1/2 \times 2$ equals 13 watt-hours.

Negative energy; 5 revolutions in 412 seconds; $5 \times 1/2 \times 2$ equals 5 watt-hours.

Net energy; $13 - 5$ equals 8 watt-hours.

Net energy by wattmeter; reading 66.6 watts; $66.6 \times 1/3{,}600 \times 412$ equals 7.62 watt-hours.

A fourth test made on an inductive load of about 39 per cent power factor.

Positive energy; 18.5 revolutions in 493 seconds; 18.5 watt-hours.

Negative energy; 5 revolutions in 493 seconds; 5 watt-hours.

Net energy; 13.5 watt-hours.

Net energy by wattmeter; reading 88.1 watts; $88.1 \times 1/3{,}600 \times 493$ equals 12.1 watt-hours.

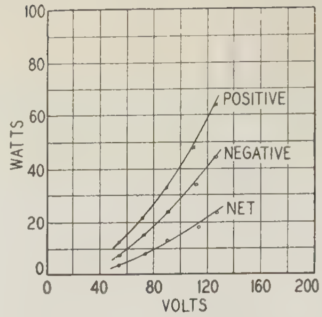
It should be noted that in the last 3 tests the net energy as measured by the watt-hour meter was 7, 5, and 10 per cent too large, respectively, but correct for the noninductive load. For discussion of the error characteristics of commutator type watt-hour meters operating on alternating currents, see reference 5.

Experimental Results on Positive and Negative Power in Single Phase Cases

In these tests 2 ordinary dynamometer type wattmeters were used across W_1W_1 and W_2W_2 of figures 1 and 2, to measure positive and negative power. A third dynamometer wattmeter was connected to measure the net power taken by the load. In figure 3, positive, negative, and net power are plotted for a particular case. No attempt was made to correct the voltage applied to the W_1W_1 and W_2W_2 wattmeter potential coils for the drop in the type 80 tube. Consequently the positive and negative power values shown in figures 3 to 6 are low by 15 to 20 per cent. The tests of figures 3 to 6 represent the variation of positive and negative power as a variable sinusoidal potential is applied to various load circuits.

Power Ratio

Although the values of power shown in figures 3 to 6 are only approximate, the ratio of W_1W_1/W_2W_2 will have but little error, and may be measured accurately without correcting for the tube drop, or for such small errors as go with the use of copper oxide rectifier units.[6] This ratio for the purpose of the present paper is defined as the "power ratio" and is equal to (positive power)/(negative power). It is a very convenient index number to use in the diagnosis of the characteristics of a load circuit. For single phase sinusoidal cases this ratio has a fixed relation to power factor, thus



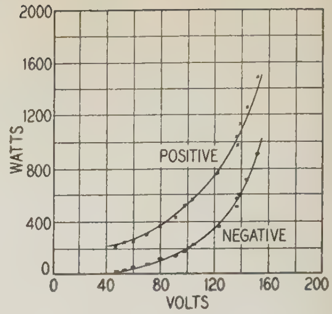
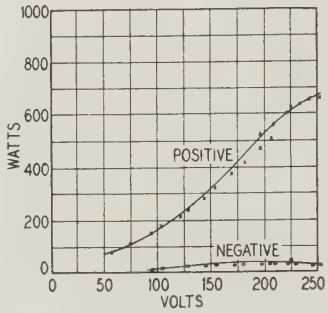


Fig. 3 (left)—For a long solenoid with iron core

Fig. 4 (right)—For a 3-phase 7.5-horsepower 110-volt induction motor operating single phase



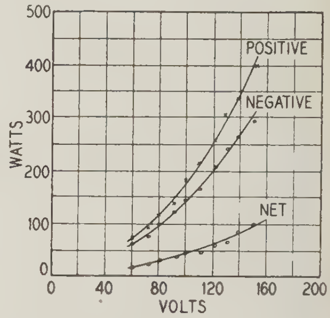


Fig. 5 (left)—For a 2-horsepower single-phase 220-volt capacitor induction motor

Fig. 6 (right)—For a single phase transformer, 3 kva 110/2,200 volts supplying a capacitance load

Figs. 3–6. Positive and negative power curves with variable sinusoidal electromotive force applied. Net power by an indicating wattmeter

$$\frac{\text{positive power}}{\text{negative power}} = \frac{(\pi - \phi)\cos\phi + \sin\phi}{\phi\cos\phi - \sin\phi}$$

where ϕ is the phase angle between the current and the voltage. This relation is plotted versus power factor ($\cos\phi$) in the curve of figure 7.

It was arranged to check the accuracy of the apparatus of figures 1 and 2 against this curve for a wide range of power factors, by providing a variable phase angle through the medium of a phase shifter. A voltage of approximately 110 volts was maintained across the E_{in} terminals of figure 1 and across the potential coil of a standard wattmeter. A current of approximately 1.33 amperes was maintained in all the wattmeter current coils. For each setting of the phase shifter, readings were made of the standard voltmeter, ammeter, wattmeter, W_1W_1, and W_2W_2 values. The observations, obtained in this test, provide values of power factor and power ratio, which are plotted as crosses in figure 7 and which show a very close check with the theoretical curve. Equipped with this rapid and fairly accurate method

of diagnosis, a wide range of apparatus was studied under a variety of experimental conditions. As a sample of such testing note the circles on figure 7. In this test readings of voltmeter, ammeter, and wattmeter fix the power factor. The readings W_1W_1 and W_2W_2 fix the power ratio. Figures 5 and 7 (circles) were derived from the same test. Other tests were carried out to fix the relation of power ratio to power factor for: (*a*) an ordinary 3 phase induction motor operating single phase; (*b*) an ordinary 3 phase induction motor operating 3 phase, with test data taken between one line and neutral; and (*c*) a long solenoid with a variable length iron core. Since in these tests there was little departure from sinusoidal shape in either the current or voltage wave, the observations substantially checked the single phase curve of figure 7.

To illustrate what happens when the waves are not sinusoidal, an electromotive force with a 100 per cent fifth harmonic was used to supply a variable power factor circuit consisting of a resistance in parallel with a condenser. The results of this test are shown in figure 8 (crosses), where the observations are consistently off the theoretical curve. It is safe to conclude that barring errors in reading, a departure from the curve of figure 7 is a sure indication of the presence of nonsinusoidal current or voltage waves. Furthermore, the failure of an observation to lie on the curve is an indication that the simple definition of power factor as the wattmeter reading divided by the product of the voltmeter and ammeter readings is not applicable to that case. Figure 8 (circles) is a further illustration of a nonsinusoidal case. A 500 cycle homopolar alternator with a pronounced fifth harmonic is used to supply power to a variable power factor circuit consisting of a resistance in parallel with a condenser. Here again the observations are at wide variance with the theoretical curve. Figure 8 (triangles) is a final example of nonsinusoidal cases, in which an incandescent lamp load is paralleled by a transformer supplying 2,200 volt capacitors, and in which the source of supply is sinusoidal.

After several hundred measurements of power ratio for a wide variety of cases, the authors have become convinced of the utility of this ratio for describing the characteristics of a circuit. It is superior to the term power factor on the score of: (*a*) simplicity of conception, involving the simple idea of the ratio of 2 numbers in contrast with the concept of a trigonometrical function; (*b*) easy and accurate measurement in all cases, sinusoidal or nonsinusoidal; and (*c*) definition not influenced by the presence of harmonics.

The ratio of positive power to negative power has for different power factors, values as follows:

Power factor	0	10	20	30	40	50
Power ratio	1	1.37	1.89	2.64	3.78	5.6
Power factor		60	70	80	90	100
Power ratio		8.73	15	30.4	71.4	∞

For different angles between voltage E and current I, this ratio has values as follows:

Angle, degrees	90	80	70	60	50
Power ratio	1	1.735	3.06	5.6	10.9
Angle, degrees	40	30	20	10	0
Power ratio	23.3	59	214.5	1,930	∞

Possibly some might take exception to a ratio which has a value of infinity at unity power factor. If such a feeling exists, the reciprocal ratio, or negative power divided by positive power might be suggested, in which case the variation of the power ratio would lie between unity and zero.

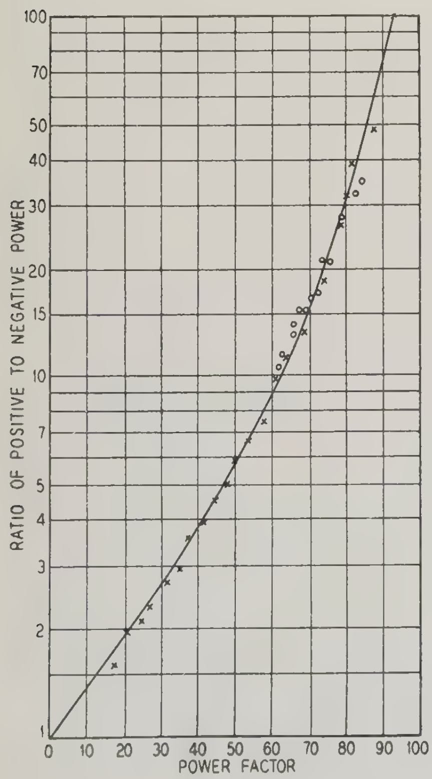


Fig. 7 (left). Relationship between power ratio and power factor, for single phase cases. Sinusoidal cases

Crosses—Measurements of the relation between power ratio and power factor obtained with the test apparatus. These points show a close check with the theoretical curves

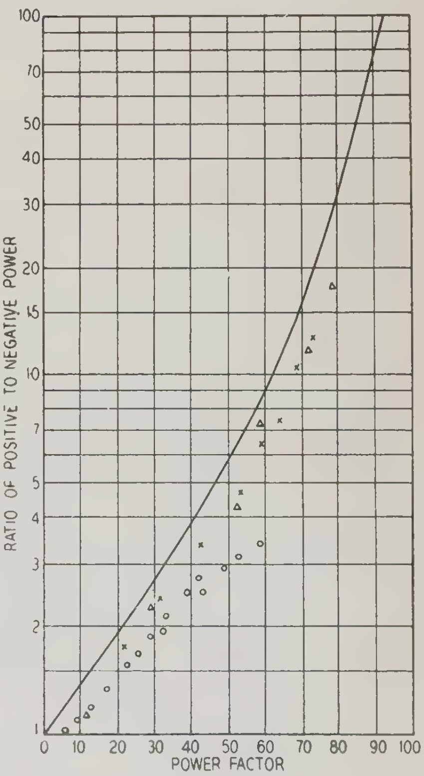
Circles—Measurements with a single-phase 2-horsepower capacitor motor, running light, with varying sinusoidal impressed electromotive force

Fig. 8 (right). Results of 3 different tests on the relationship between power ratio and power factor, for single phase cases. Nonsinusoidal cases.

Crosses—Measurements with a 100 per cent fifth harmonic electromotive force impressed

Circles—Measurements with a homopolar 500 cycle alternator, supplying a resistance-capacitance load

Triangles—Measurements with a constant sinusoidal electromotive force impressed on a single phase transformer supplying 2,200 volts capacitors, the transformer paralleled by a variable resistance load



Experimental Results on Positive and Negative Power in 3 Phase Circuits

The power ratio, having proved in the opinion of the authors such a valuable index in single phase cases, was next extended in its application to 3 phase measurements. In 3 phase measurements the 2 wattmeter method is commonly employed. The 2 wattmeters in this method have applied to them, currents and voltages which on noninductive load are 30 degrees out of phase. This peculiarity of the 2 wattmeter method leads to a different curve, relating power factor and power ratio in 3 phase measurements. Thus with a unity power factor balanced 3 phase load, the wattmeters have impressed upon them currents and voltages which have a phase angle between them of 30 degrees. Consequently, the ratio of positive to negative power at unity power factor and with 3 phase loads corresponds to an 86.6 per cent power factor with single phase loads. This correspondence between the single phase and 3 phase curves may be seen in figure 9, where a horizontal line joins the unity power factor point on the 3 phase curve with the 86.6 per cent power factor point on the single phase curve at a power ratio value of 59.

For some other value of power factor, say 70 per cent, ϕ equals 45 degrees and 34 minutes, the 2 wattmeters experience phase angles of 75 degrees, 34 minutes and 15 degrees, 34 minutes, respectively. If the line voltage is assumed to be 100 and the line current to be 10, the wattmeters will read $10 \times 100 \times \cos$ 75 degrees 34 minutes, which equals 249 watts, and $10 \times 100 \times \cos$ 15 degrees 34 minutes which equals 963 watts. The single phase power ratio for the first case is 2.23. Thus

2.49 = (positive power) − (negative power)
2.23 = (positive power)/(negative power)

Therefore

Positive power = 451
Negative power = 202

The single phase power ratio for the second case is 1,072. Thus

Positive power = 963.9
Negative power = 0.9
Total positive power of both wattmeters = 451 + 963.9 = 1414.9
Total negative power of both wattmeters = 202 + 0.9 = 202.9
Ratio of (total positive power)/(total negative power) = 6.98

A similar process is applied to other values of power factor in order to complete the 3 phase curve of figure 9.

This 3 phase curve was checked for various power factors, as shown by the crosses on figure 9. This test was similar to the one described in connection with figure 7 (crosses) and likewise involved the use of the phase shifter for obtaining various values of power factor.

With this 3 phase power ratio curve as a basis of reference, a load test was run on a 3 phase induction motor to obtain values of power factor versus power ratio. These results are plotted in figure 9 (circles) and check the theoretical curve quite satisfactorily. For a case where the 2 wattmeter power ratio is 10, then the power factor is 80 per cent. Since the single phase power ratio is 30 for this power factor, the physical nature of the induction motor for this load is such that it returns to the source one watt out of every 30 watts received.

The next 3 phase test was run on a Fynn-Weichsel synchronous induction motor. One of the field circuits was excited by a separate d-c source which provided a varying field current and thus conveniently varied the power factor of the motor. In this case the impressed voltage was sinusoidal, but the currents were nonsinusoidal. Consequently the observations lie off the theoretical curve (see figure 10). The apparatus was then used to measure the power ratio for 16 cases of unbalanced 3-phase star-connected and delta-connected loads with various leading and lagging currents in each branch. These ratios were 8.35, 2.29, 31.3, 33.2, 8.85, 8.18, 1.9, 1.682, etc. These values, although easily obtainable from the positive and negative components of the 2 wattmeter method, are in no way indicative of the character of the load. They do, however, give an indication of what is taking place at the boundary where the wattmeter measurements are made. In the case of the 31.3 and 33.2 power ratios the load was predominantly noninductive, consisting of 2 wall lamp boards taking several kilowatts each and in the third phase a single phase induction motor running light and taking a 0.5 kw. In the case of the 1.9 and 1.682 power ratios the load was predominantly inductive, consisting of 3 loads taking about the same apparent power, 1 branch at unity power factor and the other 2 at about 6 per cent power factor lagging. In contrast with the unbalanced 3 phase case, in the single phase and balanced 3 phase case the power ratio not only gives an indication of what is happening at the boundary

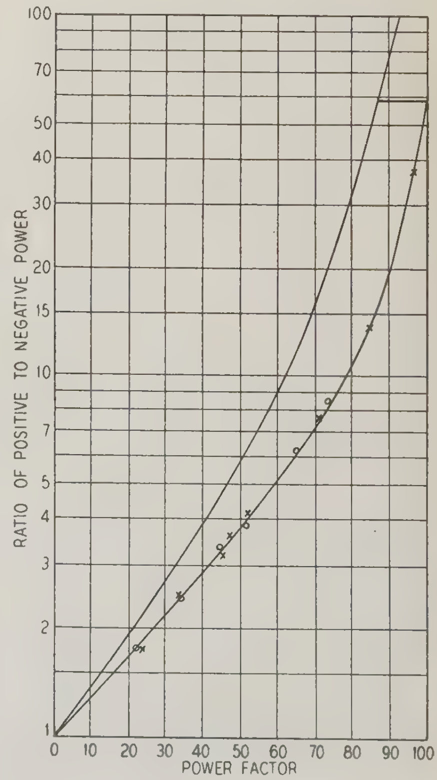


Fig. 9. Relationship between power ratio and power factor, with 3 phase balanced loads. Sinusoidal cases

Crosses—Measurements to check 3 phase curves of the relationship between power ratio and power factor, using a 3-phase phase shifter to vary the power factor. These points show a close check with the theoretical curve

Circles—Measurements with a 7.5-horsepower 3-phase induction motor, variable load with sinusoidal impressed electromotive force

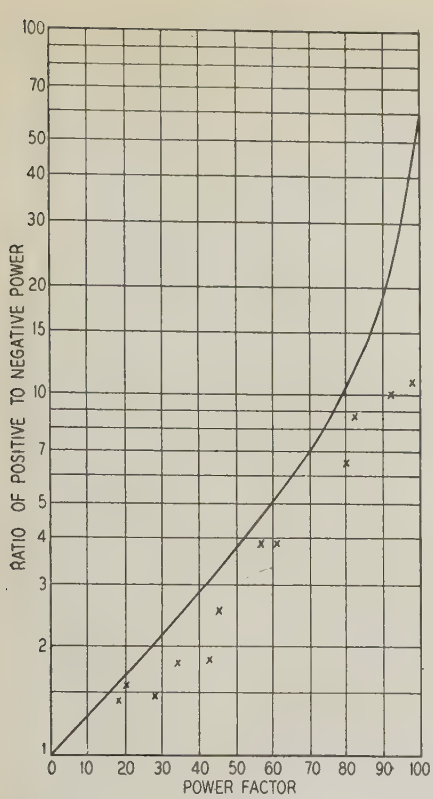


Fig. 10 (left). Relationship between power ratio and power factor with 3 phase balanced loads. Nonsinusoidal cases

Measurement with a 15-horsepower 3-phase synchronous motor, Fynn-Weichsel type, running at light load and varying field current, with sinusoidal impressed electromotive force

Fig. 11 (right). Relationship between power ratio and power factor with 3 phase balanced loads

Measurement with a 3-phase 15-kva alternator, running light as a synchronous motor, field current varied

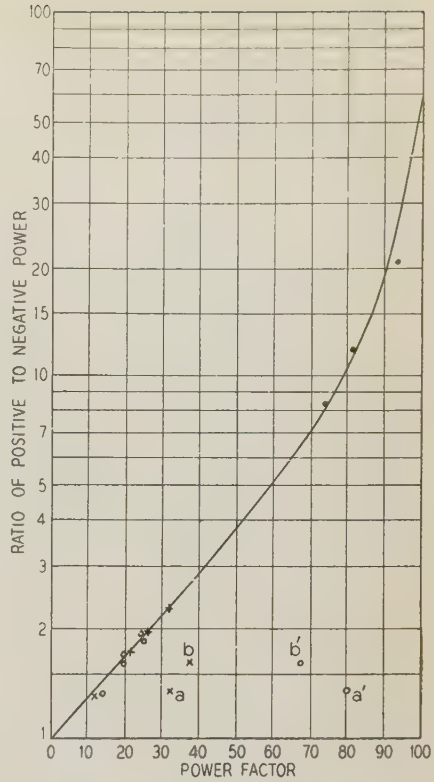
Points a, a′, b, and b′ correspond to the bottom of the V

Circles—Power factor by ratio of wattmeters

Crosses—Power factor by $\frac{\Sigma WM}{\sqrt{3}VM \times AM}$,

where *WM*, *VM*, and *AM* represent wattmeter, voltmeter, and ammeter readings, respectively

Dots—Same motor loaded with 7.5 kw load, power factor by ratio of wattmeters



where measurements are taken but in addition fixes the characteristics of the circuit.

Finally, a series of tests was run on a 15 kva alternator, operating as an ordinary synchronous motor. Both light load and load tests were made. Power factor was calculated in 2 ways: (*a*) by ratio of wattmeter readings; and (*b*) by dividing the sum of the wattmeter readings by $\sqrt{3}$ times the product of voltmeter and ammeter readings. Sinusoidal electromotive forces were impressed on the motor. Except for points near the bottom of the "V" there was a good agreement between the observations and the theoretical curve. In the region of the bottom of the "V," where the fundamental currents to supply the load are at their minimum value, harmonics have their greatest effect. It should be noted that this particular synchronous motor was subject to considerable hunting, which would introduce some distortional currents. The results of these tests are shown in figure 11.

In using the 3 phase curve of figure 9 it should be pointed out that this curve is simply an adaptation of the single phase curve to the 2 wattmeter method of measuring 3 phase power. Therefore, in describing the characteristics of a 3 phase circuit, the procedure should be: (*a*) measure the power ratio and thus the power factor, using the 2 wattmeter method, for example, a power ratio of 7 and a power factor of 70 per cent; and (*b*) record as the true power ratio of that particular circuit a value of 15, the corresponding single phase value taken from figure 9.

Summary and Recommendations

It has been the purpose of this paper to demonstrate that positive and negative energy and positive and negative power may be measured approximately with apparatus commercially available. By adjustments of potential coil resistances for tube drop, and possibly if considered desirable by shunting current coils with suitable noninductive shunts to correct for inductances in the potential coil circuits, much greater accuracy should be obtainable.

It has been demonstrated further that the power ratio is a circuit characteristic which is measurable, definable, and a readily interpreted quantity for all conditions, single phase or balanced 3 phase, sinusoidal or nonsinusoidal. Power factor, as a circuit constant, is measurable, definable, and readily interpreted only for sinusoidal cases, single phase and balanced 3 phase.

There is no disposition on the part of the authors to recommend any departure from present procedure in measuring power, energy, and power factor in the ordinary cases. When, however, through considerable distortion of wave shapes or considerable unbalance of 3 phase loads, power factor, as at present limited in its definition, is no longer a reliable indicator, recourse may be made to a set of positive and negative wattmeters or watt-hour meters for the more equitable determination of the cost of power.

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Induction Heating at Low Temperatures

Inductively heating iron parts for drying coatings discloses interesting features which may be applied to other low temperature processes. The electrical design of the oven and circuits for induction heating at low temperatures are considered in this paper. Low maintenance and operating costs, cleanliness, and reduced processing time are among the features available.

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INDUCTION heating as related to melting and very high temperature work has been repeatedly reported in the publications of the Institute, and the knowledge of the design and operation of both core and coreless melting furnaces is familiar and well understood by the electrical engineer. Induction heating as related to relatively low temperature work and for the unique purpose of baking paint or rapidly drying wet metal parts, apparently has not been previously reported in ELECTRICAL ENGINEERING. Drying paint from the "inside out," expelling the volatile products from the layer of paint next to the metal before the surface seals over, produces an exceptionally firm smooth film. Induction heating seems to be practically the only way of accomplishing this kind of drying on metal parts painted all over.

The efficiency of this type of heating is high, as the part to be processed has the heat generated directly in itself. The use of a relatively moderate frequency and voltage results in a satisfactory power input and rate of temperature rise in thin sheet metal parts, and also permits the use of practical commercial equipment.

The continuous conveyor type of oven lends itself particularly to this form of heating; polyphase windings and movement of the processed parts result in even heat being obtained. The oven is largely self-regulating in its action, as the power decreases when the parts pass out of the coil.

The fire risk in the oven is low, as the hottest point is in the part being baked. The inductor coils run relatively cool. Also, economy is effected due to the fact that the necessary air discharged to carry away the volatile products of the baking is 30 to 50 degrees lower in temperature than the baking temperature. Reduced time in baking, extremely low maintenance, together with exceptionally clean uniform work, have been obtained in the production units. A total capacity of over 600 kw of these units has already been installed.

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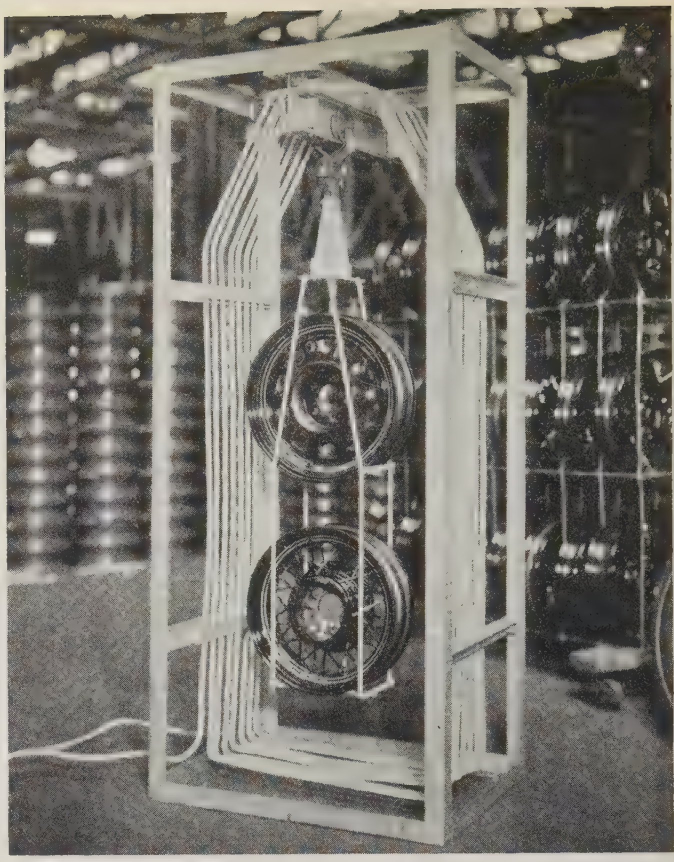


Fig. 1. A typical section of an induction heating oven for low temperature processes

Description of Oven

An induction oven suitable for paint baking makes use of no new fundamental principles, but rather applies the well-known laws of electromagnetic induction to a particular industrial thermal process. The oven consists essentially of a group of coils or solenoids so arranged that the metal parts to be heated are passed through them by some type of continuous conveyor. The coils are excited from a polyphase circuit to produce continuously varying magnetic fields both as to time and direction, the field strength and frequency being such as to produce the rate of temperature rise required by the process. Figure 1 shows a typical section. An enclosure of thermal insulation is provided for conveyor and coils in order to reduce radiation from the heated parts and conserve the I^2R loss of the inductor, as well as provide means of controlling the movement and disposition of the volatile products driven off the painted parts being baked. Figure 2 shows the relation of these items in the completed induction

oven designed for baking enamel on automobile wheels.

A coil through which an automobile fender may be carried on a single chain conveyor must be about 7 feet high by 4 feet wide, while the coil for an entire automobile body is 6 feet square. Such large coils can be made of such a wire size that the *IR* drop is small but the *IX* drop is enormous with the resulting power factor (without work in the coils) of 15 to 17 per cent. Static condensers are used to bring the power factor up to nearly unity at no load, the worst condition on the circuit.

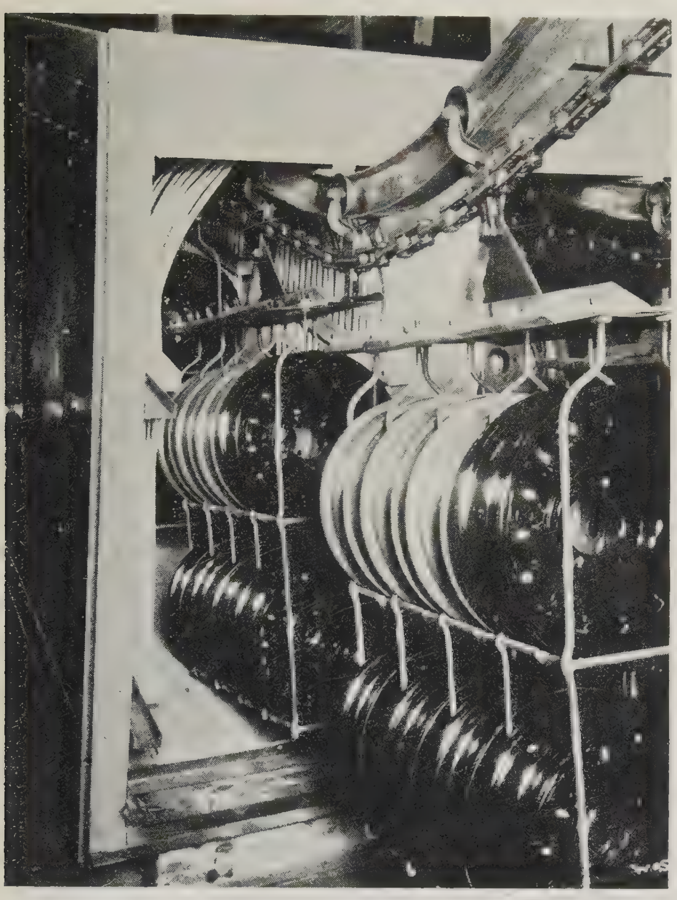


Fig. 2. Completed induction oven for baking enamel on automobile wheels

Polyphase is used for the groups of coils, which may be mounted either horizontally or vertically or both, in order to produce a more widely varied direction of flux in the irregularly shaped parts and thus produce more even heating effects. Single phase fields on fixed position parts gave very uneven heating and proved entirely unsatisfactory for paint baking where fairly uniform temperatures are important.

The selection of the proper frequency for the low temperature induction process involves satisfying 2 electrical requirements. In order to produce the watts loss required in the "work" at a rate that will raise the temperature in the time allowed, the frequency must be high enough to give a sufficiently rapid change of flux in the work, and still it must be low enough to permit the use of high currents in the primary conductors so as to give a sufficient quantity of flux in the work. The use of a relatively moderate frequency and voltage, 360 cycles and 440 volts, 3 phase, permits a satisfactory power input and rate of temperature rise even in thin sheet metal parts, and also permits the use of practical commercial generators and static condensers and low voltage insulation in the oven coils and circuits. Higher frequencies would produce the same hysteresis and eddy current losses in the metal parts at a lower flux density but the field produced by the inductor or primary coils at the lower ampere turns would have to be at a voltage too high for easy insulation.

The rate of temperature rise in sheet metal parts for various field strengths and frequencies was determined by tests, as the large air gaps and constantly changing radiation from the heated part made calculation difficult. Figure 3 shows the heating of a 6 by 12 inch panel of number 18 ordinary sheet steel similar to that used in automobiles, held in 180-cycle-per-second fields of different strengths. Figure 4 shows the effect of change of frequency at a fixed field strength. These are test values obtained on a sheet held in a vertical position in free air and show about what may be expected on work going into an oven hung on a conveyor.

Figure 5 shows the calculated power absorption

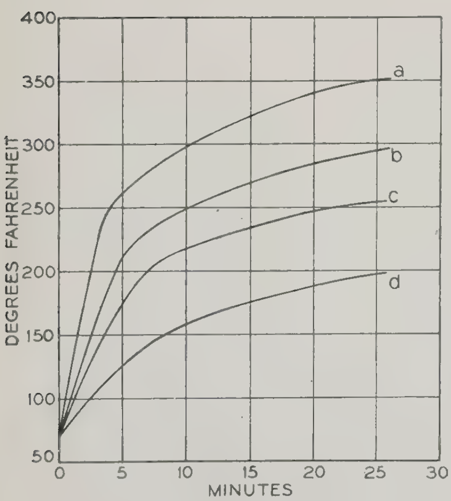
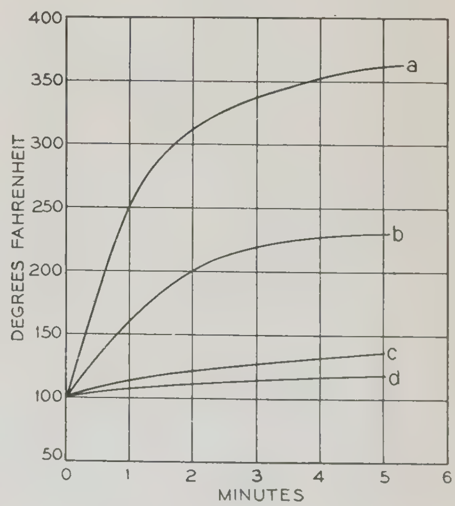


Fig. 3. Temperature-time curves showing effect of different field densities on a number 18 gauge sheet steel, at 180 cycles per second

a—300 ampere turns per inch
b—250 ampere turns per inch
c—150 ampere turns per inch
d— 66 ampere turns per inch

Fig. 4. Temperature-time curves showing effect of different frequencies on sheet metal hung vertically in flux fields

a—35 amperes at 360 cycles per second
b—35 amperes at 180 cycles per second
c—70 amperes at 60 cycles per second
d—35 amperes at 60 cycles per second



of a piece of 40 mil sheet at a flux density of 90,000 lines per square inch; it is very different from the possible absorption in an actual induction oven due of course to the low flux densities possible. The curve is given to show the small possibility of heating thin sheets of metal by 60 cycles.

Figure 6 shows the time-temperature effect on a sheet of steel made into a very shallow pan and filled with water when put into the magnetic field. Note that the temperature of the wet pan rises rapidly to the temperature where the work done to evaporate the water balances the input, and the temperature remains practically constant until all the water is evaporated. The sudden drop in temperature indicated at the moment all the water is evaporated is probably due to the wet bulb effect. All traces of water are entirely removed at a temperature well below 200 degrees Fahrenheit, also a large part of the work put into the circuit goes directly to evaporating the water, not heating the surrounding air.

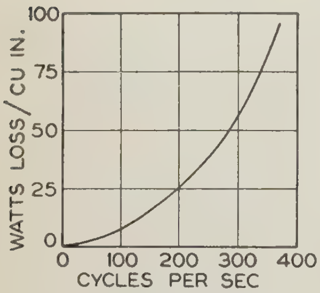
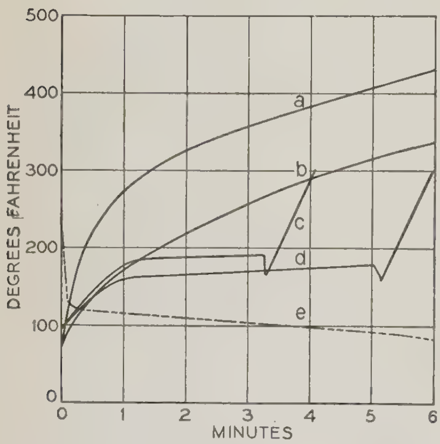


Fig. 5. Loss-frequency curve showing power absorption of a number 19 gauge body steel at a flux density of 90,000 lines per square inch, for different frequencies



a—Maximum field on bare metal pan

b—1/2 maximum field on bare metal pan

c—Maximum field on water covered pan

d—1/2 maximum field on water covered pan

e—Cooling curve, water covered pan

Fig. 6. Temperature curves showing heating effect on sheet steel made into a very shallow pan, with and without water

Coil Design

From the tests accumulated, the number of ampere turns and time necessary to raise the temperature of a part of known weight to a desired heat is determined, and coils are then designed to produce this average heating over a distance of the oven length equal to the travel of the conveyor in this time. For example, if the conveyor moves one foot per minute and it takes the painted part 5 minutes to reach the baking temperature, the first group of coils will be 5 feet long. After the baking temperature is reached the flux density in the remaining length of the oven is reduced to that necessary simply to hold the heat. The power to be absorbed is known by the weight of the metal parts to be heated. The frequency and voltage of this circuit having been selected, and the power factor and ampere turns having been determined by test, there remains simply the calculations for impedance.

From the equations $X = 2\pi LF$ the inductance required is determined, and substituting the value of L in the air core inductance formula $L = a\, n^2\, Q2.54 \times 10^{-9}$, the number of turns for the coil is easily found. The fact that there are to be iron parts in the coil affects the calculations only slightly, the air gap reluctance being so great. Twenty to 30 turns carrying 250 to 300 amperes makes an average coil covering 3 or 4 feet of length along the oven.

Regulation of Heat

The regulation of the bake is determined by the flux density and the speed of the conveyor so that when the correct values are once determined, the regulation is inherently automatic. For when there is nothing going into the coils no heat is generated except the I^2R loss of coils, which is made to balance oven wall radiation.

The production ovens have a thermostat arranged to increase the ventilation from practically nothing when no "work" is going into the oven to the maximum required when the heat generated in freshly painted parts raises the air temperature in the oven. Thus the volatile products from the baking are removed as they are released, and power input is kept down to a minimum when no baking is required.

Since the work being processed furnishes the heat for the oven, naturally the "work" is the hottest spot in the oven. The air surrounding the parts in the oven generally is 30 to 50 degrees lower in temperature than the work, so that the heat loss from radiation through the oven walls and from ventilation by necessary air to scavenge the oven of volatile products is much lower than for any other type of oven. Most paints can also be processed more quickly than with other forms of baking, so that the oven is shorter and radiation losses further reduced.

Records kept on a large production oven showed 160,000 metal automobile wheels weighing 4,800,000 pounds took only 54,500 kilowatt-hours, a production efficiency of 87 1/2 pounds per kilowatt-hour for baking synthetic enamel normally processed one hour at 250 degrees.

Low maintenance, low fire risk, low operating cost, exceptional cleanliness, reduced time in processing with reliable high grade work seem to be features of induction heating in the low temperature field; and many processes in addition to paint baking, cleaning, and drying should be developed using this convenient form of electric heat.

Vibratorily Commutated Stationary Conversion

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A system of stationary conversion of electric energy from a-c to d-c form is described in this paper in which commutation is effected by harmonically vibrating contacts actuated electromagnetically. Combined with the vibratory commutative system is a special transformer with closely coupled primary and secondary windings which are subdivided into numerous phases corresponding to the coils of a Gramme ring; the travel of the electromagnetic field that actuates the vibrating contacts is in synchronism with the travel of flux polarity in the respective transformer members. A model converter embodying these principles is described and illustrated.

THE CONVERTER here described consists of the combination with a vibratory commutative system, of a stationary transformer having a single core system and a winding subdivided into numerous phases corresponding to the coils of a Gramme ring. In synchronism with the travel of the flux polarity in the transformer member is an invisible rotation of the zones of contact among the elements of the vibratory commutator, each unit of which serves a coil. The performance is conversion rather than rectification, in that there is a flow of polyphase current (i. e., continuous power) into the magnetic reservoir of the machine and a corresponding output flow of continuous current and power.

The vibratory commutation here described consists of the effectuation by means of harmonically swinging contacts, of qualitatively the same events that occur while the bars of an ordinary commutator slide under their brushes. These occurrences include in sequence the short-circuiting of each coil through the commutative period, the reversal of the direction of its external connection, and, finally, by the termination of the short circuit, the reinsertion of the coil in series with the others. In the new conversion, such commutation is effected dually, i. e., separately adjunct to the primary winding as well as in the secondary.

Where the input is of alternating currents, as ordinarily, the primary commutator operates in parallel with the winding and its supply; but where the input is of direct current, this commutator stands in the familiar series relationship as a "turnstile." In the usual case of a-c supply, the exciting current does not go through the "turnstile"; and the fact that it slips in otherwise (wattlessly) accordingly does not disturb the commutation. Nor does the small wattless current pass through the commutator in inversion or double conversion, being separately supplied by 3-phase leads.

In apparatus based upon these principles there must be included a correlating electromagnetic means for swinging each moving contact in unison with and in progressive phase relation to the polyphase transformation. The new commutative unit calls for a new name; accordingly the term "transmuter" is offered to designate the compact assembly, separate from the transformer, in which functionally are joined the stationary and moving contacts, the tuned reeds carrying the vibrating contacts on their ends, the reed actuating system, and the common supporting structure.

The type of transformation required with this conversion, which is mechanically nonrotatory but electromagnetically synchronous, is that wherein ordinary 2 or 3 phase power is translated into a magnetic field of poles rotating or traveling in the core. Therefrom the power is passed back into the electric state in a winding subdivided into numerous (*e. g.*, 12 to 60) primary-secondary units, in which the magnetic travel produces voltages of a corresponding number of timings or phases. The currents in the successively spaced coils are led by taps, as in the ordinary commutator, to the corresponding members of the vibratory system where transpire the commutative steps just outlined. The inductive aspects of commutation are discussed more fully later; but it should be noted here as essential that the load current inductive effects are neutralized by the combination of the mentioned dual commutation with close coupling of the primary-secondary coils.

While the new converter may be typified somewhat as a locked-rotor polyphase induction motor having a secondary winding provided with many taps brought out to the electromagnetically actuated commutative system, or as a rotary converter made stationary and having the excitation of its d-c field replaced by that of the a-c supply system, these comparisons are less simple and correct than the modified-transformer concept outlined. Extension of the rotatory analogies easily may distort the true picture of the stationary converter as a transformer, because of the large quantitative deviations in these comparisons to an apparatus having no whirl, no slip, no kinetic instability, no d-c field excitation, no high gap reluctance, and no separation of primary and secondary turns by their being mounted upon

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stator and rotor or even across an air gap in stationary apparatus. The new converter does not have differential armature currents as does a rotary converter, but it does not require separate transformers as does the latter for standard voltages.

Schematic Description

Figure 1 is a system diagram of a complete converter, comprising the transformer below and the transmuter above. For convenience of subsequent explanations and comparisons, the lower half of the transmuter diagram depicts the elements of one form of reed support and actuation, and the upper half shows another form differing only in these 2 aspects of the reeds. The contact system is the same in both. The lower form is designated as one of "xylophone reeds," and the upper as the "cantilever reed form." Attention is called to the reference numbers, which are mainly common to the 2 forms. The 2 sets of co-ordinated contacts, right and left, form the 2 separate but co-operative commutators. In each, the successive contactings of the moving members overlap in time as do those of the segments in ordinary commutation. The number and joint functioning of the parts thus distinguish the vibratory commutator from the vibratory rectifier.

The present development has been directed partly toward proportioning the contacts for current capacities sufficient for industrial scale operations, i. e., with working surfaces of the order of major fractions of a square inch, and larger. Accordingly, a 3-phase 60-cycle model has been built having a rating suitable for input from ordinary panel circuits, i. e., about 30 amperes at 208 volts. Its output rating is 10 kw in d-c series-parallel combinations of 180 amperes at 56 volts or 360 amperes at 28 volts, or the corresponding 3-wire values. Its transmuter is of the xylophone reed form, the earlier of the 2 designs.

This presentation does not detail the application of vibratory commutation to other than synchronous conversion, although mention should be made of inverted conversion and of double conversion, that is, from direct current to direct current. In inversion or double conversion the transformer may receive the small currents of its polyphase excitation from an auxiliary source, such as a little motor generator or an oscillatory circuit system tuned to the proper frequency. Dual commutation should be employed in inversion as in conversion, i. e., the operation is reversible.

The design yields mainly to rational calculation and, largely through the neutralizing of inductive effects, has left little for new empirical determination, either of an electrical or mechanical nature. For the vibrating system the starting point is the dimensioning of the reeds according to well established formulas in terms of space, density, and elasticity. As a manufacturing convenience, Young's modulus for steel is practically unvarying as between the annealed state during forming and the spring-hardened final condition. Of greater importance, the elastic property is unchanging with age and is affected too little by temperature to alter

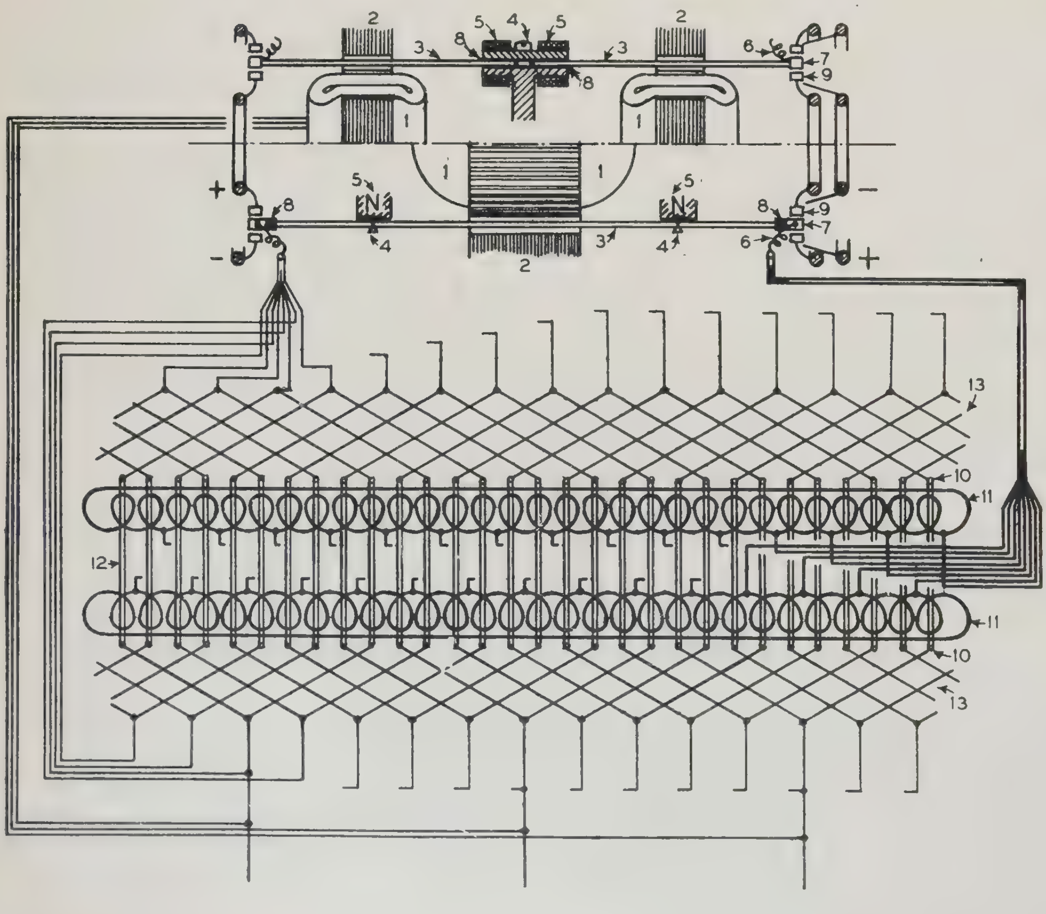

Fig. 1. Connection diagram of the new converter

1. Reed actuator
2. Flux-return ring
3. Reed
4. Reed pivot in xylophone form; reed clamp in cantilever form
5. Reed polarizer
6. Transformer contact lead-in (blade)
7. Transformer or reed contact
8. Reed insulation
9. Bus contact or V block
10. Close-coupled primary-secondary transformer coil
11. Two low voltage windings, each of one turn per coil
12. High voltage winding, 12 turns per coil
13. High voltage end connections, single wires
14. Transformer laminations
15. Thin air gaps in transformer core

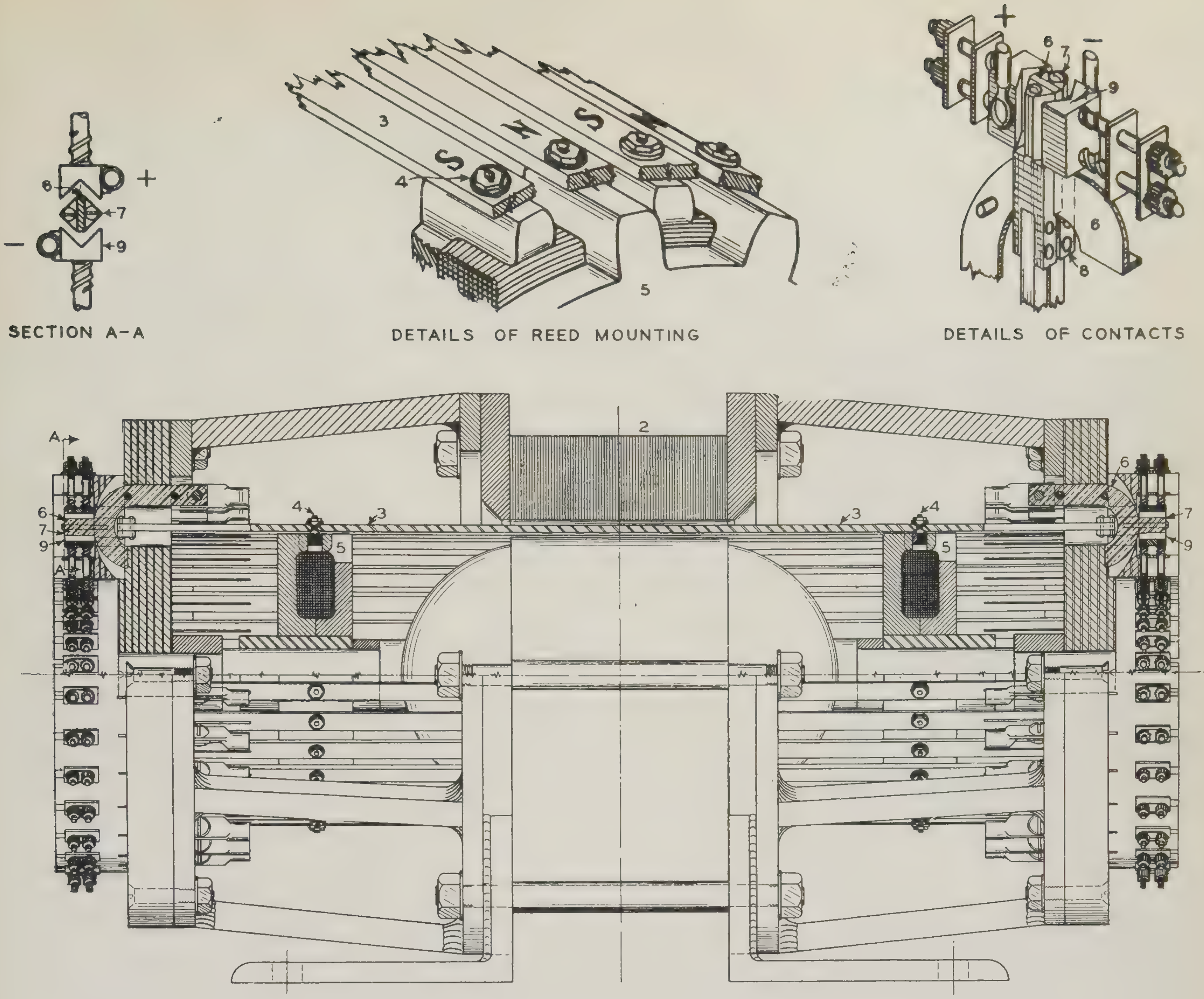

Fig. 2. Details of a 60 cycle transmuter of the "xylophone reed" form

Over-all length, 20 inches. For part numbers see caption of figure 1

its response to the electric supply frequency.

In any commutation, full flexibility of adjustment of the time-phase and duration of contact is desirable and should be available both to the designer and to the finishing and testing departments of a company manufacturing such apparatus. Vibratory commutation is adaptable to very close working adjustment for each commutated coil separately. For regulating the starting and ending of contact, adjustment is made by turning finely threaded nuts that fix the limiting positions of the resiliently mounted stationary contacts. By these means it is possible very closely to regulate the period of commutative short circuit, and, in collaboration with the unchanging steel reeds, to hold such adjustment throughout the working life of the contact members. The resilient floating of the otherwise stationary contacts is carried into the vibrating members in the forked ends described hereinafter.

Since commutative action includes the shunting of at least 2 coils at all times and in continuous succession, with proper timing for dividing the current to moderate surface density and for effecting coil-current reversal, the mechanical requirements thus readily may be defined for the new operation with its electrical and mechanical cycles so simply coupled together. Included among these requirements are: that the swings of the contacts must be reliably constant; that to this end they must be free of interference by friction comparable in magnitude with the force of their actuation; that the stationary contacts shall be positioned correctly (adjustably) with reference to the swings of the moving members; that the stationary parts shall receive the impact of the vibrating members with some resilience; and that in their return strokes following the moving contacts they shall come sharply to accurately positioned stops.

Of the 2 vibratory forms discussed herein, the one of xylophone reeds satisfies these requirements when newly adjusted. The cantilever reed transmuter is not subject to this time limitation because it does

not contain the one imperfectly stable element included in the other, i. e., the reed supporting pivots, but has a solidly built reed system and a corresponding permanence of adjustment.

The Transmuter

A 60-cycle xylophone-reed transmuter is shown in figure 2, an assembly drawing appearing as an elevation in its lower half. The unit here illustrated is the commercial sized transmuter built as a developmental model. Its basic rating is that of the stated frequency, which mainly determines its space dimensions; it otherwise is rated one or more tens of kilowatts, based upon about 400 amperes of natural-cooled contact capacity and upon voltage determined by the transformer. In the middle of this transmuter is the electromagnetic actuator, an ordinary drum-wound armature with 3-phase supply leads, surrounded at a rather large air-gap distance by a flux-return ring of clamped transformer-steel laminations. The armature-like actuator has 30 teeth, while the flux ring is smooth. Midway across the gap and opposite each tooth of the actuator, and flatwise thereto, is a steel reed 0.125 inch (0.317 centimeter) thick, 0.312 inch (0.8 centimeter) wide, and 19.5 inches (49.5 centimeters) long, measured over the ends of its contacts. The double amplitude of the swing is 0.188 inch (4.8 millimeters); the resultant maximum stress is about $^1/_4$ of the fatigue limit of spring steel. The reed assembly resembles a squirrel cage, but with respect to an induction motor the analogy does not extend far beyond visual likeness. The bars are not permitted to have any rotatory urge, they being insulated to prevent torque-producing currents.

Fig. 3. View showing details of reeds, reed pivots, polarizer, and terminals of blade contactors of a "xylophone reed" transmuter

The reeds are designed for a natural periodicity close to the electric value, and in this form they are supported at the points where occur the nodes of the fundamental, i. e., at about 22.4 per cent of the total length from each end. The reed pivots are round-shouldered small studs with round-shouldered nuts. The studs are insulated electrically from their supports, and the nuts are cushioned from the reeds with washers of soft rubber. This mounting has little damping effect when perfectly adjusted, but, as aforementioned, it will not always remain so. The members upon which the reed pivots are mounted are also those which supply the polarizing flux to the reeds.

The polarizing structure appears in the middle isometric insert of figure 2 and in figure 3, both of which show the alternating arrangement of the teeth, as well as the reed pivots. Each polarizer consists of a pair of castellated plates of cast iron with their teeth interspersed and with a d-c coil placed between the 2 toothed plates to excite all of the odd numbered teeth with, say, north polarity and all of the even numbered teeth with south polarity. A given reed is excited with the same polarity at both of its nodal supports, and its neighbor on each side is excited with the opposite polarity.

In the xylophone reed form the movement of the outward or contactor ends of the reed naturally is opposite that in the actuator zone, and it is apparent that the movements of the 2 ends are exactly in mutual phase. As in the present example, there are 30 teeth on the actuator and 30 reeds; and since the actuator winding is bipolar, there will be at any instant approximately 15 contiguous actuator teeth having varying intensities of north polarity, according to sinusoidal ordinates, and 15 diametrically opposite teeth of correspondingly varying south polarity. In the actuator north zone, all of the north polarized reeds will be repelled and all of the south polarized reeds attracted, and the converse will be true in the south polar region. Thus as the poles of the actuator rotate intangibly, there are 2 interlaced and opposite pairs of waves of reed motion, harmonically controlled both electrically and mechanically with sinusoidal phase and amplitude. According to the alternate vibratory phasing, the mating stationary contacts are connected in zigzag fashion to the positive and negative busses. While it is not necessary that the polarization be alternately north and south from tooth to tooth, this arrangement is convenient and economical.

The contacts are divided into 2 sets, those of the transformer winding and those of the busses or positive and negative mains. Except for mechanical convenience, it is immaterial which are reed-mounted and which are stationary. In the present design the transformer contacts are the vibratory ones. In some earlier designs, including examples in the author's issued patent,[1] the line contactors are the vibratory ones. Coin silver is the contact metal preferred for ordinary use in the new converter. Contact materials are discussed further in the section "Commutation and Inductance," with references.[2–8]

For connection to the moving contactors the familiar shunt or "pigtail" is not employed, but is replaced by a stationary copper blade making with the reed contact what may be called a semi-sliding contact. In such action there is firm pressure be-

1. For all numbered references see list at end of paper.

tween the blade and its reed contact at, and only at, the end of the stroke. This gripping of the blade occurs during, and is caused by, the contact of the moving member with its mating V block; both pressures last only through the commutative period—about 15 per cent of the time. The interaction of the stationary blade, the moving contact, and the stationary V block results in a strong braking or snubbing of the vibratory swing at its extremity. The right-hand isometric inset of figure 2 shows this relatively important detail. The working part of the blade member is an extended keystone rising from the body of the arch shaped portion, and it should be plated with a noble metal. Figures 3 and 4 show this member from opposite ends. The whole blade member is made from rolled copper $^1/_{16}$ inch (1.6 millimeters) thick, with the arched portion thinned somewhat. The arch passes, with clearance to avoid friction, through the correspondingly slotted insulating coupling that supports the moving contact upon its reed.

The moving contact may be considered as a square prism set diamondwise with reference to its motion in such fashion as to mate with stationary contacts of the shape of 90 degree V blocks. By the slot provided for the internal contact with the blade member, each vibratory contact actually is divided into 2 triangular prisms, with axes parallel to the length of the reed. The triangular halves of the divided contact are mounted upon the reed by couplings of insulating material.

In the model, each reed was made into a fork of 2 prongs by running a fine slitting saw through the coupling and back into the reed for an indefinite distance—about 2.5 inches (6.4 centimeters) measured from the outer end of the reed contactor. This is one of the important elements of the resilient mounting of the contacting members, whereby the 2 half contacts are free to move separately in both the transverse and the torsional directions. In a further improvement they even are relieved of the springy torsion, by being slipped upon round forked ends of the reeds. The triangular moving members fit their blades loosely, until driven in upon them by their wedging impact with the stationary contacts. This arrangement makes the contacts readily replaceable, and leaves them fully exposed for ventilation.

Each stationary V block mates, when pressed, with the oblique faces of its split-square moving contact. Each block contact is mounted upon 2 small studs, and each stud is provided with a spring between the block and its insulating support and with a nut on the outside. In the transmuter under discussion, the nuts are made of pinion rod (see figure 3) and are provided with a locking arrangement. When at the beginning of its return stroke a moving contact starts to retreat from a V block, it is followed by the latter until the nuts return to their bearing. Thus the only element that is rigid is the desirably sharp limitation of this resilient return, for precision of parting time. The V blocks are connected to their busses, alternately positive and negative as aforesaid, by flexible leads. On the low voltage side there are 2 pairs of bus rings thus connected to 2 interspersed but independent commutative groups, for the 3 wire output of the machine.

All the stationary contacts, inner and outer, are mounted upon a single bakelite laminated ring or crown, which can be removed with these V blocks in place by withdrawing 3 screws. Each individual stationary contact merely is snapped into its place in this crown, and any one may be removed individually. As may be understood from figure 4, they are exposed on all sides for ventilative cooling. The crown ring is supported upon a platform plate of bakelite laminated which, being carried upon an end cage, supports the actuator shaft and the blade contacts. As a whole the transmuter structure resembles a motor, but there is no rotation except that of the magnetism. The actuator winding could be placed in the ring instead of the core.

The transmuter illustrated in figure 5 is improved over the xylophone reed form in respect to the mentioned tendency of the pivots, by their elimination, and the supporting of the reeds instead as cantilevers rooted at the middle of the structure. For the same thickness and loading of the reeds, their length to give the stated frequency is somewhat less than in the form supported at the nodes. There is only one node of the fundamental, at the clamped end; the length is computed from the formula:[9]

$$p = \frac{1}{2\pi}\sqrt{\frac{3EIg}{\left(W + \frac{33}{140}\,wl\right)l^3}}$$

where p is the periodicity or frequency of the loaded reed in cycles per second, l its length from the clamp to the mass center of the load (the contact), w the weight of the reed per unit length, W the weight of the load, E is Young's modulus of elasticity of the steel, I the moment of inertia of the reed section, and g the acceleration of gravity. Much simplification and a good approximation may be had by considering the concentrated weight W to be negligible, as it nearly is in the transmuter of figure 5 if l be measured over the end of the contactor. Overtones may be

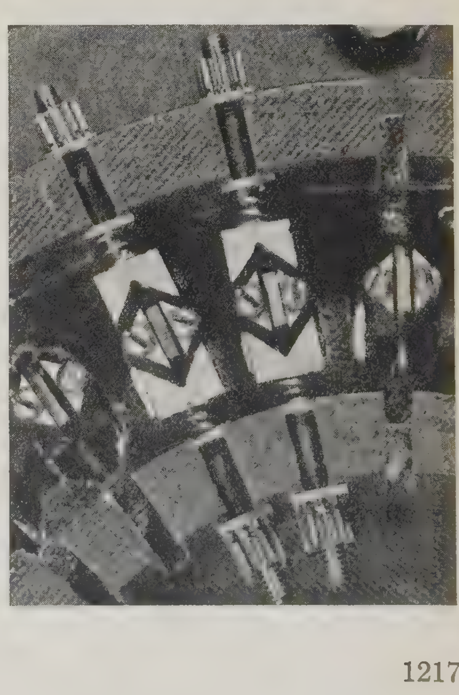

Fig. 4. Closeup of assembled V blocks, reed contacts, and lead-in blades

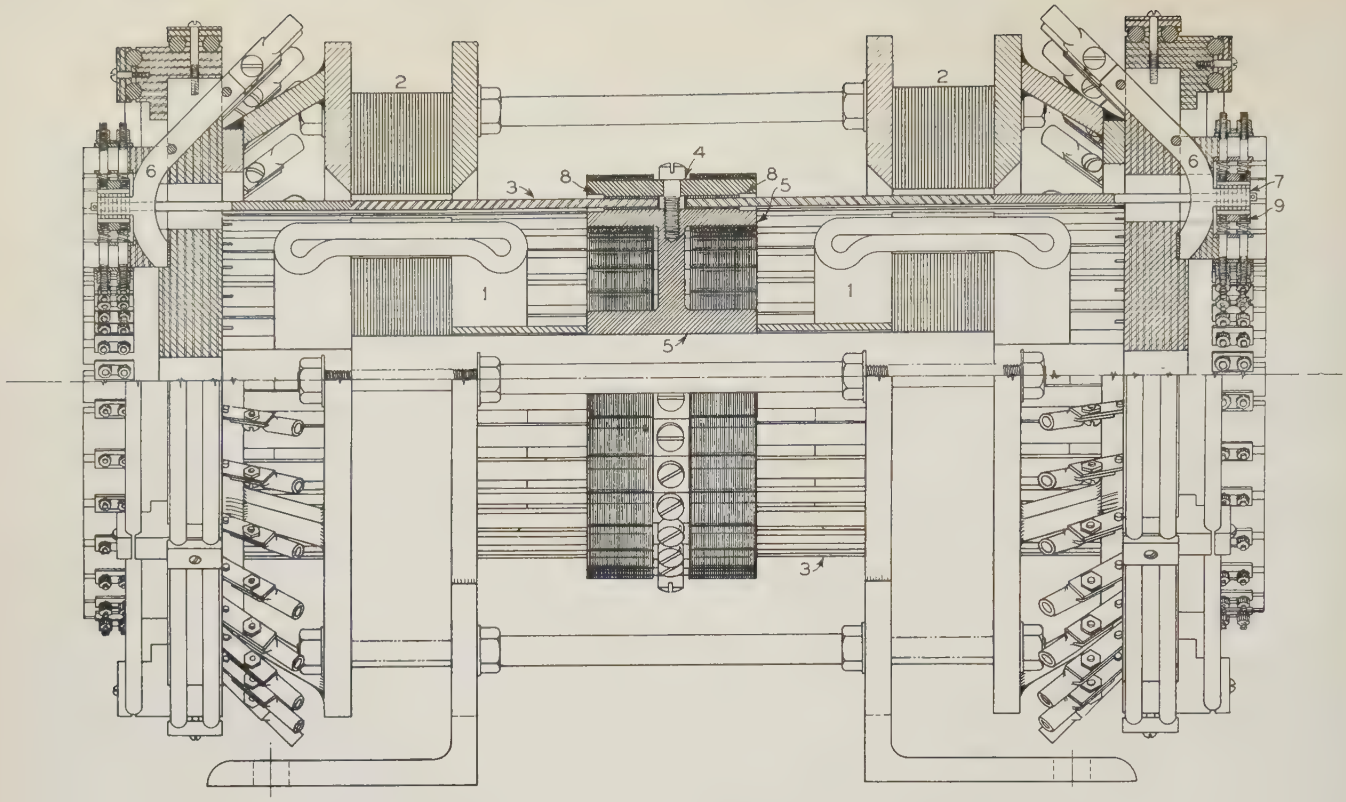

Fig. 5. Details of a 60 cycle "cantilever reed" transmuter
Over-all length, 18 inches. For part numbers see caption of figure 1

computed with coefficients, but their smaller energies can be suppressed successfully by damping and by lack of excitation at their frequencies.

The commutators of the cantilever form of transmuter are identical with those of the earlier form. Since the reeds may be divided where rooted at the middle of the structure, they are insulated there with thin material; and the insulating couplings at the contacts ordinarily become superfluous. Accordingly, the studs supporting the triangular halves of the vibratory contacts are made simple extensions of the forked reeds, as die-forged round prongs. The root-insulated reeds may be bare for electrolytic voltages, coated for distribution potentials, and for higher voltages coupling-insulated from the contacts.

In the cantilever form, the central polarizer is excited alternately north and south from tooth to tooth by separate small coils on each. The 2 actuators are essentially halves of the one in the xylophone reed design. The 2 are excited symmetrically from the power supply, so that the reed halves carrying the primary and secondary (left and right) contacts move in mutual phase relationship. The essence of the mechanical operation of the transmuter throughout is permanent precision in the timing of the metallic contacting, irrespective of electrical tolerance therein.

The Transformer

The transformer, which forms the major electromagnetic structure of this converter, is shown in the lower part of figure 6, an assembly drawing of the 10-kw model. The transmuter rests atop the transformer, and for convenience as an experimental unit the present outfit is provided with casters.

Viewed from the end, it may be seen that the transformer resembles a shell type of unit, and in the side elevation that the winding is subdivided into many (in this instance, 30) coil units. These units are interspersed with the teeth of the iron core in the same manner as are the coils of an armature winding. Each unit comprises both primary and secondary conductors, close coupled by their joint construction. In the illustrated model, the coupled construction is that of a cable of internal wires and an external copper tube, with intervening and external insulation. In this intimate relation, all of the lines of flux that link the high voltage wires link also the low voltage tubular conductor; and since for the load component current of the primary and secondary members the magnetomotive forces (ampere turns) are equal and opposite, there will be no magnetic flux caused by these load currents. The sole net flux is that of the primary exciting current.

The low-voltage secondary winding is connected in Gramme ring fashion. The high-voltage winding resembles a Gramme ring in that it is an assembly of coils that surround a central core; but the interconnection between these primary coils follows a diagram resembling that for the drum armature of a 3-phase bipolar rotary converter. The transformer is thus an extension of the conventional shell-type three-phase design, with the distinction that the windings being close coupled are especially adapted to the dual commutation.

As a convenience in building the model, its core had no air gaps in the lateral or tooth paths; and for

assurance of closeness of primary-secondary coupling the windings were formed of the mentioned concentric cable. These 2 design details have been altered by experience to a subsequent construction procedure comprising a joining of the ordinary practices in making transformers and induction motors. In motor fashion, the iron is provided with very thin air gaps in the teeth at their junctions with the central core, thus changing the coil holes to slots facing inward from the outer yokes and requiring one long slotted shape for the 2 sides and a plain strip for the center. In transformer fashion, the coils are form-wound as flat rectangles, and are slipped over the core as primary-secondary units, or couples, one for each core slot. Each of these couples is made of flat coils of the 2 windings, sheet-insulated and spaced so closely that the leakage path is negligible. The subdivision of the winding for the multiple phasing is in the nature of such large interspersing of the primary-secondary turns that, in combination with the closeness of the flat coils, there is attained a very small value of calculated flux leakage and reactance.

The everyday service of the perforating trade in punching desired rectilinear patterns in a widely tooled variety proved a boon in the construction of the model. It probably would be capital-saving and otherwise economical in manufacture, since the progressive punching method is applicable to making slots as well as holes. Figure 7 shows the partly completed model transformer, with the clamped core and the close coupled cables inserted, the inner layer connected into coils, and the outer layer ready for connection.

The transformer may be oil immersed, except where objectionable as a fire hazard as in some basements. With such oil immersion there may be combined the enclosure of the surmounted transmuter in a hood arranged as a "diving bell," for isolation of the commutators from dirt and moisture. The hood may be filled with inert gas such as nitrogen or helium, the latter as a powerful deionizing agent if needed for special high-voltage equipment.

Commutation and Inductance

Technically, the heart of this development has been the perfecting of the commutation, as in the history of generators and rotary converters. The means to that end differ in some very important essentials from those feasible in rotary commutation of noncoupled windings. Accordingly, the following analysis of the prevention of contact sparking relates largely to principles familiar in transformer design, in combination with only a portion of those that are dominant in ordinary commutation of self-inductive coils. The major omission is the resistive shunting by carbon brushes required in ordinary commutation. The important addition is an action borrowed from transformation, of which it is the core and essence. That action is the opposing by the load current in the primary winding of ampere-turns instantaneously equal to those of the load current in the secondary, so that if the 2 be coupled without leakage the only magnetism is the mutual flux caused by the primary exciting current. Since the exciting current is not commutated, the transformer action may be exploited through the dual contacting to give commutation of neutralized inductance, as discussed in other terms in succeeding paragraphs.

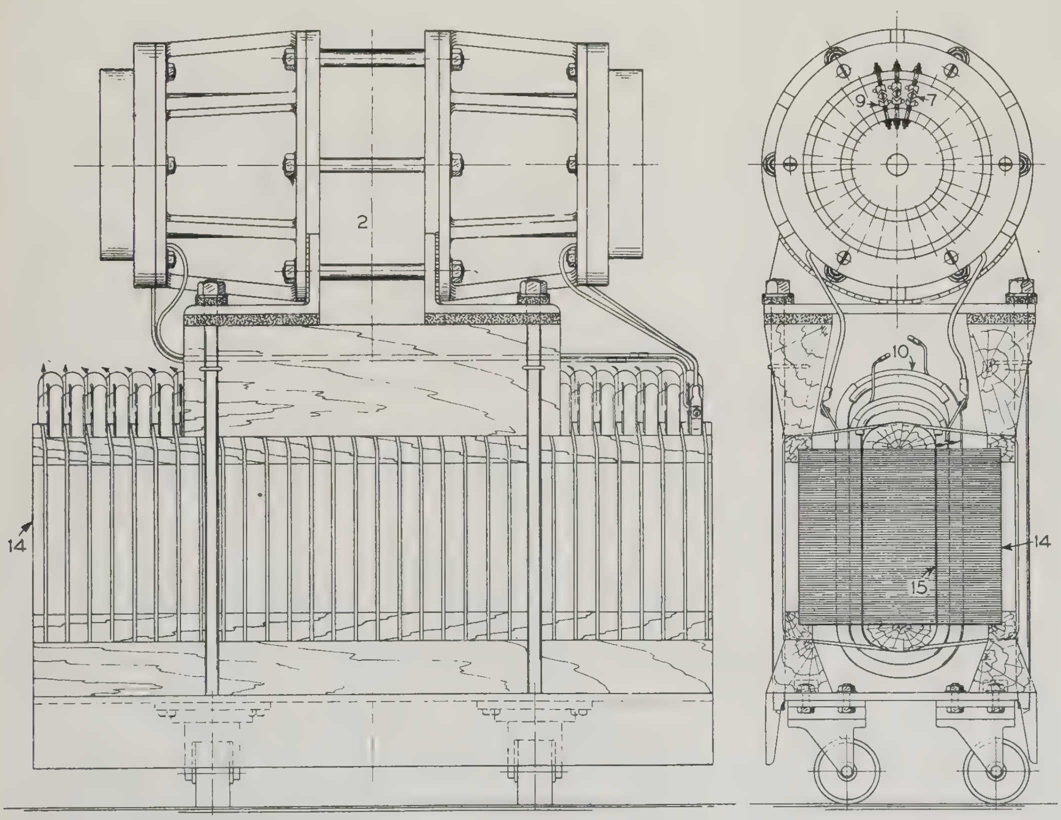

Fig. 6. Assembled converter with transmuter mounted directly over transformer

Over-all length, 25 inches; width, 10 inches. For part numbers see caption of figure 1

The causes of commutative sparking are mainly the following 3: First, there may be a breaking of substantial current in an inductive circuit, in which event there will be a spark whether the parting be quick or slow. Second, there may be a concentration of contact heating at points or edges (usually the trailing edge of a brush) whenever a large current remains until break and the final conduction is localized in the small areas of those points or edges, as distinguished from parting over nearly the whole surface of a but-

ton. In silver contacting, this concentration would have to be extreme in order alone to be deleterious.[2–8] Third, there may be heavy current interrupted, a slow separation of the contacts after their actual parting, and sufficient voltage to maintain an arc for a portion of the slow separation. During its persistence, the last is analogous to arc welding, and, of course, is aggravated if to it is added the first or inductive condition.

Among these potential causes of sparking, the second and third are countervailed in vibratory commutation by designing and adjusting the contacts to cause substantially full-faced parting at the right times, and rapid separation thereafter. By the means already described, such adjustment may be made accurately to maintain moderate surface concentrations of the currents, at full working pressures until the instants of parting.

For the inductive analysis it may be noted that by and during its commutative short circuit a coil in the secondary of the converter is shut out from series relation to its external circuit (that of the other coils in the winding) and that its local current falls and reverses without hindrance from external inductance. If the coil have practically zero flux leakage and there be negligible inductance in its commutator leads, then the only material inductive consideration is that of its own primary coil and of the circuits in which the latter may be included. By definition and construction the flux leakage of the primary coil is also nearly zero and its leads may be made of negligible inductance. Hence, if by short circuit it too can be isolated simultaneously from the inductance of its own external circuit (the other primary coils), no appreciable reactance will be left to influence the commutation. This is the dual commutation, with its dependence on close coupling.

These actions are functionally analogous to those in a motor-dynamo or dynamotor, wherein 2 windings are laid in the same armature slots, are tapped to input and output commutators, and have the same motion relative to the common field. Brief discussions of such rotary transformations and of their dual commutation are available in general electrical literature, to the effect that the magnetomotive forces of the 2 windings are equal and opposite and there is thus no armature reaction, that if the brushes are positioned so that the input and output coils in the same slots are short-circuited simultaneously the commutation is exceedingly good, and that low resistance brushes may be used and heavy currents delivered.[10–11] Mechanically, the new converter differs from a dynamotor in the all-important change from mass rotation to the intangible travel of flux poles. An interesting early proposal is described in "Edison's System of Continuous-Current Transformers."[12] While the transformation in that design was stationary, the dual commutation was still rotary; and the coupling was far from close.

Since armature reaction is absent, the opening of a secondary breaker on short-circuit does not cause the danger of flashover from voltage induction by sudden change of cross-magnetization as in generators,[13] or by distortion combined with hunting as in some rotary converters.[14] As the new converter is a (commutating) transformer with no armature to restart, the breaker often may be placed on the primary side.

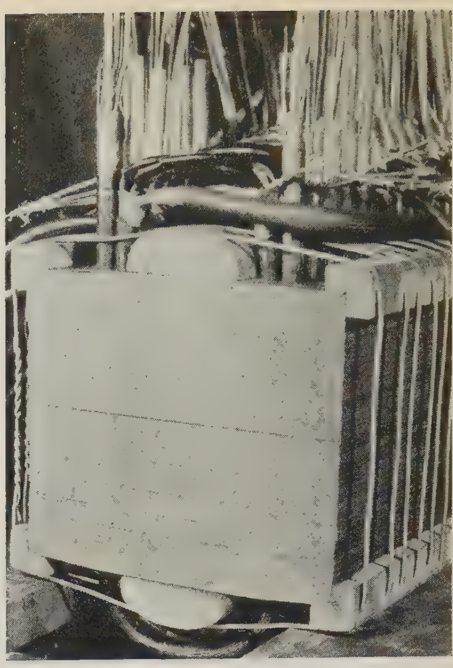

Fig. 7. Model transformer partly assembled

Concerning the precision of commutation, there is in the dual metallic method an important condition tolerant of some deviation from perfectly synchronous timing. If for a given primary-secondary coil couple the termination of short-circuit on either side lags that on the other, then the ending of the first not only will confirm the building of current in its own coil, but also because of short-circuit of the second coil will set the rise of current in it at the same rate by transformation, in the normal direction opposite that in the first. When the short-circuit thereupon is ended at the lagging contact, the current of its coil thus put in series with the others has only to adjust itself by a negligible difference, without material sparking. For similar reasons there will be little or no sparking if some bouncing of a single contact momentarily breaks the short-circuit of a coil on one side while the other coil of the couple remains short-circuited by its own contacts. This does not mean that bouncing can be permitted to be general.

The responsive travel of the spring-backed bus contact or V block is about one thread-pitch of its studs, i. e., about 0.02 inch or 0.5 millimeter. Accordingly, by whatever fraction of 360 degrees the nuts are adjusted in one setting, say 45 degrees or $12^1/_2$ per cent, by that precision the duration of the short-circuit easily may be regulated. With the elimination of commutative inductance as described, the remaining energy to be considered is the ohmic evolution. Each commutative cycle lasts about 15 per cent of the power half cycle; and practically all of this heating occurs in the coils, because of the low contact resistance. Hence, the foregoing figure of contact adjustment or deviation would affect the winding for 1.9 per cent of the total time or by about 1 per cent of the ohmic heating, the coil current being

constant except when commutating, and passing then through zero.

Reversing voltages may be employed in vibratory as in ordinary commutation, by slightly shifting the timing of the contacting. Except for higher potentials, their service and need are not important. In the absence of reactance voltage they have only to compensate the ohmic drop of each coil and its short leads. In generators and other such machines the value of this ohmic drop is about $^1/_{10}$ the reactive voltage per coil, i. e., as 0.15 to 1.5 volt in a typical 240 volt machine. Accordingly, if the reversing voltage were deficient by half or even entirely, this uncompensated ohmic voltage would be capable of only negligible sparking in the non-energy-storing operation.

Much information is now available on contact metals.[2-8] Only a limited portion of it is required directly for at least the moderate and lower voltage applications of the new converter. Since "It isn't commutation if there is substantial inductive sparking," superior ability to withstand arcing may be eliminated as a normal requirement of the contact material. Certain high grade contact alloys possess this property, but it is for other reasons that they are of important potential utility herein. The silver-tungsten and silver-molybdenum alloys combine durability with high surface conductivity. Except for its softness, prohibitive according to present indications, fine silver is the ideal material because of its high "contactivity" and stability thereof. In general coin silver is so satisfactory and economical that it seems to require little improvement for the semi-sliding contact. For the working faces of the copper blade contacts, palladium plating is indicated favorably.

Some Advantages of Vibratory Conversion

Experience with the model unit has suggested a simple method of performing routine tests of the converter, with only a small proportion of the rated input. As the divided secondary is served by independent halves of the same commutator, its 2 output voltages may be externally opposed to each other and connected in series with a low resistance source of low voltage, such as a storage battery. The system thus may be made to work at rated secondary current and with full input and output voltages, i. e., with normal primary excitation. Thus the iron and primary copper losses may be fed on the one side and the secondary copper loss measured on the other.

More important, it is now evident that the new converter has the benefit of an economic convenience peculiar to a few other classes of apparatus, of which the electrostatic precipitator is an example. That is the feasibility of prebuilding as a pilot to each new rating a group of about 3 full sized units of which the full number such as 30 would constitute the complete machine. This pilot group may be subjected to tests that demonstrate proportionately the realization of all the functions and faults of the particular design. If the full machine be assembled from these parts including those of the transmuter arranged straightaway, the leads can be made very short. In the transmuter the principal parts can be standardized for a few frequencies, voltage ranges, and current capacities for a wide variety of converter ratings.

Little mention has been made of higher-voltage applications, but these are within the possible range of service of the apparatus. As shown in the issued patent,[1] it is a peculiarity of vibratory as distinguished from rotatory commutation, that the winding of a converter can be subdivided into as many sections as there are pairs of poles and these sections connected in series rather than in parallel with each other. For low and medium voltages there seems to be no occasion to build the new converter in other than bipolar form.

The general economy of the vibratory converter has been the incentive to its development. From the manufacturing viewpoint, its design is unhampered by rotation, by centrifugal force, by the necessity for rigidity of the commutator therefor and for thermal stresses, by the visual isolation of rotation, by sliding friction and brush wear, by carbon and copper dusting, by bodily contagion of trouble from one segment to the next and corollary thereto in large and high voltage units, by the lurking flashover. As to operation, the new machine requires no attendance and promises to need little maintenance—in the nature of occasional conditioning or replacing of contactors. There is no lubrication dependent on either machine reliability or human vigilance and, as compared with the mercury vapor rectifier, no pumps or other auxiliaries to be maintained. Adjustments for correct performance can be made in the factory; but where field repair is required, its nature is simple. Since the converter is essentially a transformer practically unaffected by its commutation, it is evident that its over-all power efficiency is high.[14]

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14. Standard Handbook for Electrical Engineers. McGraw-Hill Book Co., New York, sec. 8 and 9. In addition to giving excellent general explanations of d-c machines, commutation and conversion, these sections have as appendixes ample bibliographies which are largely relevant hereto.

A Generalized Infinite Integral Theorem

A generalized infinite integral theorem and its transform and applications are presented in this paper in such form that it becomes unnecessary to use Duhamel's theorem. The treatment embodies extensions to Heavisides methods of circuit analysis, thereby simplifying materially the mathematics of operational methods and giving a clearer insight into the connection between the physics and mathematics of transients. Extensions to Heavisides expansion theorem also are given.

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WHEN Oliver Heaviside developed his operational methods of solving differential equations, he devoted his attention primarily to circuits that:

1. *Are energized by a continuous electromotive force.*
2. *Are started from rest and have neither charge nor current.*
3. *Have constant parameters (characteristics) of such values that the roots of the operational equations representing the circuits are unequal.*

These are rather severe restrictions which limit the application of Heaviside's methods to a very few problems arising in practice. Hence workers in the field of operational analysis have endeavored to eliminate as many of these restrictions as possible—in a word, to generalize Heaviside's method. Let us see what generalizations are possible and desirable.

Applied Electromotive Force. The sources of electromotive force that engineers meet in practice may or may not be continuous. In fact, sine wave and non-sine wave periodic electromotive forces commonly are met with in practice. Can Heaviside's methods be extended to any source of electromotive force? The answer is yes. Indeed such extensions are so well known that they form a part of all textbooks on the subject. There is this, however, to be said about them: They lack elegance and are artificial and awkward. In fact, Heaviside's results for continuous electromotive forces (*the unit function*) generally are extended to other functions by a step-by-step process using the principle of superposition (Duhamel's theorem).[1,2,3]

A paper recommended for publication by the A.I.E.E. committee on electrophysics, and scheduled for discussion at the A.I E.E. winter convention, New York, N. Y., Jan. 28–31, 1936. Manuscript submitted May 31, 1935; released for publication Sept. 10, 1935.

1. For all numbered references, see list at end of paper.

In this paper a different mode of extending Heaviside's methods to other than continuous electromotive forces is proposed. The procedure is to disregard the unit function as the fundamental electromotive force and to derive relations that are general and with respect to which the unit functions as well as any other electromotive force function is but a special case. The mathematics of operational methods thus are simplified materially, and a clearer insight into the connection between the physics and mathematics of transients is attained.

Initial Conditions. Circuits met in practice rarely ever start from rest. Disturbances generally occur on circuits that already have been energized. Can Heaviside's methods be extended to circuits starting from any arbitrary condition whatever? Again the answer is yes. The extension is shown in this paper under heading "Extension to Nonindicial Circuits."

Circuit Parameters. Many circuits have variable parameters. Can Heaviside's operational methods be extended to such circuits? The answer is yes in some very special cases. The present paper, however, is not concerned with this problem.

In general the parameters of circuits encountered in practice are arbitrary and need not satisfy the relation that the roots of the operational equation are unequal. Can Heaviside's methods be extended to the cases where all the roots are equal, or where some of the roots are equal? The answer is yes. These extensions, however, have been known only for the case where the applied electromotive force is continuous. This paper gives extensions to the case where the electromotive force is of any form whatever (which may be met in practice) and specifically derives results for the exponential and sine wave electromotive forces.

Derivation of the Theorem

Consider the differential equation

$$[a_n p^n + a_{(n-1)} p^{(n-1)} + \dots + a_0] i(t) = e(t) \tag{1a}$$

or

$$\sum_{k=0}^{n} a_k p^k \, i(t) = e(t) \tag{1b}$$

Multiply[4] both sides of equation 1*b* by $\epsilon^{-ut}dt$ and integrate between zero and infinity thereby obtaining

$$\int_0^\infty \sum_{k=0}^{n} a_k \epsilon^{-ut} p^k i(t) dt = \int_0^\infty \epsilon^{-ut} e(t) dt \tag{2}$$

where u is any complex number whose real part is positive and sufficiently large. Consider a typical term of the summation on the left of equation 2. Let this be the kth term and let it be integrated by parts giving

$$a_k \int_0^\infty \epsilon^{-ut} [p^k i(t)] dt = \Big[a_k \epsilon^{-ut} \{ p^{(k-1)} i(t) \} \Big]_0^\infty + a_k u \int_0^\infty \epsilon^{-ut} [p^{(k-1)} i(t)] dt \tag{3}$$

If it be assumed that $i(t)$ and all its derivatives except the nth are zero when $t = 0$, then the first

term on the right of equation 3 vanishes at the lower limit; but it also vanishes at the upper limit because $\epsilon^{-ut}p^{(k-1)}\ i(t) \to 0$ as $t \to \infty$. Hence equation 3 reduces to

$$a_k \int_0^\infty \epsilon^{-ut}[p^k\, i(t)]\, dt = a_k u \int_0^\infty \epsilon^{-ut}\, [p^{(k-1)}\, i(t)] dt \tag{4}$$

Integrating successively by parts until the number of integrations is equal to k,

$$a_k \int_0^\infty \epsilon^{-ut}\, [p^k i(t)]\, dt = a_k u^k \int_0^\infty \epsilon^{-ut}\, i(t)\, dt \tag{5}$$

Each term on the left of equation 2 yields, after repeated integration, an expression similar to equation 5. Thus equation 2 becomes

$$\sum_{k=0}^{n} a_k u^k \int_0^\infty \epsilon^{-ut}\, i(t) dt = \int_0^\infty \epsilon^{-ut} e(t) dt \tag{6a}$$

or

$$\int_0^\infty \epsilon^{-ut} i(t) dt = Y(u) \int_0^\infty \epsilon^{-ut}\, e(t) dt \tag{6b}$$

where

$$Y(u) = 1 \Big/ \sum_{k=0}^{n} a_k u^k = 1/Z(u) \tag{6c}$$

Equation 6b is the generalized infinite integral theorem of equation 1. Its solution[5,6] is

$$i(t) = \frac{1}{2\pi j} \int_{b-j\infty}^{b+j\infty} [\epsilon^{ut}\, Y(u) \int_0^\infty \epsilon^{-ux}\, e(x) dx] du \tag{7}$$

Equation 7 gives the current in a circuit that, starting from rest, is energized by any electromotive force whatever. As an illustration of the use of equation 7 let it be required to find the current in a series circuit with resistance R and inductance L when such a circuit is established by impressing the unit function **1** on it. The differential equation for such a circuit is

$$Ri + Lpi = \mathbf{1} \tag{7a}$$

whence

$$Y(p) = 1/(R + pL);\ \ Y(u) = \mathbf{1}/(R + uL) \tag{7b}$$

Substituting equation 7b in equation 7

$$i(t) = \frac{\mathbf{1}}{2\pi j} \int_{b-j\infty}^{b+j\infty} \frac{\epsilon^{-ut}}{u(R + ub)} du = \frac{\mathbf{1}}{2\pi j} \int_{b-j\infty}^{b+j\infty} \frac{1}{R}\left[\frac{1}{u} - \frac{1}{u + (R/L)}\right] \epsilon^{ut}\, du \tag{7c}$$

Now it can be shown that

$$\frac{1}{2\pi j} \int_{b-j\infty}^{b+j\infty} \frac{u^m}{u} \epsilon^{ut} du = t^{-m}/\Gamma(1 - m) = t^{-m}/(-m)! \tag{7d}$$

Substituting equation 7d in equation 7c,

$$i(t) = \frac{\mathbf{1}}{R}(1 - \epsilon^{-Rt/L}) \tag{7e}$$

Extension to Systems

The foregoing analysis may be extended readily to systems of differential equations such as:

$$Z_{11}(p)\, i_1(t) + Z_{21}(p)\, i_2(t) + \ldots + Z_{n1}(p)\, i_n(t) = e_1(t) \tag{8a}$$

$$Z_{12}(p)\, i_1(t) + Z_{22}(p)\, i_2(t) + \ldots + Z_{n2}(p)\, i_n(t) = e_2(t) \tag{8b}$$

$$\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots$$

$$Z_{1n}(p)\, i_1(t) + Z_{2n}(p)\, i_2(t) + \ldots\ Z_{nn}(p)\, i_n(t) = e_n(t) \tag{8n}$$

In equations 8 the functions $Z_{ks}(p)$ are polynomials in p and represent the operational impedances of the various branches in an electric circuit. The function $i_k(t)$ is the current in any branch k of the circuit, and it satisfies the "indicial" conditions stated following equation 3, namely, if N be the highest degree of Z_{ks} in any column k then $i_k(t)$ and all its derivatives up to the $(N - 1)$st shall be zero when $t = 0$. Finally $e_k(t)$ is the electromotive force in branch k and can be any function whatever provided that the integral on the right of equation 2 exists for each value of $e_k(t)$ in equations 8.

Multiplying each of equations 8 by $\epsilon^{-ut}dt$ and integrating between zero and infinity as was done in equations 1 to 5,

$$\sum_{k=1}^{n} Z_{k1}(u) \int_0^\infty \epsilon^{-ut}\, i_k(t) dt = \int_0^\infty \epsilon^{-ut}\, e_1(t) dt \tag{9a}$$

$$\sum_{k=1}^{n} Z_{k2}(u) \int_0^\infty \epsilon^{-ut} i_k(t) dt = \int_0^\infty \epsilon^{-ut} e_2(t) dt \tag{9b}$$

$$\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots$$

$$\sum_{k=1}^{n} Z_{kn}(u) \int_0^\infty \epsilon^{-ut} i_k(t) dt = \int_0^\infty \epsilon^{-ut} e_n(t) dt \tag{9c}$$

Equations 9 now can be solved as any set of linear equations by means of determinants, the unknowns being the integrals in $i_k(t)$. Thus:

$$\int_0^\infty \epsilon^{-ut} i_k(t) dt = \sum_{s=1}^{n} (\Delta_{ks}/\Delta) \int_0^\infty \epsilon^{-ut} e_s(t) dt \tag{10}$$

where

$$\Delta = \begin{vmatrix} Z_{11} Z_{21} \ldots\ldots\ldots\ldots Z_{n1} \\ Z_{12} Z_{22} \ldots\ldots\ldots\ldots Z_{n2} \\ \\ Z_{1n} Z_{2n} \qquad\qquad Z_{nn} \end{vmatrix} \tag{11a}$$

Δ_{ks} = the minor [with the correct sign $(-1)^{(k+s)}$] of Δ obtained by omitting column k and row s (11b)

Observe that equation 10 is of the same form as 6b. Hence its transform is

$$i_k(t) = \sum_{s=1}^{n} \frac{1}{2\pi j} \int_{b-j\infty}^{b+j\infty} \left[\epsilon^{ut}\, (\Delta_{ks}/\Delta) \int_0^\infty \epsilon^{-ux} e_s(x) dx\right] du \tag{12}$$

The expression Δ_{ks}/Δ is the quotient of 2 polynomials. Moreover Δ_{ks} is of lower degree than Δ because Δ_{ks} is a minor of Δ.

Extension to Nonindicial Circuits

The foregoing treatment applies only to indicial circuits, that is, to circuits starting from rest and containing neither charge nor current. Mathematically 6 and 7, and 10 and 12 apply to the case where $i_k(0) = i'_k(0) = \ldots = i_k^{(n-1)}(0) = 0$

where for equations 6 and 7 n is the order of the differential equation 1, and for equations 10 and 12 n is the highest degree (see equations 8) of all the values of Z_{ks} in any column k of the determinant Δ. When these conditions are not satisfied the circuit is nonindicial.

It is the object here to derive infinite integrals and their transforms for nonindicial circuits, that is, for circuits in which $i_k(0)$, $i'_k(0)$ ..... $i_k^{(n-1)}(0)$ are not necessarily zero. Consider equation 1. Multiply each side by $\epsilon^{-ut}dt$ and integrate between zero and infinity thereby obtaining expressions similar to equations 3. Since by assumption $p^{(k-1)}\,i(0)$ is not necessarily equal to zero, the first term in equation 3 does not necessarily vanish at the lower limit but gives some definite constant. Let

$$a_k\epsilon^{-ut}\{p^{(k-1)}\,i(t)\}_0 = A_{k1} \tag{13}$$

Then equation 3 becomes

$$a_k\int_0^\infty \epsilon^{-ut}\,[p^k i(t)]dt = A_{k1} + a_k u\int_0^\infty \epsilon^{-ut}\,[p^{(k-1)}\,i(t)]dt \tag{14}$$

Integrating successively by parts until the number of integrations is equal to k,

$$a_k\int_0^\infty \epsilon^{-ut}\,[p^k i(t)]dt = A_{k1} + A_{k1}\,u + \ldots\; A_{kk}u^{(k-1)} + a_k u^k\int_0^\infty \epsilon^{-ut}\,i(t)dt \tag{15}$$

Each term on the left of equation 2 yields an expression similar to equation 15. Hence equation 2 becomes

$$\sum_{k=0}^{n}\left[A_{k1} + A_{k2}u + \ldots + A_{kk}u^{(k-1)}\right] + \sum_{k=0}^{n} a_k u^k\int_0^\infty \epsilon^{-ut}\,i(t)dt = \int_0^\infty \epsilon^{-ut}\,e(t)dt \tag{16a}$$

or

$$A(u) + Z(u)\int_0^\infty \epsilon^{-ut}\,i(t)dt = \int_0^\infty \epsilon^{-ut}\,e(t)dt \tag{16b}$$

Thus equation 16b and its transform are

$$\int_0^\infty \epsilon^{-ut}\,i(t)dt = Y(u)\left[\int_0^\infty \epsilon^{-ut}\,e(t)dt + A(u)\right] \tag{17a}$$

$$i(t) = \frac{1}{2\pi j}\int_{b-j\infty}^{b+j\infty} \epsilon^{ut}Y(u)\left[\int_0^\infty \epsilon^{-ux}\,e(x)dx - A(u)\right]du \tag{17b}$$

Similarly for a nonindicial circuit involving a system of n differential equations the infinite integral theorem and its transform are:

$$\int_0^\infty \epsilon^{-ut}\,i_k(t)dt = \sum_{s=1}^{n}(\Delta_{ks}/\Delta)\left[\int_0^\infty \epsilon^{-ut}\,e(t)dt - A_k(u)\right] \tag{18a}$$

$$i_k(t) = \sum_{s=1}^{n}\frac{1}{2\pi j}\int_{b-j\infty}^{b+j\infty}\left\{\epsilon^{ut}(\Delta_{ks}/\Delta)\left[\int_0^\infty \epsilon^{-ux}e(x)dx - A_k(u)\right]\right\}du \tag{18b}$$

Outline of the Problem

The treatment will be confined to indicial circuits for 2 reasons: (1) They are easier to treat; and (2) the generalization from indicial to nonindicial circuits is obvious. The results of the first 2 sections of the paper may be summarized in the following 2 sets of equations which are identical with equations 6b, 7, 10, and 12, respectively:

$$\int_0^\infty \epsilon^{-ut}\,i(t)dt = Y(u)\int_0^\infty \epsilon^{-ut}\,e(t)dt \tag{19a}$$

$$i(t) = \frac{1}{2\pi j}\int_{b-j\infty}^{b+j\infty}\left[\epsilon^{ut}\,Y(u)\int_0^\infty \epsilon^{-ux}\,e(x)dx\right]du \tag{19b}$$

$$\int_0^\infty \epsilon^{-ut}\,i_k(t)dt = \sum_{k=1}^{n}(\Delta_{ks}/\Delta)\int_0^\infty \epsilon^{-ut}e_s(t)dt \tag{20a}$$

$$i_k(t) = \sum_{s=1}^{n}\frac{1}{2\pi j}\int_{b-j\infty}^{b+j\infty}\left[\epsilon^{ut}\,(\Delta_{ks}/\Delta)\int_0^\infty \epsilon^{-ux}e_s(x)dx\right]du \tag{20b}$$

Hereafter equations 19a and 20a will be referred to as the infinite integrals and 19b and 20b as their transforms for indicial circuits. Here are 2 separate cases: (1) a single differential equation 1 whose infinite integral is equation 19a and whose transform is equation 19b; and (2) a system of differential equations 8 whose infinite integral is equation 20a and whose transform is equation 20b. In each of these cases 2 distinct types of problems arise from the reciprocal relation between the infinite integrals and their respective transforms: (a) given $Y(u)$ or Δ_{ks}/Δ, required $i(t)$; and (b) given $i(t)$, required $Y(u)$ or Δ_{ks}/Δ.

Problems of the first type (a) are solved by the use of equations 19b and 20b, respectively. They have been treated in articles and textbooks[1,2,3,6] for the unit function. Such treatments, however, have not been extended to other electromotive force functions. In other words, no theorem has been given corresponding to Heaviside's expansion theorem, which applies, for example, to the exponential or the sine electromotive force. One object of this paper is actually to develop such theorems.

Problems of the second type (b) are solved by the use of equations 19a and 20a, respectively. Such problems apparently have received no attention whatever. This type of problem will not be treated in the present paper because the process, in its most general form, is important and involved enough to be the subject of a separate paper. The remainder of the paper is concerned with only the first type of problem.

Application to Exponential Electromotive Forces

Consider a circuit having exponential sources of electromotive force of the form

$$e(x) = E\epsilon^{cx}\,\mathbf{1} \text{ or } e_s(x) = E_s\epsilon^{c_s x}\,\mathbf{1} \qquad c \neq 0 \tag{21}$$

Substituting equation 21 in 19 and 20, respectively,

$$\frac{Y(u)}{u-c} = \int_0^\infty \epsilon^{-ut}\,i(t)dt \tag{22a}$$

$$i(t) = \frac{1}{2\pi j}\int_{b-j\infty}^{b+j\infty}\frac{Y(u)}{u-c}\,\epsilon^{ut}\,duE\,\mathbf{1} \tag{22b}$$

$$\sum_{s=1}^{n}\Delta_{ks}/(u-c_s)\,\Delta = \int_0^\infty \epsilon^{-ut}\,i(t)dt \tag{23a}$$

$$i_k(t) = \sum_{s=1}^{n} \frac{1}{2\pi j} \int_{b-j\infty}^{b+j\infty} \frac{\Delta_{ks}}{(u - c_s)\Delta} \epsilon^{ut} du\, E_s \mathbf{1} \tag{23b}$$

Expressions 22 and 23 are special forms of 19 and 20. The former are used for simple differential equations such as 1, while the latter are applicable to systems such as 8. Now a single differential equation is but a special case of a system of such equations. Hence a discussion of equations 23 is all that would be required to cover the general case. Therefore, equations 22 will not be discussed at length, but their use will be illustrated by solving a problem.

Let it be required to find the current $i(t)$ in a circuit, with R and L, which is subjected suddenly to any electromotive force, $e(t) = \epsilon^{ct} E \mathbf{1}$. By Kirchoff's law,

$$(R + pL)i = e^{ct} E \mathbf{1} \text{ or } i = \frac{1}{R + pL} e^{ct} E \mathbf{1} \tag{24}$$

whence

$$Y(u) = \frac{1}{R + uL} \tag{25}$$

Substituting equation 25 in 22*b* and integrating,

$$i(t) = \frac{E \mathbf{1}}{Lc + R} [\epsilon^{ct} - \epsilon^{-Rt/L}] \tag{26}$$

Extension of Heaviside's Expansion Theorem

Heaviside's expansion theorem may be extended readily to any form of electromotive force for which equations 19*b* and 20*b* exist, by the use of these equations. However, it will be extended here only to the exponential electromotive force. The problem may be stated as follows: Given a circuit with constant parameters having exponential electromotive forces of the form indicated by equation 21, which circuit moreover is at rest and satisfies the "indicial" conditions; what is the value of any current $i_k(t)$? For convenience the following 4 cases will be considered, which naturally yield 4 separate extensions of Heaviside's expansion theorem.

Case a. The determinant Δ (see equations 11) has no repeated roots and no roots equal to any c_s in equation 21. It can be shown[7] that under these conditions

$$\frac{\Delta_{ks}}{\Delta} = \sum_{r=1}^{m} \frac{b_{rs}}{u - u_r} \qquad b_{rs} = \frac{\Delta_{ks}(u_r)}{\Delta'(u_r)} \tag{27}$$

where m is the degree of the polynomial Δ and u_r is any of its roots. Substituting equation 27 in 23*b* and integrating,

$$i_k(t) = \sum_{s=1}^{n} \sum_{r=1}^{m} b_{rs} E_s (\epsilon^{u_r t} - \epsilon^{c_s t})/(u_r - c_s) \tag{28}$$

Equation 28 is the first extension of Heaviside's expansion theorem and applies to indicial circuits with constant parameters, subjected to exponential electromotive forces of the type designated by equation 21. Moreover, the parameters of the circuits are so related that the determinant Δ (see equation 11) has neither repeated roots nor any roots equal to c_s.

Case b. The determinant Δ has no repeated roots but some of its roots are equal to some of the values of c_s in equation 21. Hence

$$\frac{\Delta_{ks}}{\Delta} = \sum_{r=1}^{h} \frac{b_{rs}}{u - c_{sr}} + \sum_{r=h+1}^{m} \frac{b_{rs}}{u - u_r} \tag{29a}$$

where

$$b_{rs} = \frac{\Delta_{ks}(c_{sr})}{\Delta'(c_{sr})} \qquad 1 \leqq r \leqq h \tag{29b}$$

$$b_{rs} = \frac{\Delta_{ks}(u_r)}{\Delta'(u_r)} \qquad h + 1 \leqq r \leqq m \tag{29c}$$

$$c_{sr} = \text{that } c_s \text{ which is equal to } u_r \tag{29d}$$

Substituting equation 29*a* in 23*b* and integrating

$$i_k(t) = \sum_{r=1}^{h} \sum_{\substack{s=1 \\ c_{sr} \neq c_s}}^{n} b_{rs} E_s \frac{\epsilon^{c_{sr}t} - \epsilon^{c_{sr}t}}{c_{sr} - c_s} +$$

$$\sum_{r=h+1}^{m} \sum_{s=1}^{n} b_{rs} E_s \frac{\epsilon^{u_r t} - \epsilon^{c_s t}}{u_r - c_s} + \sum_{r=1}^{h} b_{rs} E_{sr} t \epsilon^{c_{sr} t} \tag{30}$$

In equation 30 the summation of c_{sr} is to be carried only over r (not over s). Equation 30 is the second extension of Heaviside's expansion theorem and applies to indicial circuits subjected to exponential electromotive forces. The parameters of the circuits are so related that the determinant Δ, while not having any repeated roots, may have roots equal to some of the c_s's.

Case c. The roots of the determinant Δ are such that u_k is repeated m_k times and none of the roots is equal to any c_s. Here if m is the degree of Δ and if Δ has g distinct roots that are repeated $m_1, m_2 \ldots m_g$ times, respectively,

$$\frac{\Delta_{ks}}{\Delta} = \sum_{r=1}^{g} \sum_{v=1}^{m_r} \frac{b_{vrs}}{(u - u_r)^v} \tag{31a}$$

where

$$m_1 + m_2 + \ldots + m_g = m \tag{31b}$$

Substituting equation 31*a* in 23*b* and integrating,

$$i_k(t) = \sum_{r=1}^{g} \left\{ \sum_{v=1}^{m_r} \sum_{s=1}^{n} b_{vrs} E_s \frac{\epsilon^{u_r t}}{(c_s - u_r)^v} R_v[(c_s - u_r)t] \right\} \tag{32a}$$

where

$$R_v[(c_s - u_r)t] = \epsilon^{(c_s - u_r)t} - \sum_{q=1}^{v} [(c_s - u_r)t]^{(q-1)}/(q - 1)! \tag{32b}$$

Now in equations 31*a* and 32*a* the values of b_{vrs} may be expressed as follows:

$$b_{vrs} = \frac{\phi_r^{(m_r - v)}(u_r)}{(m_r - v)!} \tag{33a}$$

where

$$\phi_r(u) = \frac{[\Delta_{ks}(u)](u - u_r)^{m_r}}{\Delta(u)} \tag{33b}$$

Equation 32*a* is the third extension of Heaviside's expansion theorem and applies to indicial circuits subjected to exponential electromotive forces. The parameters of the circuits are so related that Δ has roots $u_1, u_2 \ldots u_g$ which are repeated $m_1, m_2 \ldots m_g$ times, respectively.

Case d. The determinant Δ has repeated roots some of which are equal to c_s. Here the decomposition yields:

$$\frac{\Delta_{ks}}{\Delta} = \sum_{v=1}^{m_1} \frac{b_{v1s}}{(u - c_{s1})^v} + \sum_{v=1}^{m_2} \frac{b_{v2s}}{(u - c_{s2})^v} + \ldots + \sum_{v=1}^{m_w} \frac{b_{vws}}{(u - c_{sw})^v} + \sum_{v=1}^{m(w+1)} \frac{b_{v(w+1)s}}{(u - u_{w+1})^v} + \ldots + \sum_{v=1}^{m_g} \frac{b_{vgs}}{(u - u_g)^v} \quad (34)$$

Substituting equation 34 in 23*b* and integrating,

$$i_k(t) = \sum_{r=1}^{w} \left[\sum_{v=1}^{m_r} \frac{b_{vrs} E_{sr} t^v \epsilon^{c_{sr}t}}{v!} \right] + \sum_{\substack{r=1 \\ c_s \neq c_{sr}}}^{w} \left\{ \sum_{v=1}^{m_r} \sum_{s=1}^{n} \frac{b_{vrs} E_s \epsilon^{c_{sr}t}}{(c_s - c_{sr})^v} R_v \left[(c_s - c_{sr})t\right] \right\} + \sum_{r=w+1}^{g} \left\{ \sum_{v=1}^{m_r} \sum_{s=1}^{n} \frac{b_{vrs} E_s \epsilon^{u_r t}}{(c_s - u_r)^v} R_v \left[(c_s - u_r)t\right] \right\} \quad (35)$$

Equation 35 is the fourth extension of Heaviside's expansion theorem and applies to indicial circuits subjected to exponential electromotive forces. The parameters of the circuits are so related that Δ has roots $u_1, u_2 \ldots u_g$ which are repeated $m_1, m_2 \ldots m_g$ times, respectively. Moreover, some (say w) of these roots have values equal to some values of c_s. Thus u_r is equal to c_{sr} for $r = 1,2,\ldots w$.

This concludes all the possible extensions of Heaviside's expansion theorem. The exponential electromotive force has been chosen not only because of the facility with which it can be handled, but also because of its practical application in actual engineering problems. Indeed all the trigonometric functions are reducible to the exponential; moreover, most electromotive force functions encountered in engineering practice are expressible as Fourier series and hence are also reducible to exponential functions.

Extension of Heaviside's Expansion Theorem to Sine Wave Electromotive Forces

The extension of Heaviside's expansion theorem to sine wave electromotive force functions follows directly from its extension to exponential functions. Thus let the electromotive force $e_s(t)$ be

$$e_s(t) = E_{ms} \sin(\omega t + \alpha_s)\,\mathbf{1} = E_{ms} \left[\frac{\epsilon^{j(\omega t + \alpha_s)} - \epsilon^{-j(\omega t + \alpha_s)}}{2j} \right] \mathbf{1} \quad (36a)$$

$$= \epsilon^{j\omega t} \left[\frac{E_{ms}\,\epsilon^{j\alpha_s}}{2j} \right] \mathbf{1} - \epsilon^{-j\omega t} \left[\frac{E_{ms}\,\epsilon^{-j\alpha_s}}{2j} \right] \mathbf{1} \quad (36b)$$

Now observe that each term on the right of equation 36*b* has a form similar to equation 21. Thus $c_s \sim \pm j\omega$, $E_s \sim E_{ms}\,\epsilon^{\pm j\alpha_s}/2j$. The extensions, equations 28, 30, 32, and 35, therefore, may be written directly for sine wave electromotive forces by performing the necessary algebraic substitutions. Thus the first extension of Heaviside's expansion theorem to sine wave electromotive forces is (see equations 28 and 36)

$$i_k(t) = \sum_{s=1}^{m} \sum_{r=1}^{m} \frac{b_{rs} E_{ms}}{2j} \left[\frac{\epsilon^{j\alpha_s}(\epsilon^{u_r t} - \epsilon^{j\omega t})}{u_r - j\omega} - \frac{\epsilon^{-j\alpha_s}(\epsilon^{u_r t} - \epsilon^{-j\omega t})}{u_r + j\omega} \right] \quad (37)$$

Similarly expressions may be written corresponding to equations 30, 32, and 35.

Equation 37 gives the current in a circuit in which the parameters are such that none of the roots of its operational equation are equal. Moreover the circuit, starting from rest, is energized by a sine wave electromotive force. As an illustration, let it be required to determine the current in a series circuit with resistance R and inductance L which is subjected suddenly to the sine wave electromotive force of equations 36*a* and 36*b*. The differential equation for such a circuit is

$$Ri + Lpi = E_m \sin(\omega t + \alpha)\,\mathbf{1} \quad (37a)$$

whence

$$Y(u) = R + Lu, \text{ and therefore } u_r = -R/L \quad (37b)$$

Now equation 37 cannot be used in this instance because $\Delta = 0$ (in other words, here is an example where a single differential equation rather than a system of differential equations is involved). Hence equation 22*b* is resorted to. Substituting $\pm j\omega$ for c, and $E_m \epsilon^{\pm j\alpha}/2j$ for E, and integrating,

$$i = \left[\frac{E_m\,\epsilon^{j\alpha}}{2j(R + j\omega L)} \left\{ \epsilon^{j\omega t} - \epsilon^{-Rt/L} \right\} - \frac{E_m\,\epsilon^{-j\alpha}}{2j(R - j\omega L)} \left\{ \epsilon^{j\omega t} - \epsilon^{-Rt/L} \right\} \right] \mathbf{1} \quad (37c)$$

$$= \left[\frac{E_m}{2j} \left\{ \frac{\epsilon^{j(\omega t + \alpha)}}{Z\epsilon^{j\phi}} - \frac{\epsilon^{-j(\omega + \alpha)}}{Z\epsilon^{-j\phi}} \right\} - \frac{\epsilon^{-Rt/L} E_m}{2j} \left\{ \frac{\epsilon^{j\alpha}}{Z\epsilon^{j\phi}} - \frac{\epsilon^{-j\alpha}}{Z\epsilon^{-j\phi}} \right\} \right] \mathbf{1} \quad (37d)$$

$$= \left[\frac{E_m}{Z} \sin(\omega t + \alpha - \phi) - \frac{E_m\,\epsilon^{-Rt/L}}{Z} \sin(\alpha - \phi) \right] \mathbf{1} \quad (37e)$$

Notations

a_k—a constant coefficient of the differential equation 1.
A_{ks}—a constant defined by equation 13.
$A(u)$—a polynomial in u defined by equation 16.
b—any positive real number.
b_{rs}—a constant obtained by expanding the function Δ_{ks}/Δ and defined by equation 29*b*.
b_{vrs}—a constant obtained by expanding the function Δ_{ks}/Δ and defined by equation 33.
c or c_s—coefficient of t in the exponential electromotive force function.
c_{sr}—that c_s which is equal to u_r.
$e(t)$—any instantaneous electromotive force such that the integral of equation 2 exists.
$e_s(t)$—exponential electromotive force defined by equation 21.
E_s—value of $e_s(t)$ corresponding to $t = 0$.
E_{sr}—that E_s whose exponent has for its coefficient c_{sr}.
E_{ms}—amplitude of the sine wave electromotive force $e_s(t)$.

g—number of roots of Δ that are repeated.

$z_{ks}(u)$—operational impedance of branch k on branch s.

$i(t)$—instantaneous current.

r—any integer.

$i', i''\ldots i^{(n-1)}$—1st, 2d....($n - 1$)th derivatives of $i(t)$.

k—any integer.

s—any integer.

h—number of roots of Δ that are equal to some values of c_s (see equation 30).

$m_1, m_2\ldots m_g$—the number of times that the distinct roots ($u_1, u_2\ldots u_g$) of the polynomial Δ are repeated.

p—differential operator d/dt.

m—degree of the polynomial Δ.

n—number of sources of electromotive force in a circuit (see equations 28 and 30).

t—time.

u—any complex number whose real part is positive.

u_r—any root of the determinant Δ.

w—number of roots u_r that are equal to some of the values of c_s (see equations 34 and 35).

$Y = Y(u)$—operational admittance.

$Z = Z(u)$—operational impedance.

$i_k(t)$—current in branch k of a circuit.

Δ, Δ_{ks}—impedance determinant and its minor defined by equation 11.

α_s—initial angle of the sine wave electromotive force $e_s(t)$.

ω—angular velocity of the sine wave electromotive force $e_s(t)$.

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Parallel Inverter With Resistance Load

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Calculation of inverter performance and analysis of the circuit apparently have not been treated to any great extent in technical literature. This paper presents a method of calculating in terms of 2 parameters the characteristics of an inverter supplying a pure resistance load, and discusses the various modes of operation, including that in which the direct current supplied to the inverter flows in pulses.

THE APPLICATION of inverters has become of noticeably greater interest during recent years. A number of papers[1] of a descriptive nature have been written but no papers, with the exception of several[2,3,4] in Germany, have been published, to the author's knowledge, which deal with the analysis of the circuit and the calculation of the inverter performance. It is the purpose of this paper to present a method of calculating the characteristics of an inverter with a pure resistance load and to discuss the different modes in which inverters may operate. The treatment differs from that of previous investigators in that different parameters are used, the results are plotted in more usable form, and the case is discussed in which the direct current flows in pulses starting from zero each half cycle. It is shown that parallel inverters with resistance load can operate in 2 different fashions depending upon the relative values of the constants. The relations between the constants are conveniently expressed in terms of 2 parameters K and J which represent respectively the ratio of capacitor to load volt-amperes and the ratio of ballast reactance to load resistance.

The circuits under consideration are intended to operate with any of the grid-controlled mercury-vapor discharge tubes or the conventional grid-controlled rectifier. In these devices current conduction may be initiated at any instant that the anode-cathode voltage has the proper polarity, but current extinction must be accomplished through some function of the circuit.

Description of Operation

In figure 1 is shown a schematic diagram for a single-phase inverter with a resistance load. The constant potential E is applied from the d-c source and tubes A and B are alternately made conducting through the functioning of their control devices, the frequency of the a-c load being determined by the

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1. For all numbered references, see list at end of paper.

frequency of the control. Assume for the moment that tube A is conducting. Capacitor C assumes a potential of approximately $2E$, the right-hand side being positive. Now if tube B is rendered, conducting capacitor C tends to discharge through the 2 tubes, causing the current in tube A to decrease to zero. Tube A thus becomes open-circuited. The anode-cathode potential across tube A is then equal to the potential across the condenser minus the arc drop in tube B. Since the left-hand side of the capacitor is negative in potential relative to the right-hand side, the anode becomes negative relative to the cathode. With the new conduction paths, the capacitor tends to charge with the opposite polarity and as it becomes charged the anode-cathode potential increases from a negative value to a positive value as shown by the curve in figure 2. When tube A is made conducting, the capacitor discharges in the reverse direction extinguishing the arc in tube B. In this manner tubes A and B alternately are made to conduct current, and pulses of current are made to pass alternately through the 2 sides of the transformer which produce an alternating current in the secondary of the transformer and in the load resistance. The other curves in figure 2 show how other quantities vary with time. The symbol T represents one period of the control frequency.

Modes of Operation

In the foregoing explanation of operation of the circuit it was assumed tacitly that the ballast inductance was sufficiently large that the current in the d-c circuit did not reach zero at any instant. As the ballast inductance is made smaller and smaller the variations in the direct current become greater until finally a condition is reached in which the direct

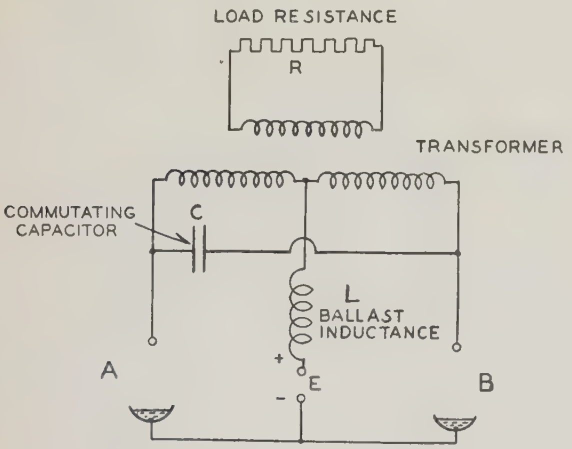
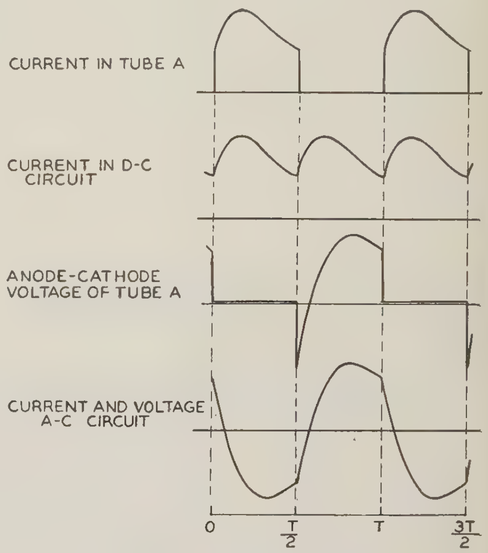


Fig. 1. Schematic diagram of parallel inverter with resistance load

current flows in pulses. The variations in the principal quantities for this condition are shown in figure 3.

In the development which follows it will be found that the results can be expressed conveniently in terms of 2 dimensionless parameters, namely, $K = 4YR$ and $J = X/R$. The former gives approximately the ratio of the kilovolt-ampere capacity of the commutating capacitor to the load kilowatts. Thus, assuming that all 3 transformer windings have the same number of turns, for the fundamental component of voltage across the load the load in watts is E^2/R. Similarly, since the voltage across the capacitor is twice that across the load, the volt-amperes of the capacitor are $2\pi fC\ (2E)^2$. The ratio is thus $\dfrac{2\pi fC4E^2}{E^2/R} = 2\pi fC\ 4R = 4YR = K$. The other parameter J is equal to the ratio of the reactance of the ballast inductance at *control* frequency $(2\pi fL)$

Fig. 2. Current and voltage relations when inductance in d-c side is large



to the load resistance. Expressed as a function of these 2 parameters, the division between the 2 modes of operation exemplified by the curves of figures 2 and 3 is given by the line OA in figure 4. The derivation of this division line will be given later.

Part or all of the capacitance of C in figure 1 can be placed in parallel with the load. In some instances it may also be desirable to arrange that capacitance be switched in simultaneously with additional load.

Outline of Method of Calculation

The general plan of attack will be to set up the differential equations of the circuit with one tube operating and the other tube open-circuited. This circuit is shown in figure 5*a*. It will be assumed that all 3 windings of the transformer have the same number of turns. If this is not so, R can be reduced readily to an equivalent resistance. The further assumption will be made that the transformer leakage reactance and magnetizing current are negligibly small. Calculations in which these quantities were not neglected have shown these assumptions to be justifiable. The circuit can be simplified further to that shown in figure 5*b*. All circuit resistances except the load will be assumed to be equal to zero. Since the tube arc drop is essentially constant, inde-

pendent of the current, it may be subtracted from the applied continous voltage and the difference regarded as the actual applied voltage.

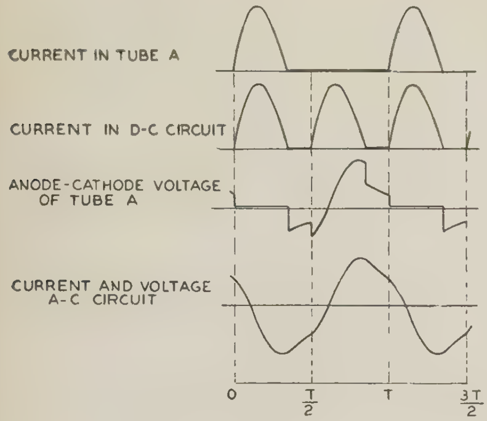


Fig. 3. Current and voltage relations when the direct current flows in pulses

For the figure shown, the following 3 differential equations may be written:

$$Ri_2 - \frac{q}{4C} = 0 \tag{1}$$

$$L\frac{di_1}{dt} + \frac{q}{4C} = E \tag{2}$$

$$-\frac{dq}{dt} = i_2 - i_1 \tag{3}$$

in which

i_1 = current in the circuit connected to the supply
i_2 = current in the load
q = charge on the capacitor of such sign as to make the right-hand side positive

Now let

$$i_1 = I_1\epsilon^{mt} \tag{4}$$

$$i_2 = I_2\epsilon^{mt} \tag{5}$$

$$q = Q\epsilon^{mt} \tag{6}$$

and substitute in equations 1 to 3, letting the right-hand member of equation 2 be zero. Then

$$R\,I_2 - \frac{Q}{4C} = 0 \tag{7}$$

$$mL\,I_1 + \frac{Q}{4C} = 0 \tag{8}$$

$$-mQ = I_2 - I_1 \tag{9}$$

From the determinant of equations 7 to 9 the following auxiliary equation is obtained:

$$4CR\,m^2 + m + \frac{R}{L} = 0 \tag{10}$$

The roots of this equation give the values of m which when used in connection with equations 4 to 6 give the complementary functions. These roots are

$$m = \frac{1}{2(4CR)}\left[-1 \pm \sqrt{1 - 4(4CR)\frac{R}{L}}\right] \tag{11}$$

Now introducing the parameters $Y = 2\pi fC$ and $X = 2\pi fL$, there results

$$m = \frac{\pi f}{K}\left[-1 \pm \sqrt{1 - \frac{4K}{J}}\right] \tag{12}$$

It is evident from this equation that 2 distinct (if the special boundary case between the 2 be neglected) solutions result: first, the case in which the 2 roots are real, the transient terms then consisting of 2 exponentials; and second, the case in which the 2 roots are complex, the transient terms then consisting of 2 damped sinusoids. The division between these 2 cases is obtained by equating the quantity under the radical to zero. Thus

$$K = \frac{J}{4} \tag{13}$$

The straight line in figure 4 shows graphically the division between these 2 cases.

Solution With Roots Real

Let the roots of equation 12 for this case be $-a$ and $-b$ where

$$a = \frac{\pi f}{K}\left[1 + \sqrt{1 - \frac{4K}{J}}\right] \tag{14}$$

$$b = \frac{\pi f}{K}\left[1 - \sqrt{1 - \frac{4K}{J}}\right] \tag{15}$$

The complete solution for the current i_1 may be written

$$i_1 = I_1'\epsilon^{-at} + I_1''\epsilon^{-bt} + \frac{E}{R} \tag{16}$$

in which I_1' and I_1'' are integration constants and E/R is the particular integral. Similarly, the charge on the capacitor may be written

$$q = Q'\epsilon^{-at} + Q''\epsilon^{-bt} + 4CE$$

in which Q' and Q'' are integration constants related to I_1' and I_1'' through equation 8. Thus

$$q = 4CaL\,I_1'\epsilon^{-at} + 4CbL\,I_1''\epsilon^{-bt} + 4CE \tag{17}$$

The instantaneous voltage across the capacitor in figure 5*b* is

$$e = q/4C \tag{18}$$

so that from equation 17

$$e = aL\,I_1'\epsilon^{-at} + bL\,I_1''\epsilon^{-bt} + E \tag{19}$$

The integration constants may be obtained by applying the terminal conditions to equations 16 and 19. Calling $T = 1/f$, the full period of the control frequency, it may be seen from figure 2 that

$$i_{1(\text{for } t\,=\,0)} = i_{1(\text{for } t\,=\,T/2)} \tag{20}$$

and

$$e_{(\text{for } t\,=\,0)} = -e_{(\text{for } t\,=\,T/2)} \tag{21}$$

After applying equations 20 and 21 to equations 16 and 19

$$(1 - \epsilon^{\frac{-aT}{2}})\,I_1' + (1 - \epsilon^{\frac{-bT}{2}})\,I_1'' = 0 \tag{22}$$

$$aL(1+\epsilon^{\frac{-aT}{2}})\, I_1' + bL\,(1+\epsilon^{\frac{-bT}{2}})\, I_1'' = -2E \tag{23}$$

From equations 14 and 15, since $fT = 1.0$.

$$g = \frac{aT}{2} = \frac{\pi}{2K}\left[1+\sqrt{1-\frac{4K}{J}}\right] \tag{24}$$

$$h = \frac{bT}{2} = \frac{\pi}{2K}\left[1-\sqrt{1-\frac{4K}{J}}\right] \tag{25}$$

and

$$aL = \frac{2gL}{T} = \frac{2g2\pi fL}{2\pi fT} = \frac{gJR}{\pi} \tag{26}$$

$$bL = \frac{2hL}{T} = \frac{2h2\pi fL}{2\pi fT} = \frac{hJR}{\pi} \tag{27}$$

Rewriting equations 22 and 23

$$(1-\epsilon^{-g})\, I_1' + (1-\epsilon^{-h})\, I_1'' = 0 \tag{28}$$

$$g(1+\epsilon^{-g})\, I_1' + h(1+\epsilon^{-h})\, I_1'' = -\frac{2\pi}{JR}E \tag{29}$$

Solving for I_1' and I_1'',

$$I_1' = \frac{1}{\Delta}(1-\epsilon^{-h})\frac{2\pi}{JR}E \tag{30}$$

$$I_1'' = -\frac{1}{\Delta}(1-\epsilon^{-g})\frac{2\pi}{JR}E \tag{31}$$

in which

$$\Delta = h\,(1-\epsilon^{-g})\,(1+\epsilon^{-h}) - g\,(1-\epsilon^{-h})\,(1+\epsilon^{-g}) \tag{32}$$

Therefore

$$\frac{i_1R}{E} = \frac{2\pi}{J\Delta}(1-\epsilon^{-h})\,\epsilon^{\frac{-2g}{T}t} - \frac{2\pi}{J\Delta}(1-\epsilon^{-g})\,\epsilon^{\frac{-2h}{T}} + 1 \tag{33}$$

Also, from equations 19, 26, 27, 30, and 31,

$$\frac{e}{E} = \frac{2g}{\Delta}(1-\epsilon^{-h})\,\epsilon^{\frac{-2g}{T}t} - \frac{2h}{\Delta}(1-\epsilon^{-g})\,\epsilon^{\frac{-2h}{T}t} + 1 \tag{34}$$

Solution With Roots Complex

Let the roots of equation 12 for this case be

$$m = -\alpha \pm j\beta \tag{35}$$

where

$$\alpha = \frac{\pi f}{K} \tag{36}$$

$$\beta = \frac{\pi}{K}\sqrt{\frac{4K}{J}-1} \tag{37}$$

The complete solution for the current i_1 may be written

$$i_1 = I_1'\epsilon^{(-\alpha+j\beta)t} + I_1''\,\epsilon^{(-\alpha-j\beta)t} + \frac{E}{R} \tag{38}$$

Since the complex integration constants I_1' and I_1'' will be conjugate, this equation may be written, if the factor E/R is absorbed in the integration constant, as

$$\frac{i_1R}{E} = \text{real part } I_1\epsilon^{(-\alpha+j\beta)t} + 1 \tag{39}$$

The integration constant Q can be evaluated in terms of I_1 from equation 8, after which e/E can be written by the aid of equation 18 as

$$\frac{e}{E} = -\text{ real part }(-\alpha+j\beta)\frac{L}{R}I_1\epsilon^{(\alpha+j\beta)t} + 1 \tag{40}$$

Again applying the terminal conditions expressed by equations 20 and 21 to equations 39 and 40,

$$\text{real part } I_1 + 1 = \text{real part } I_1\,\epsilon^{(-\alpha+j\beta)\frac{T}{2}} + 1 \tag{41}$$

$$-\text{ real part }(-\alpha+j\beta)\frac{L}{R}I_1 + 1 = \text{real part }(-\alpha+j\beta)\frac{L}{R}I_1\epsilon^{(-\alpha+j\beta)\frac{T}{2}} - 1 \tag{42}$$

Let

$$I_1 = m + jn \tag{43}$$

then expanding equations 41 and 42,

$$\left[1-\epsilon^{-\frac{\alpha T}{2}}\cos\beta\frac{T}{2}\right]m + \left[\epsilon^{-\frac{\alpha T}{2}}\sin\beta\frac{T}{2}\right]n = 0 \tag{44}$$

$$\text{real part }(-\alpha+j\beta)\left\{\left[1+\epsilon^{\frac{-\alpha T}{2}}\cos\beta\frac{T}{2}\right] + j\,\epsilon^{\frac{-\alpha T}{2}}\sin\beta\frac{T}{2}\right\}\frac{L}{R}(m+jn) = 2 \tag{45}$$

To simplify the notation, let

$$s = \frac{\alpha T}{2} = \frac{\pi}{2K} \tag{46}$$

$$\gamma = \frac{\beta T}{2} = \frac{\pi}{2K}\sqrt{\frac{4K}{J}-1} \tag{47}$$

$$k = \epsilon^{-s} \tag{48}$$

Equation 44 may then be written as

$$(1-k\cos\gamma)\,m + (k\sin\gamma)\,n = 0 \tag{49}$$

and equation 45 as

$$\text{real part }\frac{2}{T}(-s+j\gamma)\,[(1+k\cos\gamma)+j\,k\sin\gamma]\,\frac{X}{2\pi fR}(m+jn) = 2$$

or

$$[s(1+k\cos\gamma)+\gamma k\sin\gamma]\,m + [\gamma(1+k\cos\gamma) - s\,k\sin\gamma]\,n = -\frac{2\pi}{J} \tag{50}$$

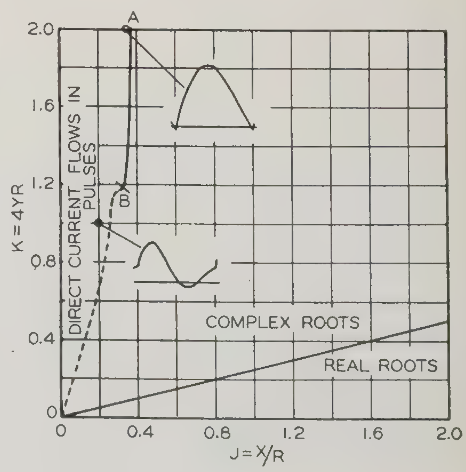


Fig. 4. Division of solutions according to relative values of J and K

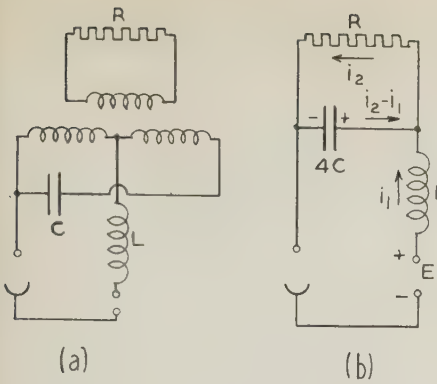


Fig. 5. Equivalent circuits for parallel inverter with resistance load

Solving for m and n in equations 49 and 50, there results

$$m = \frac{2\,\pi\,k\sin\gamma}{J\,[\gamma\,(1-k^2) - 2sk\sin\gamma]} \tag{51}$$

$$n = \frac{-\,2\,\pi\,[1-k\cos\gamma]}{J\,[\gamma\,(1-k^2) - 2\,s\,k\sin\gamma]} \tag{52}$$

From equation 39

$$\frac{i_1R}{E} = \epsilon^{\frac{-2s}{T}t}\left(m\cos\frac{2\gamma}{T}t - n\sin\frac{2\gamma}{T}t\right) + 1 \tag{53}$$

and from equation 40

$$\frac{e}{E} = -\text{ real part }\frac{L}{R}\frac{2}{T}\,[(-sm-\gamma n)+j(\gamma m - sn)]\epsilon^{(-\alpha+j\beta)t} + 1$$

$$= \frac{J}{\pi}\epsilon^{\frac{-2s}{T}t}\left[(sm+\gamma n)\cos\frac{2\gamma}{T}t + (\gamma m - sn)\sin\frac{2\gamma}{T}t\right] + 1 \tag{54}$$

Thus for any value of K or J it is possible to solve for the variation of i_1 or e by the use of equations 46 to 48 and 51 to 54. In figure 7 is shown an oscillogram and test results for a case in which the constants lie in the region which requires this type of solution.

In a large number of cases the transient terms reduce to a very small value at the end of a half cycle. Thus for $K = 0.5$, the exponential has reduced to $\epsilon^{-\frac{\pi}{2\times 0.5}}$ or 0.043. For the special case in which the exponential has reduced to zero at the end of the half cycle ($k = 0$), the solution simplifies to the following: In equations 51 and 52

$$m = 0 \tag{55}$$

$$n = -\frac{2\pi}{J\gamma} \tag{56}$$

and equations 53 and 54 become

$$\frac{i_1R}{E} = \frac{2\pi}{J\gamma}\epsilon^{\frac{-2s}{T}t}\sin\frac{2\gamma}{T}t + 1 \tag{57}$$

$$\frac{e}{E} = \epsilon^{\frac{-2s}{T}t}\left[-2\cos\frac{2\gamma}{T}t + \frac{2s}{\gamma}\sin\frac{2\gamma}{T}t\right] + 1 \tag{58}$$

Failure of Solutions to Comply With Physical Requirements of Tubes

It was explained previously that as L decreases the variations in i_1 become so great that the current is extinguished before the end of the half cycle. A general analytical expression to determine the limits at which this phenomenon occurs is difficult to obtain. However, for the special case just considered in which the transient components reduce to zero at the end of a half cycle, it is possible to obtain a simple relation between the parameters at which the phenomenon first appears. The limiting condition occurs when the minimum value of current just touches zero. This may be determined mathematically by obtaining the point at which minimum occurs and substituting this value in the expression for current and equating to zero. Equating the differential of equation 57 to zero

$$\frac{2\gamma}{T}\epsilon^{\frac{-2st}{T}}\cos\frac{2\gamma}{T}t - \frac{2s}{T}\epsilon^{\frac{-2st}{T}}\sin\frac{2\gamma}{T}t = 0 \tag{59}$$

Calling the time which satisfies this equation t'',

$$t'' = \frac{T}{2\gamma}\left[\tan^{-1}\frac{\gamma}{s} + \pi\right] \tag{60}$$

the π is added to obtain the first minimum; without it, the expression gives the first maximum.

Substituting this value of t'' in equation 57 and equating i_1 to zero,

$$\frac{i_1R}{E} = 0 = \frac{2\pi}{J\gamma}\epsilon^{-\frac{2s}{T}\frac{T}{2\gamma}\left(\tan^{-1}\frac{\gamma}{s}+\pi\right)}\sin\left[\frac{2\gamma}{T}\frac{T}{2\gamma}\left(\tan^{-1}\frac{\gamma}{s}+\pi\right)\right] + 1$$

$$= -\frac{2\pi}{J\gamma}\epsilon^{\frac{-s}{\gamma}\left(\tan^{-1}\frac{\gamma}{s}+\pi\right)}\frac{\gamma}{\sqrt{s^2+\gamma^2}} + 1$$

or

$$\epsilon^{-\frac{s}{\gamma}\left(\tan^{-1}\frac{\gamma}{s}+\pi\right)} = \frac{J}{2\pi}\sqrt{s^2+\gamma^2} \tag{61}$$

From equations 46 and 47

$$\frac{\gamma}{s} = \sqrt{\frac{4K}{J} - 1}$$

$$s^2 + \gamma^2 = \frac{\pi^2}{4K^2} + \frac{\pi^2}{4K^2}\left(\frac{4K}{J} - 1\right) = \frac{\pi^2}{KJ}$$

On substituting these values in equation 61,

$$\epsilon^{-\frac{\tan^{-1}\sqrt{\frac{4K}{J}-1}+\pi}{\sqrt{\frac{4K}{J}-1}}} = \frac{1}{\sqrt{\frac{4K}{J}}} \tag{62}$$

It will be observed that equation 62 is a function of $4K/J$ alone. By a cut-and-try process it was established that a value of $4K/J$ equal to 12.96 satisfies this equation. Thus the relation

$$K = \frac{12.96}{4}J = 3.24\,J \tag{63}$$

expresses the relation between K and J for small values of K. The relation between K and J for somewhat larger values of K which would just prevent negative values of direct current from appearing were obtained by a cut-and-try process and the results plotted by the line OB in figure 4. Below the point

B the solution fails to comply with the physical requirements of tube conduction by the appearance of a negative current at the point in the curve where the slope is zero (see the wave shape plotted for point $K = 1.0$). Above the point *B* the solution fails by the appearance of negative current at the 2 ends of the curve. The mathematical requirements are still satisfied but the physical requirements are not. To the left of the line *OA*, therefore, the direct current must flow in pulses.

Summary of Results

Figure 6 shows families of curves which were calculated by the foregoing methods. Each family represents the characteristics for a constant value of

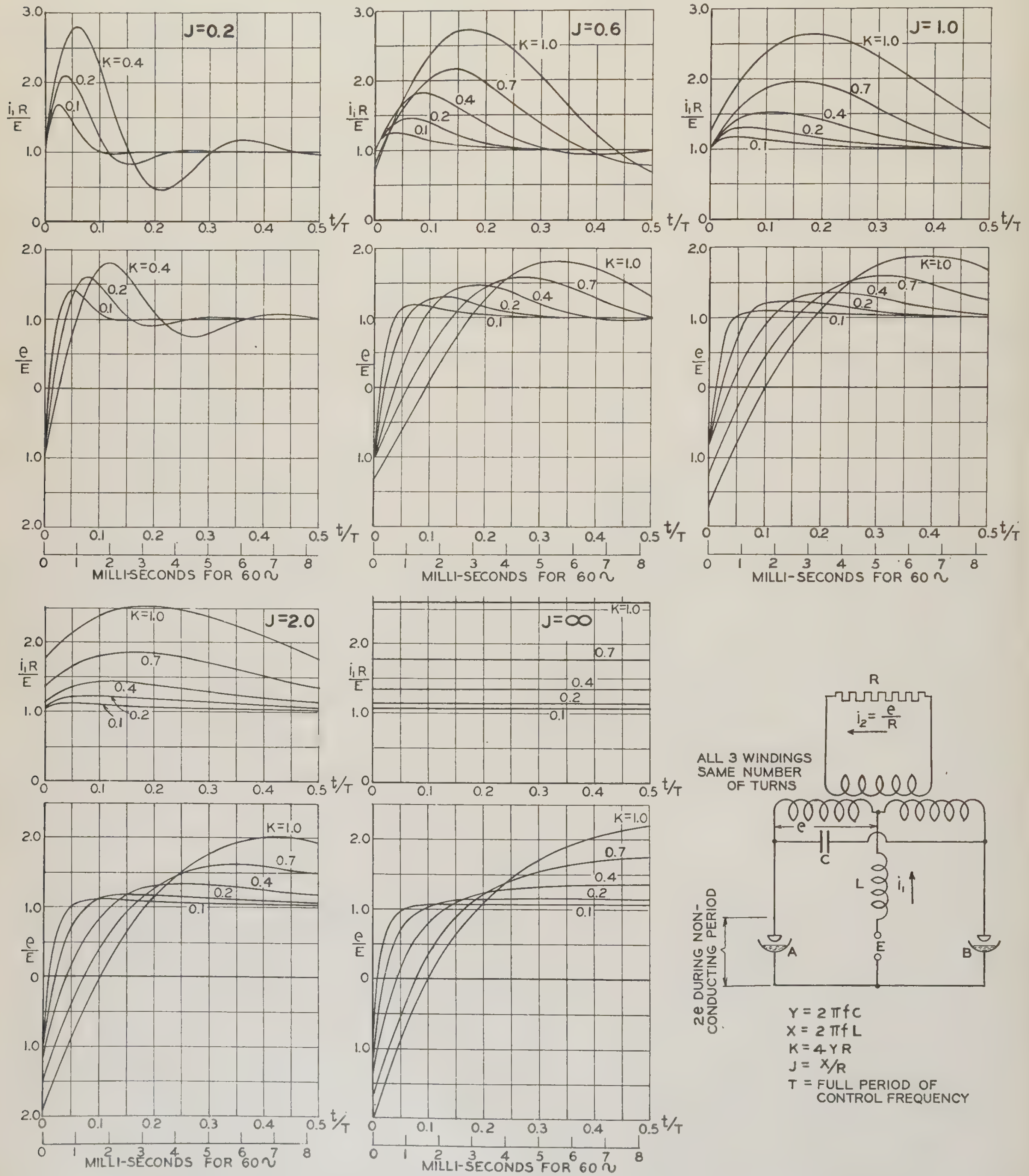


Fig. 6. Current and voltage relations for single phase inverter with resistance load

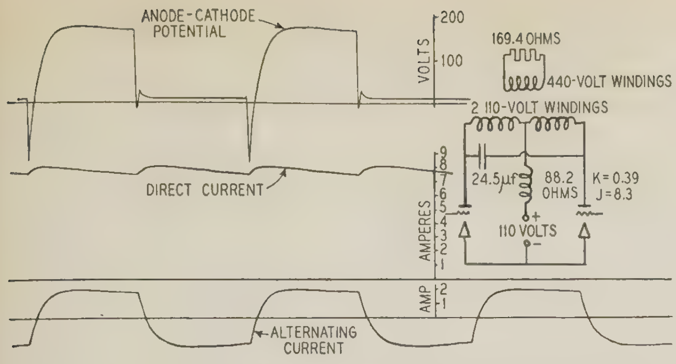


Fig. 7. Oscillogram of 60 cycle inverter operation for K = 0.39 and J = 8.3

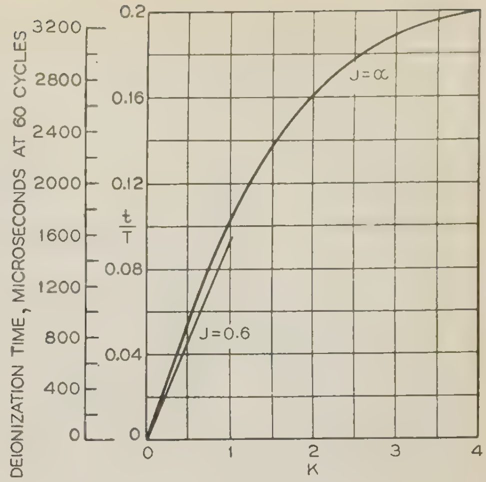


Fig. 8. Deionization time as a function of K and J

J or X/R and different values of K or $4YR$. The abscissa in all cases is the ratio t/T, which extends to only 0.5, representing a half period. The load current, i_2R/E, varies cyclically, the next half cycle being the negative of the particular half cycle plotted, thus forming a continuous alternating current. The transformer voltage curves, e/E, vary in a similar manner. It must be remembered that this is the voltage across the capacitor and also across the load in the equivalent circuit. To get the voltage across the actual capacitor connected between the outside terminals of 2 windings and which is also, except for the arc drop, the anode-cathode voltage during the nonconducting period of the tube, it is necessary to multiply the function e/E by $2E$.

Plotted in the manner indicated, the results may be used for any frequency. In addition to the time scale t/T, a corresponding time scale for the 60 cycle case is also plotted.

To approximate the effect of arc drop, E in this development should represent the actual continuous voltage minus the arc drop which is essentially constant. This will give correct values for i_1, i_2, and capacitor voltage. To get the anode-cathode voltage it is necessary to multiply $2\ e/E$ by the actual continuous voltage minus the arc drop and then to add the arc drop to the product. Figure 7 shows an oscillogram of anode-cathode potential, load current, and direct current for the case in which the parameters are $J = 8.3$ and $K = 0.39$.

Deionization Time

A knowledge of the deionization characteristics of the particular tube used is a prerequisite to the design of any inverter circuit. Actually this is not a constant but depends upon the magnitude and the rate of decay of the tube current just prior to current zero and also upon the rate of rise of anode-cathode voltage. As a measure of the circuit characteristics the time from current zero to the time positive current first appears may be taken conveniently. For satisfactory operation of the inverter it is, therefore, necessary that the actual deionization time of the tube be smaller than this quantity. The deionization time defined in this manner and taken from the curves of figure 6 is plotted in figure 8. It will be observed that this quantity is proportional to the capacitance for a large range of values, but varies little with change in J.

For a given commutating capacitor, if the load current is decreased by increasing the load resistance, then $K = 4YR$ increases. From the curve it may be seen that the deionization time also increases. It follows then that if the constants of the circuit are adjusted so that sufficient deionization time is obtained at full load the deionization time for fractional loads will be greater.

Power and Effective Value of Load Voltage

The power P taken by the load is obtained most readily from the arithmetic mean, I_{1e}, of the direct current. Thus

$$P = EI_{1e}$$

The power taken by the load may also be expressed in terms of the effective value E_e^2 of the load voltage. Thus

$$P = \frac{E_e^2}{R} = EI_{1e}$$

Multiplying through by R/E^2, there is obtained

$$\frac{PR}{E^2} = \frac{E_e^2}{E^2} = \frac{RI_{1e}}{E} \tag{64}$$

By integrating the analytical expression for Ri_1/E it is thus possible to obtain values from which E_e^2 and P may be obtained directly.

For the case in which the roots are real, after integrating equation 33 and dividing by $T/2$, there results

$$\frac{RI_{1e}}{E} = \frac{2\pi}{J\Delta}(1 - \epsilon^{-g})(1 - \epsilon^{-h})\frac{h - g}{gh} + 1 \tag{65}$$

For the case in which the roots are complex, a similar operation on equation 53 gives

$$\frac{RI_{1e}}{E} = \frac{k}{s^2 + \gamma^2}[(-ms + n\gamma)\cos\gamma + (m\gamma + ns)\sin\gamma] + \frac{ms - n\gamma}{s^2 + \gamma^2} + 1 \tag{66}$$

The results obtained by applying equations 65 and 66 are plotted in figure 9. These curves should

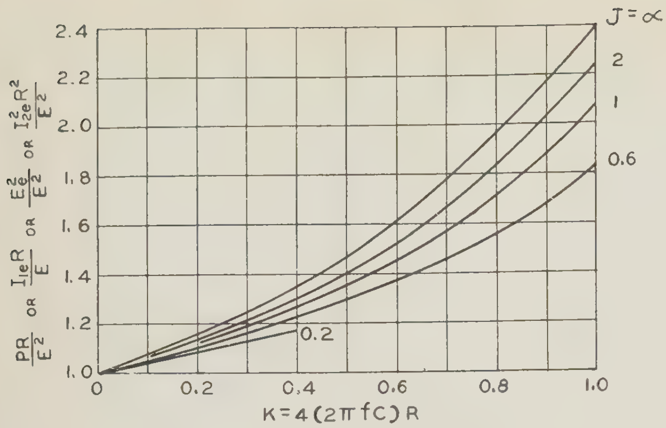


Fig. 9. Power, arithmetic mean value of i_1, and effective values of e and i_2 as a function of J and K

be useful for the determination of regulation at half load, obtained by doubling the load resistance, K is doubled and J is halved. The effect upon the voltage may be read from the curves.

Solution When Direct Current Flows in Pulses

This problem of direct current flowing in pulses is complicated by the fact that 2 circuit conditions must be considered: first, the period during which a tube is conducting, and second, the period during which no tube is conducting. During the latter period the capacitor discharges through the load resistance. The integration constants must be so chosen that the capacitor potentials match on at the boundaries of these 2 periods to form a continuous function. Because of the particular manner in which the transcendental functions are involved it is impossible to obtain a straightforward solution for this case. A cut-and-try process is necessary to determine one of the integration constants. During the conducting period the natural oscillation constants of the circuit are given by equations 46 to 48, and the expression for i_1 may be written in the form of equation 53 as

$$\frac{i_1R}{E} = \epsilon^{\frac{-2s}{T}t}\left(M\cos\frac{2\gamma}{T}t - N\sin\frac{2\gamma}{T}t\right) + 1 \tag{67}$$

and e may be written in the form of equation 54 as

$$\frac{e}{E} = \frac{J}{\pi}\epsilon^{\frac{-2s}{T}t}\left[(sM + \gamma N)\cos\frac{2\gamma}{T}t + (\gamma M - sN)\sin\frac{2\gamma}{T}t\right] + 1 \tag{68}$$

in which M and N are the integration constants.

The integration constant M may be determined by applying the terminal condition $i_1 = 0$ for $t = 0$ in equation 67. Thus,

$$M = -1 \tag{69}$$

Now designate the time at which i_1 again reaches zero by t'. Then from equation 67

$$N = \frac{\epsilon^{\frac{2s}{T}t'} - \cos\frac{2\gamma t'}{T}}{\sin\frac{2\gamma t'}{T}} \tag{70}$$

On substituting equation 69 in equation 68, then for $t = 0$

$$\frac{e}{E} = \frac{J}{\pi}(-s + \gamma N) + 1 \tag{71}$$

and for $t = t'$,

$$\frac{e}{E} = \frac{J}{\pi}\epsilon^{\frac{-2st'}{T}}\left[(-s + \gamma N)\cos\frac{2\gamma t'}{T} - (\gamma + sN)\sin\frac{2\gamma t'}{T}\right] + 1 \tag{72}$$

During the nonconducting period the capacitor discharges through the load resistance with a time con-

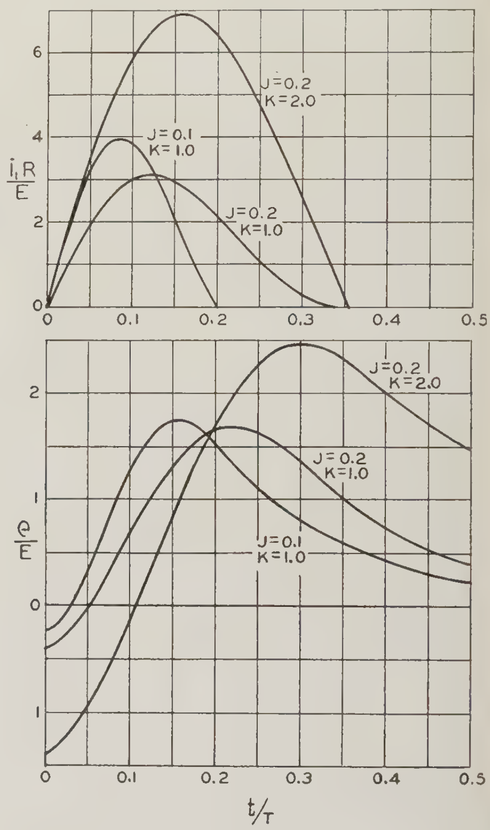


Fig. 10. Calculated curves for current and voltage relations when direct current flows in pulses

stant equal to $4RC$ or $K/2\pi f$ or $1/2\alpha$ or $T/4s$. The total decrease in capacitor voltage during this period is then

$$\epsilon^{\frac{-4s}{T}\left(\frac{T}{2} - t'\right)} \text{ or } \epsilon^{-2s + \frac{4st'}{T}}$$

Multiplying equation 72 by this factor gives the value of e/E at the end of the half cycle as

$$\frac{e}{E} = \frac{J}{\pi}\epsilon^{-2s\left(1 - \frac{t'}{T}\right)}\left[(-s + \gamma N)\cos\frac{2\gamma t'}{T} - (\gamma + sN)\sin\frac{2\gamma t'}{T}\right] + 1 \tag{73}$$

A value of t' must be arbitrarily chosen until N calculated from equation 70 gives equal but negative values of e/E when inserted in equations 71 and 73.

Equations 67 and 68 then give the instantaneous values of i_1 and e for the conducting period. During

the nonconducting period $i_1 = 0$ and e decreases exponentially with a time constant equal to $T/4s$. In figure 10 are shown the results of calculation for 3 sets of constants which fall within this region.

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Capacitor Motors With Windings Not in Quadrature

Quite often the design of a split phase induction motor is such that the so-called quadrature winding is displaced at some angle other than 90 degrees from the main winding, in which event the motor behaves differently when run in opposite directions. It is the purpose of this paper to indicate a method by which the designer can predict the performance of such a motor. While the emphasis is placed upon the capacitor motor, the treatment is applicable also to motors of other types.

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IN the design of split-phase induction motors, the number of stator slots and poles may frequently be such that the so-called quadrature winding is, in reality, other than 90 degrees from the main winding. Under such a condition the operating characteristics of the motor are modified; the torque chiefly, being different from that predicted by the usual analyses. Deliberate adjustment of the winding displacements may be used to modify the torque characteristics of the motor. In all such cases the motor displays different behavior when run in opposite directions; this is frequently a desirable characteristic.

It is the purpose of this paper to indicate a method of determining such effects for a motor of the split-phase type so as to enable the designer to predict the characteristics for those cases in which the windings are not in quadrature, and accurate torque calculations are necessary. Emphasis is placed upon the capacitor motor calculations as they represent a slightly more general case, although, as indicated later, the equations developed are suitable for the plain single phase, plain split-phase, capacitor, 2-phase, and shaded pole motors, with windings equal or unequal, and displaced any number of electrical degrees. The work is an extension of that given by Wayne J. Morrill,[1] whose analysis of the capacitor motor, based upon the revolving field theory, was found to be adaptable to these further generalizations. To aid in cross reference, the symbols used here will follow (for the most part) those given in his paper.

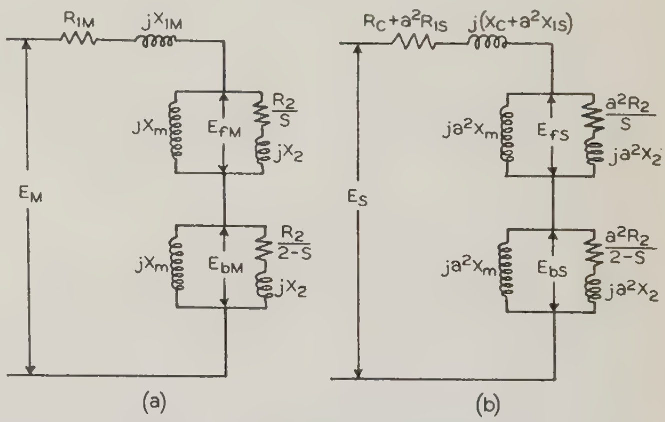

Fig. 1. Equivalent networks of the single phase motor

The method shown in this paper covers a more general process than that hitherto available for presenting the theory and calculations of a 2-phase machine in which the 2 stator windings are unequal with respect to the number of turns and distribution, are not 90 degrees apart in space, and are supplied by unequal impressed voltages not 90 degrees apart in time. The ability to displace windings other than

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1. For reference see end of paper.

90 degrees and then calculate the machine characteristics, gives the designer an additional tool to meet the multiplicity of difficult industrial specifications.

Theory

In the revolving field theory of the single phase induction motor, the alternating flux produced by the stator winding is resolved into a pair of sinusoidal waves revolving in opposite directions at synchronous speed. The effect upon the stator, produced by each of these fluxes, can be represented by the network of figure 1*a*. If the starting phase *S* is operating alone, the equivalent network is as shown by figure 1*b*.

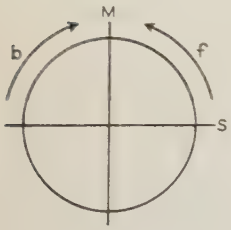

Fig. 2. Relative positions of the fields

If both the main, *M*, and the starting, *S*, phases are excited simultaneously, their fluxes superimpose without distortion and the equivalent circuit of each phase is the same as before, except that in addition to the forward and backward voltages (self-induced in each phase) there is present in each phase a forward and backward voltage due to the fluxes of the other phase.

Referring to figure 2, since *M* is 90 electrical degrees ahead of *S*, the voltage generated in *M* by the *S* forward flux must lag, by 90 time degrees, the voltage which the same flux produces in *S*; and since the ratio of effective turns of the windings, or T_M to T_S, is $1/a$, the voltage in *M* must be $1/a$ times that of *S*. Then

$$\underset{Sf}{\boldsymbol{E}_M} = -j\frac{\boldsymbol{E}_{fS}}{a} = \epsilon^{-j\frac{\pi}{2}}\frac{\boldsymbol{E}_{fS}}{a} \qquad (1)$$

in which the subscript *M* identifies the voltage as being that of the main winding, and the underscript *Sf* indicates that the voltage arises from the forward flux of the starting winding.

The voltage generated in *M* by the *S* backward flux must lead by 90 time degrees the voltage which the same flux produces in *S*. Then

$$\underset{Sb}{\boldsymbol{E}_M} = j\frac{\boldsymbol{E}_{bS}}{a} = \epsilon^{j\frac{\pi}{2}}\frac{\boldsymbol{E}_{bS}}{a} \qquad (2)$$

The voltage generated in *S* by the *M* forward flux must lead by 90 time degrees the voltage which the same flux produces in *M*, and since the ratio of effective turns of $T_S/T_M = a$, this *S* voltage must be *a* times that of *M*. Then

$$\underset{Mf}{\boldsymbol{E}_S} = ja\boldsymbol{E}_{fM} = \epsilon^{j\frac{\pi}{2}}a\boldsymbol{E}_{fM} \qquad (3)$$

The voltage generated in *S* by the *M* backward flux must lag by 90 time degrees the voltage which this same flux produces in *M*. Then

$$\underset{Mb}{\boldsymbol{E}_S} = -ja\boldsymbol{E}_{bM} = \epsilon^{-j\frac{\pi}{2}}a\boldsymbol{E}_{bM} \qquad (4)$$

The equivalent networks then become those shown in figure 3. The divided circuits of figures 1*a* and 1*b* have been replaced by series impedances of equivalent values.

If the *M* and *S* windings are not 90 degrees apart in space, equations 1 to 4, respectively, become

$$\underset{Sf}{\boldsymbol{E}_M} = \epsilon^{-j\alpha_1}\frac{\boldsymbol{E}_{fS}}{a} \qquad (1a)$$

$$\underset{Sb}{\boldsymbol{E}_M} = \epsilon^{j\alpha_1}\frac{\boldsymbol{E}_{bS}}{a} \qquad (2a)$$

$$\underset{Mf}{\boldsymbol{E}_S} = \epsilon^{j\alpha_1}a\boldsymbol{E}_{fM} \qquad (3a)$$

$$\underset{Mb}{\boldsymbol{E}_S} = \epsilon^{-j\alpha_1}a\boldsymbol{E}_{bM} \qquad (4a)$$

wherein α_1 is the angle in degrees between *M* and *S*.

In the stator there is a transformer action between *M* and *S* which, it will be assumed, produces the same effects as do these 2 forward and backward moving fluxes. This transformer action, in either direction, is not the same as that which would be produced by a transformer action between *M* and *S* if it were of equal value but outside the machine.

Examination of the circuits of figure 3 indicates the following voltage and current relationships, all values being considered vectorially:

$$\boldsymbol{E}_M = \boldsymbol{E}_{1M} + \boldsymbol{E}_{fM} + \boldsymbol{E}_{bM} + \underset{Sb}{\boldsymbol{E}_M} + \underset{Sf}{\boldsymbol{E}_M} \qquad (5)$$

$$\boldsymbol{E}_S = \boldsymbol{E}_{1S} + \boldsymbol{E}_{fS} + \boldsymbol{E}_{bS} + \underset{Mb}{\boldsymbol{E}_S} + \underset{Mf}{\boldsymbol{E}_S} \qquad (6)$$

$$\boldsymbol{E}_{1M} = \boldsymbol{I}_M(R_{1M} + jX_{1M}) \qquad (7)$$

$$\boldsymbol{E}_{fM} = \boldsymbol{I}_M(R_f + jX_f) \qquad (8)$$

$$\boldsymbol{E}_{bM} = \boldsymbol{I}_M(R_b + jX_b) \qquad (9)$$

$$\underset{Sf}{\boldsymbol{E}_M} = \boldsymbol{I}_S a\epsilon^{-j\alpha_1}(R_f + jX_f) \qquad (10)$$

$$\underset{Sb}{\boldsymbol{E}_M} = \boldsymbol{I}_S a\epsilon^{j\alpha_1}(R_b + jX_b) \qquad (11)$$

$$\boldsymbol{E}_{1S} = \boldsymbol{I}_S a^2(R_{1S} + jX_{1S}) \qquad (12)$$

$$\boldsymbol{E}_{fS} = \boldsymbol{I}_S a^2(R_f + jX_f) \qquad (13)$$

$$\boldsymbol{E}_{bS} = \boldsymbol{I}_S a^2(R_b + jX_b) \qquad (14)$$

$$\underset{Mf}{\boldsymbol{E}_S} = \boldsymbol{I}_M a\epsilon^{j\alpha_1}(R_f + jX_f) \qquad (15)$$

$$\underset{Mb}{\boldsymbol{E}_S} = \boldsymbol{I}_M a\epsilon^{-j\alpha_1}(R_b + jX_b) \qquad (16)$$

Expanded, the equations of total voltage become

$$\boldsymbol{E}_M = \boldsymbol{I}_M[(R_{1M} + jX_{1M}) + (R_f + jX_f) + (R_b + jX_b)] + \boldsymbol{I}_S[a(R_b + jX_b)\epsilon^{j\alpha_1} + a(R_f + jX_f)\epsilon^{-j\alpha_1} \pm j\omega M] \qquad (17)$$

$$\boldsymbol{E}_S = \boldsymbol{I}_S[a^2(R_{1S} + jX_{1S}) + a^2(R_f + jX_f) + a^2(R_b + jX_b)] + \boldsymbol{I}_M[a(R_b + jX_b)\epsilon^{-j\alpha_1} + a(R_f + jX_f)\epsilon^{j\alpha_1} \pm j\omega M] \qquad (18)$$

Referring to equation 17, let the first expression in the brackets be represented by Z_I; let the second be Z_{II}. In equation 18 let the first expression be Z_{III}; and the second expression be Z_{IV}. Note the quantity $j\omega M$. This is to take care of the coupling caused by local mutual leakage fluxes which link both

stator windings but do not link with the rotor windings. It can readily be measured and possibly calculated but it will, in general, be small enough to neglect here. Using the impedances, equations 17 and 18 reduce to

$$\boldsymbol{I}_M Z_I + \boldsymbol{I}_S Z_{II} = \boldsymbol{E}_M \tag{17a}$$

$$\boldsymbol{I}_M Z_{IV} + \boldsymbol{I}_S Z_{III} = \boldsymbol{E}_S \tag{18b}$$

Solving these 2 equations simultaneously for currents,

$$\boldsymbol{I}_M = \frac{\boldsymbol{E}_M Z_{III} - \boldsymbol{E}_S Z_{II}}{Z_I Z_{III} - Z_{II} Z_{IV}} = A + jB \tag{19}$$

$$\boldsymbol{I}_S = \frac{\boldsymbol{E}_S Z_I - \boldsymbol{E}_M Z_{IV}}{Z_I Z_{III} - Z_{II} Z_{IV}} = g + jh \tag{20}$$

$$\frac{\boldsymbol{I}_S}{\boldsymbol{I}_M} = \frac{\boldsymbol{E}_S Z_I - \boldsymbol{E}_M Z_{IV}}{\boldsymbol{E}_M Z_{III} - \boldsymbol{E}_S Z_{II}} = \frac{Z_I - Z_{IV}}{Z_{III} - Z_{II}} \tag{21}$$

The starting currents, with windings 90 degrees apart, are

$$\boldsymbol{I}_M = \frac{\boldsymbol{E}_M}{Z_I} \tag{22}$$

$$\boldsymbol{I}_S = \frac{\boldsymbol{E}_S}{Z_{III}} \tag{23}$$

The torque equation per inch of rotor periphery can be set up for the condition of $\alpha_1 \lesseqgtr 90$ degrees,

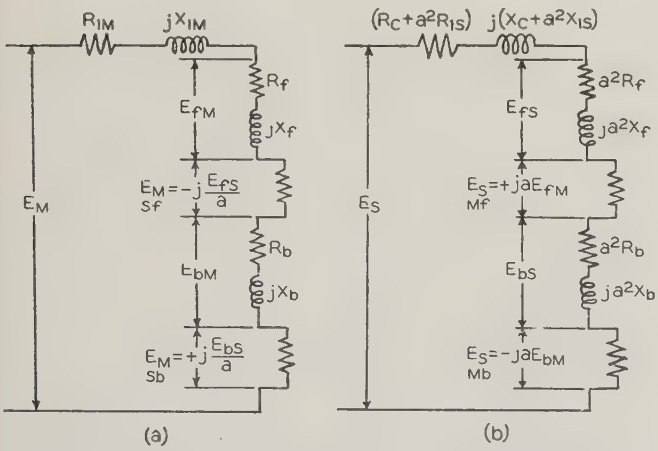

Fig. 3. Equivalent networks of the unbalanced 2 phase motor

and integrated to determine the average torque. This yields (after a long integration) the following expression for average torque in synchronous watts:

$$T_{avg.} = (\boldsymbol{I}_M^2 + a^2\boldsymbol{I}_S^2)(R_f - R_b) + 2a\boldsymbol{I}_S\boldsymbol{I}_M[R_f \cos(\alpha_1 - \varphi) - R_b \cos(\alpha_1 + \varphi)] \tag{24}$$

If α_1 equals 90 degrees this reduces to:

$$T_{avg.} = (\boldsymbol{I}_M^2 + a^2\boldsymbol{I}_S^2)(R_f - R_b) + 2a(Ah - Bg)(R_f + R_b) \tag{25}$$

$$Ah - Bg = \boldsymbol{I}_M\boldsymbol{I}_S \sin\varphi$$

where φ is the angle between $\boldsymbol{I}_M$ and $\boldsymbol{I}_S$.

The starting torque for any displacement becomes

$$T_{avg.}\ (\text{starting}) = 2a\boldsymbol{I}_S\boldsymbol{I}_M[R_f \cos(\alpha_1 - \varphi) - R_b \cos(\alpha_1 + \varphi)] \tag{26}$$

Constants Required for Calculation

1. The ratio $a = \frac{T_{Sk_wS}}{T_{Mk_wM}}$ in which k_w represents the winding factor and T the number of series turns
2. X_m = magnetizing reactance, which is half of the test or design value. (The double-field theory assumes that the motor is replaced by one stator and 2 oppositely rotating rotors, each with half constants)
3. R_{1M} = resistance of the main stator winding
4. R_{1S} = resistance of the starting stator winding divided by a^2
5. R_2 = rotor resistance (half of design or test value)
6. X_{1M} = leakage reactance of stator main winding M
7. X_{1S} = leakage reactance of stator starting winding S divided by a^2
8. X_2 = leakage reactance of rotor (half of design or test value)
9. X_c = reactance of the capacitor used with S winding. It requires the minus sign
10. R_c = resistance of the capacitor

Calculate the following combinations of terms for a slip S:

11. $R_f = \dfrac{X_m^2\left(\frac{R_2}{S}\right)}{\left(\frac{R_2}{S}\right)^2 + (X_2 + X_m)^2}$ At synchronism $S = 0$ and $R_f = 0$

12. $X_f = X_m \dfrac{\left(\frac{R_2}{S}\right)^2 + X_2(X_2 + X_m)}{\left(\frac{R_2}{S}\right)^2 + (X_2 + X_m)^2}$ At synchronism $S = 0$ and $X_f = X_m$

13. $R_b = \dfrac{X_m^2\left(\frac{R_2}{2 - S}\right)}{\left(\frac{R_2}{2 - S}\right)^2 + (X_2 + X_m)^2}$

At synchronism $R_b = \dfrac{X_m^2 R_2}{R_2^2 + (X_2 + X_m)^2}$

14. $X_b = X_m \dfrac{\left(\frac{R_2}{2 - S}\right)^2 + X_2(X_2 + X_m)}{\left(\frac{R_2}{2 - S}\right)^2 + (X_2 + X_m)^2}$

At synchronism $X_b \doteq X_m \dfrac{R_2^2 + X_2(X_2 + X_m)}{R_2^2 + (X_2 + X_m)^2}$

15. $R_{1M} = R_{1M} + R_f + R_b$
16. $X_{1M} = X_{1M} + X_f + X_b$
17. $R_{1S} = R_c + a^2(R_{1S} + R_f + R_b)$
18. $X_{1S} = X_c + a^2(X_{1S} + X_f + X_b)$
19. $R = R_f - R_b$
20. $X = X_f - X_b$

Referring to equations 17 and 18,

21. $Z_I = R_{1M} + jX_{1M}$
22. $Z_{II} = a(R_f + jX_f)\epsilon^{-j\alpha_1} + a(R_b + jX_b)\epsilon^{j\alpha_1} \pm j\omega M$
$= -ja[(R_f - R_b) + j(X_f - X_b)]$ when $\alpha_1 = 90$ degrees

Note that in items 22 and 24 with M considered, the plus sign is used when the magnetomotive forces are prevailingly additive, and the minus sign is used when the magnetomotive forces are prevailingly opposite.

23. $Z_{III} = R_{IS} + jX_{IS}$

24. $Z_{IV} = a(R_f + jX_f)\epsilon^{j\alpha_1} + a(R_b + jX_b)\epsilon^{-j\alpha_1} \pm j\omega M$

$= ja[(R_f - R_b) + j(X_f - X_b)]$ when $\alpha_1 = 90$ degrees

25. Capacitor voltage:

$$E_c = I_s\sqrt{R_c^2 + X_c^2}$$

Theoretically a capacitor is usually represented as an ideal capacitance with a shunted resistance. The capacitors utilized in practice have a comparatively high foil resistance which gives the effect of a series circuit.

Applications of Equations

The equations given here can be applied to the analysis of the following:

(*a*) Plain single-phase induction motor

(*b*) Split-phase induction motor, windings 90 degrees apart

(*c*) Capacitor induction motor, windings 90 degrees apart

(*d*) Split-phase induction motor, windings not 90 degrees apart

(*e*) Capacitor induction motor, windings not 90 degrees apart

(*f*) Shaded pole induction motor

(*g*) Two-phase induction motor, windings equal and 90 degrees apart

(*h*) Two-phase induction motor, windings equal and not 90 degrees apart

(*i*) Two-phase induction motor, windings unequal and 90 degrees apart

(*j*) Two-phase induction motor, windings unequal and not 90 degrees apart

In *g*, *h*, *i*, and *j* the terminal voltages are assumed to be 90 degrees apart in time, but the voltage of the second phase can be taken as $E_1(\cos\alpha_2 + j\sin\alpha_2)$ if such is not the case. It should also be pointed out that the shaded pole motor for which this analysis is applicable is the continuous-iron distributed-winding type, and not the usual construction.

Taking up each of these analyses separately, the equations for I_M, I_S, and T_{avg} are:

(*a*). Mutual inductance M (as here used) and ratio of transformation become zero, I_S equals zero, Z_{II} and Z_{IV} become zero, and

$$I_M = \frac{E}{Z_I} \qquad T_{avg} = I_M^2(R_f - R_b)$$

(*b*). I_M, I_S, and T_{avg} are as in equations 19, 20, and 25 with the impedances (Z terms) modified by R_c and X_c being equal to zero.

(*c*). I_M, I_S, and T_{avg} are as in equations 19, 20, and 25.

(*d*). R_c and $X_c = 0$. I_m, I_S, and T_{avg} are as in equations 19, 20, and 24.

(*e*). I_M, I_S, and T_{avg} are as in equations 19, 20, and 24.

(*f*). Z_I, Z_{II}, Z_{III}, and Z_{IV}, from items 21, 22, 23, and 24, are modified by the capacitor constants being equal to zero. Then

$$I_M = E_M\left(\frac{Z_{III}}{Z_IZ_{III} - Z_{II}Z_{IV}}\right)$$

$$I_S = -E_M\left(\frac{Z_{IV}}{Z_IZ_{III} - Z_{II}Z_{IV}}\right)$$

$$\frac{I_S}{I_M} = -\frac{Z_{IV}}{Z_{III}} \qquad E_S = 0$$

T_{avg} is obtained from equation 24.

(*g*). $E_S = jE_M$, $\alpha_1 = 90$ degrees, and $\varphi = 90$ degrees. The following assumptions are made:

$$\begin{aligned} a &= 1 \quad R_f = R_b \quad X_f = X_b \\ Z_I &= (R_{1M} + 2R_f) + j(X_{1M} + 2X_f) \\ Z_{II} &= 0 \\ Z_{III} &= Z_I \\ Z_{IV} &= 0 \end{aligned} \qquad \begin{aligned} I_M &= \frac{E_M}{Z_I} \\ I_S &= \frac{jE_M}{Z_I} \\ T_{avg} &= 4I_MI_SR_f \end{aligned}$$

(*h*). $E_S = jE_M$, $\alpha_1 \gtrless 90$ degrees, $a = 1$, $R_f = R_b$, and $X_f = X_b$, with Z_I, Z_{II}, Z_{III}, and Z_{IV} from items 21, 22, 23, and 24. $T_{avg} = 4I_MI_SR_f$

(*i*). $E_S = jE_M$, $\alpha_1 = 90$ degrees, $a \lessgtr 1$, $R_f = R_b$, and $X_f = X_b$, with Z_I, Z_{II}, Z_{III}, and Z_{IV} from items 21, 22, 23, and 24. $T_{avg} = 4aI_MI_S\sin\varphi$

(*j*). $E_S \doteq jE_M$, $\alpha_1 \gtrless 90$ degrees, $a \gtrless 1$, $R_f = R_b$, and $X_f = X_b$, with Z_I, Z_{II}, Z_{III}, and Z_{IV} from items 21, 22, 23, and 24. $T_{avg} = 4aI_MI_S\sin\varphi$

Appendix I

To aid in identifying the terms and to show the influence of winding displacement, the detailed calculations on the capacitor motor used by Morrill with some additional items will be considered.

$R_{1M} = 2.02$	$X_{1M} = 2.79$
$R_2 = 2.06$	$X_2 = 1.06$
$R_{1S} = 5.12$	$X_{1S} = 2.31$
R_c at start $= 3$	X_c at start $= -14.5$
R_c running $= 9$	X_c running $= -172$
Syn. speed $=$ 1,800 rpm	Running speed $=$ 1,740 rpm
$X_m = 33.4$	$E = 110$
$a = 1.18$	slip $= 1/30$
$a^2 = 1.39$	

Calculations:

$X_m^2 = 1{,}115 \qquad X_2 + X_m = 34.46 \qquad (X_2 + X_m)^2 = 1{,}186$

$$\frac{R_2}{S} = 61.8 \qquad \left(\frac{R_2}{S}\right)^2 = 3{,}820 \qquad \frac{R_2}{2 - S} = 1.047$$

$$\left(\frac{R_2}{2 - S}\right)^2 = 1.10$$

$$R_f = \frac{1{,}115 \times 61.8}{3{,}820 + 1{,}186} = 13.77$$

$$X_f = 33.4\,\frac{3{,}820 + 1.06 \times 34.46}{5{,}006} = 25.72$$

$$R_b = \frac{1{,}115 \times 1.047}{1.10 + 1{,}186} = 0.984$$

$$X_b = 33.4\,\frac{1.10 + 1.06 \times 34.46}{1.10 + 1{,}186} = 1.057$$

(*a*). With the running capacitor,

$Z_I = (2.02 + 13.77 + 0.984) + j(2.79 + 25.72 + 1.057) = 16.77 + j29.57$

$Z_{II} = j1.18(0.984 + j1.057) - j1.18(13.77 + j25.72) = 29.1 - j15.1$

$Z_{III} = 9 + 1.39(5.12 + 13.77 + 0.984) + j[-172 + 1.39(2.31 + 25.72 + 1.057)]$
$= 36.7 - j128$

$Z_{IV} = -Z_{II}$ (when $\alpha_1 = 90$ degrees)

$$I_M = 110\,\frac{(36.7 - j128) - (29.1 - j15.1)}{(16.77 + j29.57)(36.7 - j128) + (29.1 - j15.1)^2}$$

$= 2.31\,\lvert 64°\ 59'$ or $0.975 - j2.10$.

Similarly,

$I_S = 0.978\,\lvert 3°\ 40'$ or $0.976 + j0.0626$

$I = I_M + I_S$ or $2.82\,\lvert 46°\ 20'$

Table I

	$\alpha_1 = 90°$		$\alpha_1 = 60°$		$\alpha_1 = 120°$	
Starting						
I_M	14.2	$\overline{\lvert 40° 50'}$	13.35	$\overline{\lvert 19° 40'}$	15.64	$\overline{\lvert 4° 50'}$
I_S	6.28	$\underline{\lvert 27° 40'}$	4.26	$\underline{\lvert 62° 10'}$	8.54	$\underline{\lvert 54° 10'}$
I	17.5	..	14.56	..	21.3	
Torque (syn. watts)	760	..	446	..	908	
Torque per ampere	43.5	..	30.7	..	42.6	
Capacitor voltage	93.0	..	63.0	..	126.4	
Running (slip $^1/_{30}$)						
I_M	2.31	$\overline{\lvert 64° 59'}$	2.661	$\overline{\lvert 67° \ 7'}$	2.068	$\overline{\lvert 56° 52'}$
I_S	0.978	$\underline{\lvert 3° 40'}$	0.678	$\underline{\lvert 20° 28'}$	1.116	$\underline{\lvert 6° 16'}$
I	2.82	..	2.77	..	2.905	
Torque (syn. watts)	138.6	..	154.1	..	112.1	
Capacitor voltage	168.0	..	116.0	..	192.0	

$$T_{avg} = (2.31^2 + 1.39 \times 0.978^2)(13.77 - 0.984) + 2 \times 1.18(0.975 \times 0.0626 + 2.10 \times 0.976)(13.77 + 0.984)$$
$$= 138.6 \text{ synchronous watts}$$

(*b*). Considering next the same windings displaced so that $\alpha_1 = 60$ degrees, with slip $= {}^1/_{30}$, R_f, X_f, R_b, X_b, Z_I, and Z_{III} as before, and $M = 0$,

$$Z_{II} = 1.18(13.77 + j25.72)(0.5 - j0.866) + 1.18(0.984 + j1.057)(0.5 + j0.866)$$
$$= 33.9 + j2.74$$
$$Z_{IV} = -16.52 + j28.88$$
$$I_M = 2.661 \overline{\lvert 67° 7'} \qquad \varphi = 87° 35'$$
$$I_S = 0.678 \underline{\lvert 20° 28'}$$
$$T_{avg} = (2.661^2 + 1.39 \times 0.678^2)(13.77 - 0.984) + 2 \times 1.18 \times 0.678 \times 2.661[13.77 \times 0.886 + 0.984 \times 0.844]$$
$$= 154.1 \text{ synchronous watts}$$

(*c*). Considering $\alpha_1 = 120$ degrees, slip $= {}^1/_{30}$, etc.,

$$Z_{II} = 1.18(13.77 + j25.72)(-0.5 - j0.866) + 1.18(0.984 + j1.057)(-0.5 + j0.866)$$
$$= 16.52 - j28.88$$
$$Z_{IV} = -33.9 - j2.74$$
$$I_M = 2.068 \overline{\lvert 56° 52'}$$
$$I_S = 1.116 \underline{\lvert 6° 16'}$$
$$T_{avg} = 112.1 \text{ synchronous watts}$$

(*d*). The effect of winding displacement upon starting torque, using starting capacitor, is shown as follows:

$M = 0$. Slip $= 100$ per cent $\quad R_f = 1.93 \quad R_b = 1.93 \quad X_f = 1.14$

$$X_b = 1.14$$
$$\alpha_1 = 90 \text{ degrees}$$
$$Z_I = 5.88 + j5.07$$
$$Z_{II} = j1.18(1.93 + j1.14) - j1.18(1.93 + j1.14) = 0$$
$$Z_{III} = 15.5 - j8.12$$
$$Z_{IV} = -j1.18(1.93 + j1.14) + j1.18(1.93 + j1.14) = 0$$
$$I_M = 14.2 \overline{\lvert 40° 50'}$$
$$I_S = 6.28 \underline{\lvert 27° 40'}$$
$$\varphi = 68° 20'$$

The torque equation reduces to $2aI_MI_S[2R_f \cos(\alpha_1 - \varphi)]$

$$T_{avg} = 2 \times 1.18 \times 14.2 \times 6.28(2 \times 1.93 \times 0.9304)$$
$$= 760 \text{ synchronous watts}$$

Voltage across the capacitor $= I_S\sqrt{R_c^2 + X_c^2}$
$= 93$ volts

Starting current $= 17.5$ amperes

Starting torque, synchronous watts per ampere $= 43.5$

(*e*). With $\alpha_1 = 60$ degrees, Z_I and Z_{III} are unchanged.

$$Z_{II} = 1.18(1.93 + j1.14)(0.500 + j0.866) + 1.18(1.93 + j1.14)(0.500 - j0.866)$$
$$= 2.28 + j1.345$$
$$Z_{IV} = 2.28 + j1.345$$
$$I_M = 13.35 \overline{\lvert 19° 40'}$$
$$I_S = 4.26 \underline{\lvert 62° 10'}$$
$$T_{avg} = 2 \times 1.18 \times 13.35 \times 4.26 \times 1.93(0.929 + 0.781)$$
$$= 446 \text{ synchronous watts}$$
$$E_c = 63.0 \text{ volts}$$

(*f*). For $\alpha_1 = 120$ degrees the contrasted results are given in table I.

Losses and efficiency can be calculated by the usual methods. It should be noted that the equivalent circuit includes no resistance by which iron loss is considered. An iron loss current can be added to the in-phase component of the total current and the iron loss added to the input.

Appendix II

An analysis of the shaded pole motor, with continuous iron and distributed shaded winding, is given in the following:

Partial design data for $^1/_{20}$ hp =

$X_m = 93.8$ ohms	$R_S = 18.2$ ohms
$X_1 = 5.47$ ohms	$T_M = 1{,}112$ series turns
$X_2 = 5.47$ ohms	$T_S = 1{,}112$ series turns
$R_M = 9.3$ ohms	$k_{WM} = k_{WS} = 0.772$
$R_2 = 6.45$ ohms	$E_M = 110$ volts

Shaded winding and main windings are identical (except for wire size) with relative displacements of 60 degrees. Values for diagram or equations are:

$a = 1$	$X_m = 46.9$
$\alpha_1 = 60°$	$X_{1M} = 5.47$
$R_{1M} = 9.3$	$X_{1S} = 5.47$
$R_{1S} = 18.2$	$X_2 = 2.735$
$R_2 = 3.225$	$M = 0$

Starting conditions, slip $= 100$ per cent,

$$R_f = R_b = \frac{46.9^2 \times 3.225}{3.225^2 + (2.735 + 46.9)^2} \text{ or } 2.87$$

$$X_f = X_b = 46.9\,\frac{3.225^2 + 2.735(2.735 + 46.9)}{3.225^2 + (2.735 + 46.9)^2} \text{ or } 2.775$$

$$R_{1M} = 9.3 + 2.87 + 2.87 \text{ or } 15.04$$
$$X_{1M} = 5.47 + 2.775 + 2.775 \text{ or } 11.02$$
$$Z_I = 15.04 + j11.02$$
$$Z_{II} = 1(2.87 + j2.775)(0.5 - j0.866) + 1(2.87 + j2.775)(0.5 + j0.866) + 0$$
$$= 2.87 + j2.775$$
$$Z_{III} = 23.94 + j11.02$$
$$Z_{IV} = 2.87 + j2.775$$

$$I_M = 110 \times \frac{23.94 + j11.02}{(15.04 + j11.02)(23.94 + j11.02) - (2.87 + j2.775)(2.87 + j2.775)}$$
$$= 6.06 \overline{\lvert 35° 18'}$$
$$I_S = -0.921 \overline{\lvert 16° 1'} \text{ or } 0.921 \underline{\lvert 163° 59'}$$
$$\varphi = 199° 17'$$

Average starting torque:

$$T = 2 \times 1 \times 0.921 \times 6.06[2.87 \times (-0.7579) - 2.87 \times (-0.1859)]$$
$$= -18.35 \text{ synchronous watts.}$$

The minus sign indicates rotation opposite from that which would be brought about by separate excitation of the so-called starting winding.

Reference

1. The Revolving Field Theory of the Capacitor Motor, W. J. Morrill. A.I.E.E. Trans., v. 48, April 1929, p. 614–29.

The "Comet"—A Diesel Electric Unit Train

The electrical equipment and performance of an articulated light-weight streamlined Diesel-electric train are described in this paper. Particular attention is given to the control, which provides full utilization of the available engine power for driving the train over a wide range of train speeds. Generators, motors, and auxiliary equipment are considered briefly, and the estimated train performance is given.

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IN a Diesel electric rail vehicle, the electric power transmission system which conveys the power from the engine to the driving wheels is 1 of the 5 essential component parts which must be co-ordinated in design and location. These 5 parts are the body and running gear; the motive power plant; the transmission system; the auxiliaries; and the braking apparatus. The electrical transmission equipment and other details of the "Comet," a light weight stream-lined Diesel electric unit train now being operated by the New York, New Haven and Hartford Railroad, are described in the present paper, and calculated data on the performance of this train are given.

In the electrical system of power transmission, each engine drives a generator and power is transmitted by electrical conductors to motors geared to the driving wheels. A system of control is provided for regulating the engine output and for insuring correct generator voltage for the varying propulsion needs. For the sake of simplicity and ease of regulation, the electrical power is generated and used as direct current rather than as alternating current.

Duplicate Power Plants

In this unit train, there are 2 duplicate power plants, one located at each end of the train, with duplicate sets of control and auxiliaries, both being controlled simultaneously from either of the 2 operating stations, which also are located one at each end of the train. Operation at full train speed in either direction is thereby facilitated. These power plants are housed in engine rooms which are likewise in duplicate, except that the single storage battery for the train is located in one engine room while the single train heating boiler is located in the other engine room. Two traction motors are mounted on each car truck immediately under the engine rooms, these motors receiving electrical energy from the plant above.

The main items of the transmission equipment consist of:

- 2 d-c generators.
- 2 d-c auxiliary generators.
- 4 railway type traction motors.
- 2 controllers.
- 6 pneumatically operated contactors for motor circuits.
- 2 pneumatically operated reversers for motors.
- 2 load regulating relays.
- 2 sets of contactors and resistors for fields, load regulation, motor field shunting, and engine starting.
- 1 storage battery for engine starting.

The auxiliary equipment includes 4 radiator blower motors, 2 airbrake compressor motors, 2 air conditioning compressor motors, motors for car circulating fans, and such miscellaneous equipment.

Generators, Auxiliary Generators, and Traction Motors

Each generator is of the railway type, being built with a single bearing bracket at the commutator end. The armature is carried by a ball bearing in this bearing bracket and by the engine crankshaft bearings, the armature shaft being coupled solidly to the crankshaft. The generator frame is a rolled steel ring to which are welded the supporting feet. The bearing bracket is bolted onto one end of this ring. Class *B* (high temperature) insulation is used for windings, thus permitting high rating with a minimum of weight and space. The characteristic curve of this generator when coupled to a 400 horsepower engine operating at 900 rpm is shown in figure 1. This machine has 3 field windings: a separately excited winding for voltage control; a series winding

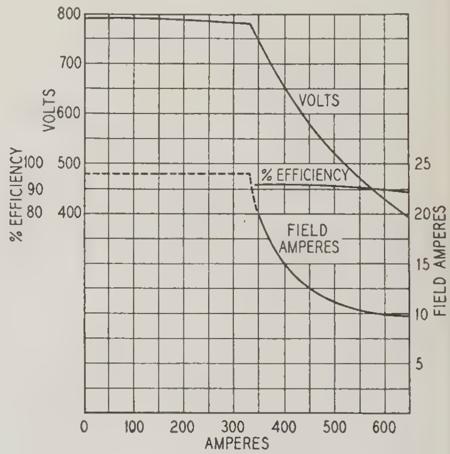


Fig. 1. Characteristic curves of generator coupled to 400 horsepower engine at 900 rpm

Load of auxiliary generator on same shaft, at 140 volts: 12 amperes for control and battery, 75 amperes for 2 radiator blowers

A paper presented before a meeting of the Institute's Connecticut Section, New Haven, May 22, 1935; recommended for publication by the A.I.E.E. committee on transportation; and scheduled for discussion at the A.I.E.E. winter convention, New York, N. Y., Jan. 28–31, 1936. Manuscript submitted July 3, 1935; released for publication Sept. 23, 1935.

used for engine starting purposes; and a commutating field winding.

The auxiliary generator has no bearings, the armature being carried on an extension of the main generator shaft and the field frame being bolted to the bearing bracket of the main machine. This generator is used for auxiliary purposes only, supplying power at 110 volts (nominal voltage) for battery charging, lighting, air brake compressor operation, air conditioning operation, main generator field excitation, and cooling fans, as will be described more in detail later. This generator is also built with class *B* insulation for weight and space economy.

The series type traction motors are of standard railway construction, being supported by a spring support at the motor nose and by bearings on the axle. The armature is geared to the axle with a 24 to 47 ratio and its performance with this gear ratio and with 36 inch wheels is shown by figure 2. The windings are insulated with class *B* insulation.

Performance Curves

Figures 1 and 2 show the performance of one generator and of one motor. The performance of the train, with 2 400 horse power engines, each driving a generator and each generator supplying power for 2 traction motors, is shown by figure 3. The dotted curve shows the available power at the engine shaft, the difference between this and the actual power at the rims of the driving wheels representing power consumed by the auxiliary devices and losses in the transmission equipment. The performance curve includes series connections of the motors, parallel connections of the motors, and motor field shunting to obtain successively higher speed conditions under the limitation of maximum generator voltage. It may be noted that the curve up to 100 miles per hour is approximately hyperbolic in shape, whereas above 100 miles per hour, the curve breaks to a different slope. This is due to the fact that the generator voltage has reached its maximum at 100 miles per hour and above that speed the voltage on the motors is constant.

The sequence of operations during the acceleration of a train consists of first connecting the motors in series relation across their respective generators with the engine turning over at an idling speed (slightly below 400 rpm), then bringing the engine speed up gradually until it is operating at full speed. This operation is with relatively low generator voltage. With the engine at full speed, the voltage of the generator is automatically raised gradually to keep the engines fully loaded until the maximum generator voltage is reached. With maximum generator voltage across 2 motors in series, further advance in train speed is made by reconnecting the traction motors in parallel relation, this reconnection automatically reducing the generator voltage to approximately half of full voltage, with its subsequent gradual increase to full voltage with further increase in train speed. The shunting of the traction motor field again reduces the generator voltage, which again increases automatically, reaching full voltage at 100 miles per hour.

Fig. 3. Performance of New Haven "Comet"

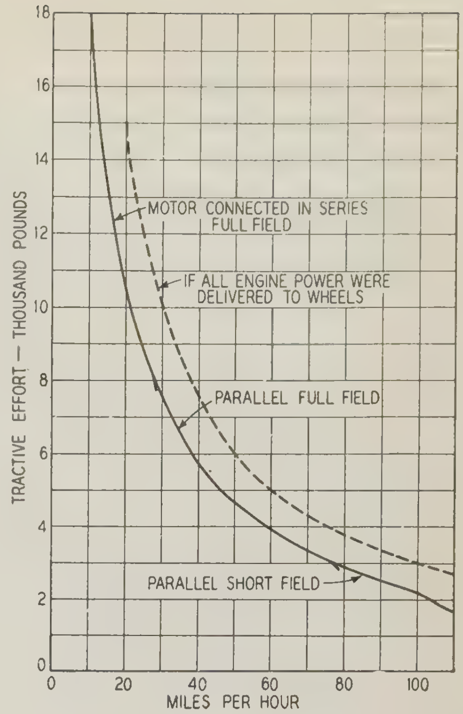
2 Diesel engines each rated 400 horsepower, 900 rpm

2 railway generators

4 railway motors

24 to 47 gear ratio

36 inch wheels



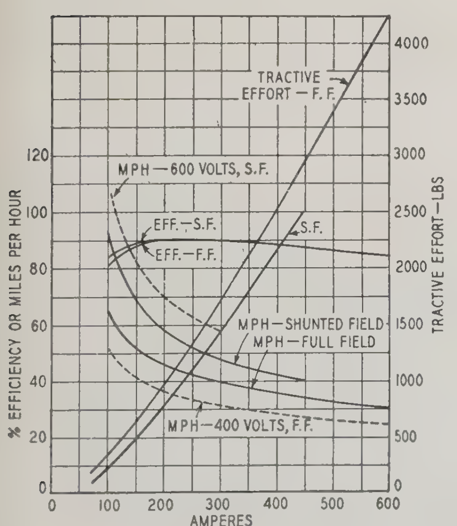


Fig. 2. Performance curves of series type railway traction motors

24 to 47 gear ratio

36 inch wheels 500 volts

Torque Control

The "torque control" system described in the following paragraphs is used for obtaining full utilization of engine power and for insuring an adequate supply of power for the auxiliary devices. The application of propulsion power is governed by the movement of a control lever located at the engineman's position at either end of the train, and the motor connections may be selected by the same lever. A second lever controls the direction of movement of the train by the remote control of reversing drums to reverse the direction of current flow through the traction motor fields. Both engines operate simultaneously and may be started or stopped individually from these control stations. The engines are accelerated from idling to full speed in 6 steps.

Torque control of internal combustion engine pro-

pulsion equipments fulfills the 2 fundamental requirements for this class of motive power, namely:

1. Full utilization of available power over as wide a range of train speed as possible.

2. Adequate supply of auxiliary power at relatively constant voltage.

It is obvious from a study of figure 1 that the main generator voltage must vary over a wide range in order to keep from overloading the engine under the

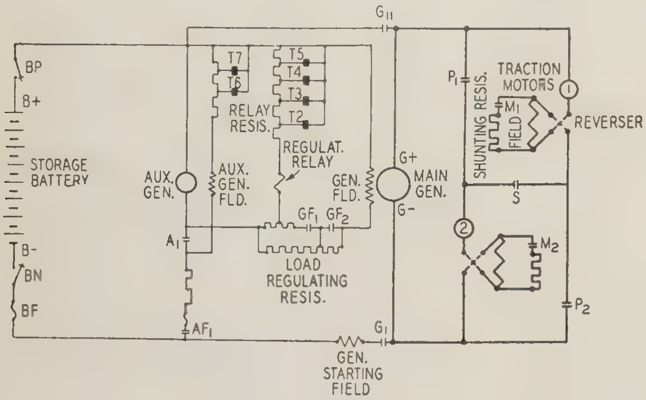


Fig. 4. Schematic diagram of torque control

widely varying current demands of the traction motors. Thus, with the traction motors requiring a total of 600 amperes for acceleration or for climbing a grade, the generator voltage must be limited to 430 volts in order to keep from overloading the engine, yet with the motors requiring only 350 amperes total, the voltage may rise to 743 volts, still within the limits of engine power. Such variable voltage regulation may be approximated by the use of a

means of a sensitive, yet substantially constructed relay which is responsive to very slight variations in engine speed.

Power Application by Torque Control

Figure 4 shows the general scheme of power control for one power plant with its 2 motors. The main power circuits are indicated by heavy lines, while the auxiliary generator circuits, field circuits, and regulating circuits are shown by lighter lines. Engines are started by closing contactors G_1 and G_{11}, which causes power to flow from the storage battery through the main generator armature and the starting field, which causes the generator to act as a motor and spin the engine until it fires. With the engine firing, power is applied to the traction motors by closing switch S for series connections of the traction motors or switches P_1 and P_2 for parallel connections of the motors. Contactor AF_1 is also closed to energize the auxiliary generator field from the storage battery. The auxiliary generator then develops a voltage for supplying excitation current for the main generator field, this current flowing through the load regulating resistance.

The engine speed is advanced step by step by means of a governor operator which resets the engine governor for higher speed operation. This governor operator is controlled by the engineman's operating lever (circuits not shown). Contacts T_2 to T_7 are carried by this governor operator, these various contacts being closed for various positions of the governor operator.

As a matter of fact, the engine governor, as set by the governor operator, does not actually control the engine speed except when idling. Correct engine speeds are maintained automatically slightly below the governor speed setting by varying the generator

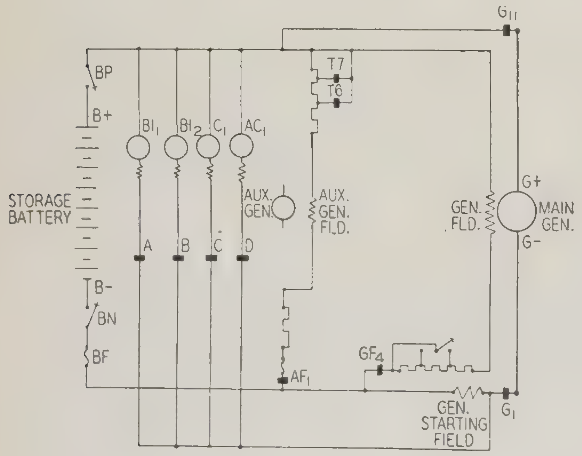


Fig. 5. Schematic diagram of auxiliary power supply with engine idling

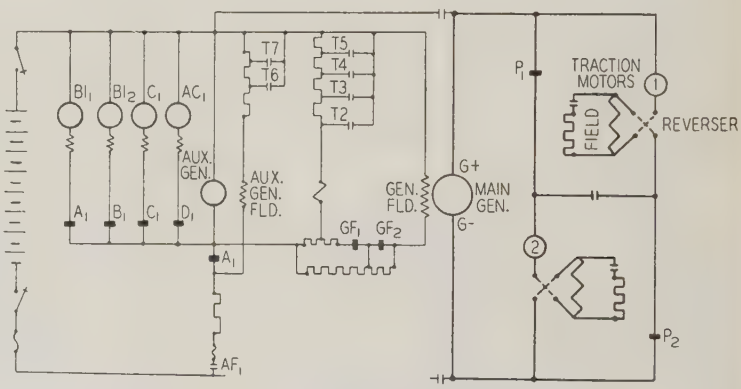


Fig. 6. Schematic diagram of auxiliary power supply with propulsion power on

differentially connected series field winding in the generator, but full utilization of the available power may be secured only over a relatively small range of operating speeds, and such a scheme has other disadvantages. Torque control, however, regulates by

load to hold the engine down to the selected speed, which is accomplished by varying the main generator field to load or unload the engine according to whether its speed is above or below the selected operating speed. This is accomplished through the regulating relay.

With the auxiliary generator field excited from the battery, its generated voltage is directly proportional to engine speed. As soon as the engine speed is raised by the operation of the engineman's controller, the auxiliary generator voltage comes up to a point where its voltage is sufficient to operate the regulating relay. When this relay closes its contacts, the field regulating contactors GF_1 and GF_2 close to reduce the resistance in the main generator field circuit, thus resulting in a higher main generator field and voltage and an increase in the current flowing through the traction motors. Thus the increase in power delivery from the main machine causes the engine to slow down slightly, resulting in a corresponding reduction in auxiliary generator voltage which causes the regulating relay and field regulating contactors to open. The reduction of engine load then results in an increase in engine speed. This cycle is repeated rapidly, resulting in practically constant engine speed as determined by the speed setting selected.

Higher Speed Steps With Torque Control

In order to raise the engine speed for the next higher speed setting, the contact T_2 is opened by the advancement of the engine governor operator. The opening of T_2 inserts additional resistance in the relay circuit and insures that the relay and regulating contactors remain open. Thus, with minimum generator field and load, the engine speed advances rapidly until the auxiliary generator voltage reaches a new value sufficiently high to again cause the regulating relay to go into action and control the engine speed. This procedure is repeated step by step until all resistance is in the relay circuit and the auxiliary generator voltage is in the neighborhood of 130 volts (sufficient to charge the battery fully). Further increases in engine speed are then accomplished by opening contacts T_6 and T_7 successively to reduce the auxiliary generator voltage. As this voltage is reduced, the effect on the regulating relay is the same as the insertion of resistance in its circuit, thus unloading the engine to allow its speed to rise until the auxiliary generator voltage is again back to 130 volts. Thus, on the last 3 controller notches the auxiliary generator is generating practically constant voltage.

The control system of the "Comet" keeps the engines fully loaded over the operating speed range. As the train speeds up and the current demand of the traction motors reduces, the regulating system automatically increases the generator voltage to keep the engines fully loaded. Any change in the traction motor connections which increases the current demand instantly reflects in reduced generator voltage to maintain full load.

The power supply for the auxiliary equipment is obtained from the main generator when the engine is idling and from the auxiliary generator when the main generator is delivering propulsion power. Figure 5 shows the connections when idling and figure 6 shows the running connections. Under idling conditions, contactors G_1 and G_{11} are closed and the main generator field is energized from the battery to

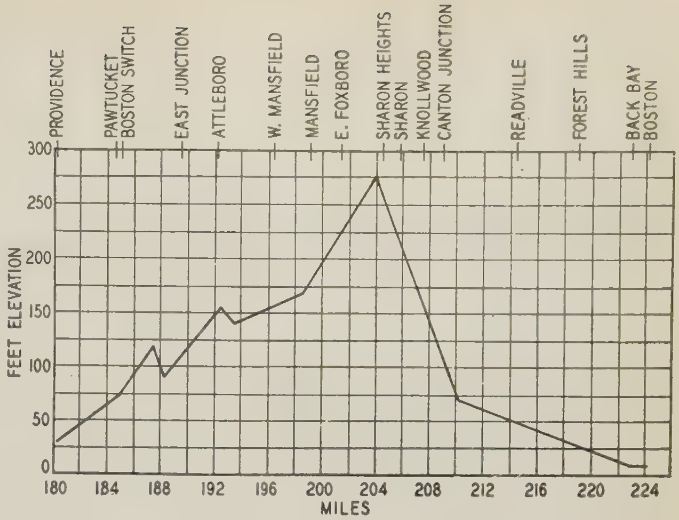


Fig. 7. Condensed profile of line of New York, New Haven and Hartford Railroad Between Providence, R. I., and Boston, Mass.

Maximum curve, high speed zone, 1 degree 31 minutes (curve resistance negligible)

give the correct generator voltage for battery charging. The auxiliary equipment is then connected to this main machine to operate at normal auxiliary voltage. When propulsion power is required, however, the main generator voltage is too high and too variable for auxiliary operation, so the auxiliary equipment is connected across the auxiliary generator which operates at the correct voltage for battery charging and auxiliary circuits. Contactor A_1 shown in figures 4 and 6 is a reverse current switch, closing when the auxiliary generator voltage reaches battery voltage, and dropping out at lower voltages.

Auxiliary Equipment

The auxiliary requirements of the "Comet" are unusually heavy. Among the auxiliaries which must be supplied with power are:

1 battery.
2 airbrake compressor motors.
2 air conditioning compressor motors.
12 air conditioning fan motors.
2 engine fuel pump motors.
4 radiator cooling fan motors.
1 heating boiler blower motor.
2 regulated lighting systems.
1 control system.

Air for the braking system is supplied by 2 air compressors requiring an estimated average of 4.5 horsepower during train operation. Air from the braking system is used for the operation of electropneumatic contactors, traction motor reversers, engine trip and reset devices, door control, and similar devices.

Two compressor and condensing sets are provided for air conditioning purposes. Each compressor is of 2 cylinder construction using di-chloro di-fluoro methane as a refrigerant. The compressed gas is

cooled by 2 condensers located at the sides of the engine room, air being first drawn through these condensers and then through the engine lubricating oil radiators by the radiator fans, after which it is forced out through the engine jacket water cooling radiators located in the roof of the engine room. The liquid refrigerant is then carried through the train to 6 air conditioning units, 2 of which are located in each power car and 2 in the center car (over the vestibules), and returns to the compressor as a gas. Each unit consists of finned tube cooling (and also steam heating) coils with 2 motor driven fans for circulating air through these coils, this air then being forced through the upper tubular members of the passenger compartment structure and discharged uniformly along the car in a horizontal direction through slots at the base of the tubular members. These air conditioning units include expansion valves and electrically operated fluid valves used for control. The units in the power car are each of 2 tons capacity, and those in the center car are of 2.5 tons capacity each. They will maintain the inside temperature 15 degrees Fahrenheit below outside temperatures if desired, with full passenger load.

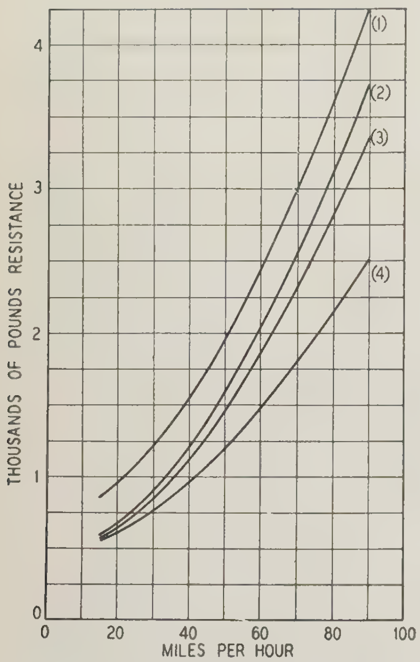


Fig. 8. Train resistance values of the New Haven "Comet" compared to conventional train

Based upon Davis formula

3 car train; 2 power plants, one at each end

Cross sections, 125 and 107.5 square feet

Streamlining to 65 per cent of "K"

Curve 1—Conventional cars, 210 ton train, 12 axles
Curve 2—Conventional cars, 136 ton train, 8 axles
Curve 3—Cars with reduced cross section, 136 ton train, 8 axles
Curve 4—Cars with reduced cross section, 136 ton train, streamlined

The lighting of this train constitutes a marked improvement over any passenger train installation to date, and is the result of considerable study and many tests. Of the indirect type, it is arranged to diffuse the light evenly so as to minimize glare and create a restful illumination. Designed to give a light intensity of approximately 8 foot candles on a 45 degree plane 33 inches from the floor, the ultimate results were even more satisfactory than had been anticipated. The lighting load amounts to approximately 8.3 kw, exclusive of the headlights and vertical beam headlight.

Ample room is provided to insure proper inspection and maintenance of the various items of equipment. This presented a problem, since the low floor height and shrouding below the floor for the reduction of air resistance necessitated locating the bulk of the equipment above the car floor. With the power plant located in the forward center section of the engine room, the space above the generator is occupied by the engine cooling water reservoir and a control cabinet. The radiators and radiator fans are located at the rear of the engine room on each side, with the air compressor under one fan housing and the air conditioning compressor under the other. The air-brake equipment is in one of the forward corners.

Estimate of Train Performance

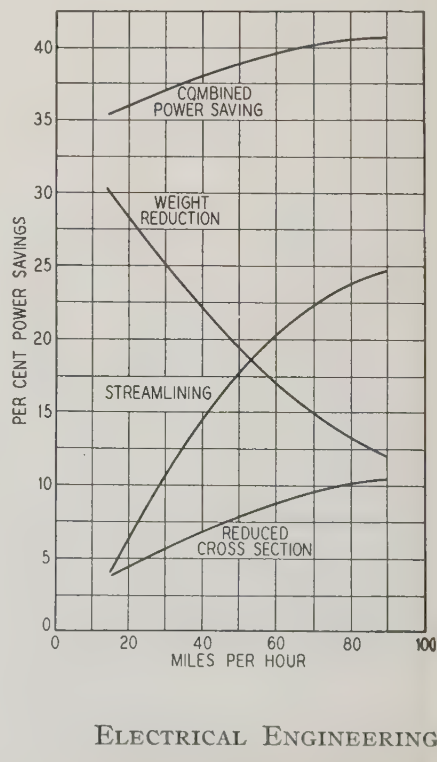
The first step in the determination of any proposed train performance is a study of the physical characteristics of the proposed run—grades, curves, speed restrictions, necessary stops, and similar pertinent data. Figure 7 is a condensed profile of the line between Providence, R. I., and Boston, Mass., over which the "Comet" is scheduled to operate. The specification was to stop at Pawtucket (4.51 miles out of Providence) and at Back Bay (1.28 miles out of South Station, Boston) with the following time limits: 10 minutes from Providence to Boston Switch (4.9 miles out of Providence), including the stop time at Pawtucket; and 4 minutes from South Station to Back Bay, including stop time at Back Bay. A speed limit of 90 miles per hour was effective over the whole line.

There have been a vast number of tests made to determine the resistance of trains of various weights and types and at various speeds. Out of this mass of data, the application engineer has generally accepted

Fig. 9. Effect on power requirements of car alterations (weight reduction, reduced cross section, reduction of wind resistance) on level track

3-car train, 210 ton conventional type, 2 engines, 125 square feet cross section, 12 axles, altered to 136 tons streamlined type, 107.5 square feet cross section, 8 axles

(Streamlining to "K" = 65 per cent, Davis formula)



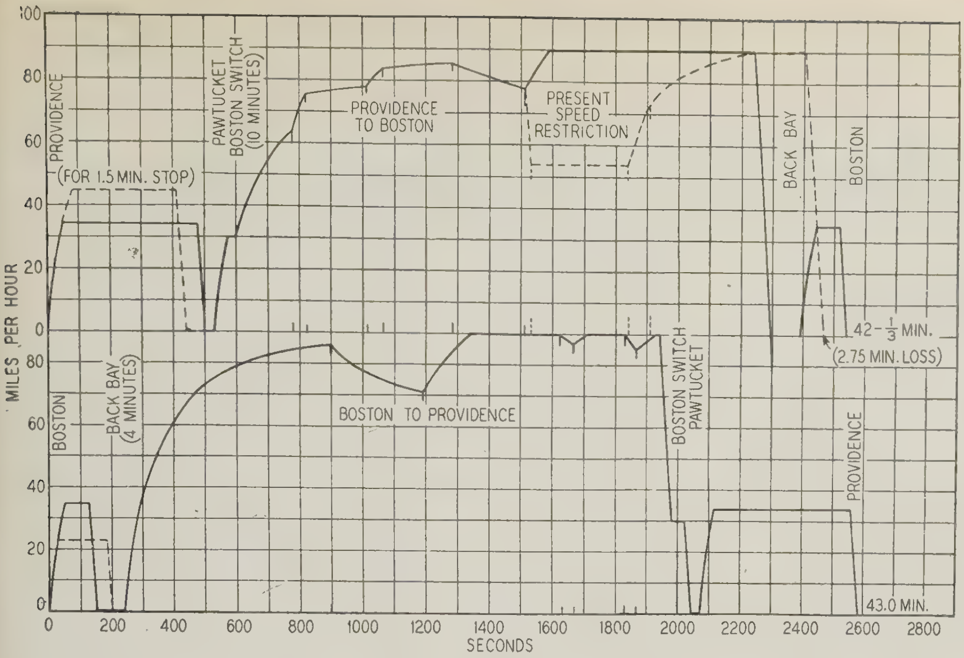


Fig. 10. Continuous speed-time curves of the "Comet" between Providence and Boston

Calculated performance of high-speed 3-car unit train, 136 tons loaded, 2 400 horsepower Diesel engines

the Davis formula as representing reasonable values of resistance, and, from frequent use of this formula, has been possible to prognosticate a train performance within reasonable accuracy. This formula, however, has been confirmed only for conventional types of equipment and up to 60 or 70 miles per hour. This formula is:

$$\text{Resistance} = \underset{\text{(Journal friction)}}{1.3 + \frac{29}{W}} + \underset{\text{(Flange resistance)}}{0.045V} + \underset{\text{(Windage)}}{\frac{K}{WN}AV^2}$$

where

W = tons per axle
V = speed in miles per hour
N = number of axles
A = frontal area of train in square feet
K = windage constant
= 0.0024 for head car (conventional type)
= 0.00034 for trail cars (conventional type)

No verified information is available for the close determination of high speed train performance where the train exterior has been rounded and smoothed for the purpose of minimizing the air resistance. Various wind tunnel tests and computations have been made with varying results. It was felt, therefore, that a closer determination of results would be possible by the use of the known and tried formula with adjustments in the factor K to compensate for the reduced air resistance. An examination of existing data indicated that the value of K would range from 40 to 60 per cent of the values for conventional cars. To be on the safe side, the actual value used was 65 per cent.

The estimated weight of the "Comet" was 136 tons loaded, on 8 axles. Since this train has no "head car" and "trailer cars," the front portion and the rear portion, which have slightly greater cross section than the center, were assumed to equal a "head car" and the balance constituted the "trailer car." From this segregation, a resistance for the complete train was calculated to be as shown by curve 4 on figure 8. As a matter of interest, the resistance of a conventional train of 210 tons and 125 foot cross section with 12 axles is shown by curve 1, while this same train reduced to 136 tons (the estimated weight of the "Comet") would have a resistance as shown by curve 2. Reducing both the weight and cross section of the conventional train would give curve 3 and the further reduction in resistance by aerodynamic design of exterior gives curve 4. Expressed in another way, figure 9 shows the power savings which may be effected by the various alterations from a conventional train to a unit train such as the "Comet."

Using line 4 of figure 8 as a basis of train resistance, a continuous speed-time curve was prepared covering the run from Providence to Boston and from Boston to Providence, calculating the performance on each grade section. This is shown by figure 10. From this, it may be determined that the run may be made within the time specified, i. e., 43.78 miles in 44 minutes.

Road tests have shown that the calculated performance is on the safe side and that there is a reasonable margin to insure maintenance of schedule, the actual run between Providence and Boston without stops having been made in 37 minutes.

Reasons for Diesel Motive Power

A great many railroad men wonder why the Diesel engine is used for propulsion of fast trains such as the "Comet" instead of using steam. The answer is economy of operation.

The "Comet" is 207 feet, 4 inches in length, with engine room space of approximately 37 feet. A rough estimate of the train weight increase by the inclusion of Diesel propulsion equipment is 50 tons. A steam locomotive for this purpose would add approximately 85 feet to the train length and 200 tons to its weight. This would require very considerably more power for the run.

The round trip between Providence and Boston requires 50 gallons of Diesel fuel at a cost of approximately \$2.50 and the total expense of operation is less than half of that of a steam hauled train. In addition, considerable time is saved at terminals because of the fact that the "Comet" may operate in either direction without turning.

Anode Materials for High Vacuum Tubes

Continued development of the high vacuum tube has played an extremely important part in the development of radio communication. Since the power output of a tube is proportional to the amount of heat that may be dissipated safely from its anode, the anode is one of the most vital parts of high vacuum tubes especially those used for transmitting purposes. Several materials have been found suitable for the anodes of transmitting tubes, depending upon the requirements of specific applications and the type of cooling adopted; these materials and their characteristics are discussed in this paper.

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DEVELOPMENT of radio communication has been marked by a series of periods, each of which has been the result of the application of some new device or devices. Without any question, the high vacuum tube is the device that has affected most profoundly every branch of the art. Development of the high vacuum tube has been continuous from the time the triode first was used. Today there are available in the United States tubes of all sizes ranging from the "acorn tube" to tubes capable of delivering 100 kw of radio frequency power. Tubes have been developed for many special applications to permit the desired results to be accomplished better, more easily, or more cheaply. To attempt even a hasty review of these developments would require far more space than is available here. For this reason the discussion will be limited to a more detailed consideration of one phase of the development of transmitting tubes, namely, the advances that have been made in the design and construction of anodes.

As is well known, in one of its simpler forms the vacuum tube consists of a filament or cathode which serves as a source of electrons, a plate or anode, and a grid consisting of a wire mesh or grating surrounding the cathode so that it may control the flow of electrons to the anode. The chief application of this device is to amplify a-c power to the levels desired. The action of the vacuum tube amplifier may be described as follows: The electrical impulse to be amplified is applied to the grid of the tube and thus controls electrostatically the flow of electrons from the cathode to the anode. The energy required to draw the electrons to the anode comes from a high voltage d-c supply in the anode circuit. The power required by the grid to vary this electron stream from the cathode to the anode ordinarily is only a fraction of the power flowing in the anode circuit; hence the tube is regarded as an amplifier. Strictly speaking, however, the action of the tube is that of a valve, the d-c power of the anode high voltage supply being converted into a-c power in the load. The efficiency of this energy conversion is never 100 per cent since there always must be some voltage drop between the anode and cathode of the tube, and hence some loss in the tube. Quantitatively, the energy loss is equal to the average energy with which the electrons bombard the anode. As a result of this bombardment the anode becomes heated and assumes some temperature at which the thermal loss by radiation, convection, and conduction equals the energy dissipated in the anode.

For any given tube there is a maximum amount of power that can be dissipated safely by the anode, if reasonable tube life is to be obtained. Then, as the conversion efficiency is determined fairly well by the mode of operation, it is seen that the a-c power output of the tube is proportional to the safe anode dissipation. Thus, in transmitting tubes, the anode dissipation rating is one of the most important factors in determining the amount of power the tube will deliver.

Anodes may be classified according to the principal method of cooling employed. In some types of tubes the anodes are cooled almost entirely by radiation, in others by convection, and in the third type by conduction.

Radiation Cooled Anodes

Historically, radiation cooled anodes were the first developed. In this type of construction, the anode is operated at some fairly high temperature and heat is radiated directly by the anode to and through the glass walls of the bulb. It has been necessary to design such anodes to operate at fairly high temperatures in order to dissipate reasonable amounts of power in anodes of the size demanded by the electrical characteristics desired of a tube. With some types of anode materials it has been desirable to operate at temperatures as high as 1,000 degrees centigrade.

Operation of anodes at such high temperatures immediately brought up a host of problems. One of the most important was that presented by the liberation of gases from the anode. All materials suitable for anodes contain gases in the raw state. These gases are mainly hydrogen, nitrogen, carbon monoxide, and carbon dioxide. They are present throughout the body of the material. When the material is heated in a vacuum, these gases are liber-

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ated at a varying rate, depending on the temperature and the time of heating. The major portion of these gases must be driven out of the anode during the manufacture of the tube so that in subsequent normal operation no appreciable amounts of gases are liberated. Residual gases in vacuum tubes become ionized under impact by electrons. If enough ions are generated in this way they partially neutralize the electron space charge that limits the flow of electron current between the cathode and anode. Increased current then flows through the tube, resulting in greater anode heating and more rapid gas liberation. This process quickly may become cumulative and lead to an arc discharge between the cathode and anode. If the tube be not protected by overload devices, such a discharge may result in an abrupt termination of the life of the tube by the melting of the cathode, grid, and anode.

The gases contained in anodes commonly are driven off by heating during one of the last stages of manufacture. The assembled tube is sealed to a vacuum system where the glass bulb can be baked out to free it of adsorbed gases. The anodes are heated by 2 processes. One method is to apply a high positive voltage to the anode and bombard it with electrons from the cathode. Another method is to place a coil carrying high frequency currents around the glass bulb in such a way that the anode acts as the short-circuited secondary of a transformer. The induced currents then heat the anode.

Another factor that is important in the choice of anode materials for radiation cooled tubes is the total heat radiation emissivity. It is desirable to have this approach as nearly as possible the ideal of a black body, since for a given anode and anode operating temperature (determined by gas liberation) it permits the highest dissipation rating. At first thought it would appear that the size of the anode could be increased to get the desired dissipation rating in the event that a material of low emissivity were used. However, this would result in an increase in the electrostatic capacitances to the other electrodes of the tube. Because of the definite trend to higher frequencies in radio communication, it is absolutely necessary to keep interelectrode capacitances to a minimum so that capacitance charging currents, which of course entail losses, can be limited to reasonable values.

A third set of factors in the choice of anode material consists of the mechanical properties. The material must be capable of being worked into the desired shapes without undue difficulty or expense. It must maintain these shapes at the highest temperatures necessary during the manufacture of the tube. Only a very small amount of warping can be tolerated at the normal anode temperature as warping results in a change of electrical characteristics.

A fourth factor is the vapor pressure of the anode material. This must be low enough to avoid noticeable metallic deposits in the tube during manufacture. Deposits on insulators in a tube may result in excessive interelectrode leakage or excessive radio frequency losses in the insulators. Deposits on the glass bulb result in higher glass temperatures, because of increased radiation absorption and radio frequency losses, and may lead to gas evolution and strain cracks.

One of the first materials used for anodes in radiation cooled transmitting tubes was tungsten. A typical 250 watt tube made commercially about fifteen years ago had an anode in the form of 2 sheets of tungsten about $1^3/_4$ by $2^1/_2$ inches arranged about the grid. From the standpoint of gas content, ease of degassing, vapor pressure, and maintenance of mechanical shape at high temperatures, tungsten is a fairly desirable material for anodes. It has been found that the initial gas content of tungsten is of the order of 0.6 per cent by volume. The main constituent of the evolved gas is nitrogen, although there is considerable carbon monoxide and some carbon dioxide and hydrogen. The absence of oxygen undoubtedly is attributable to the ease with which this element combines with other elements, such as carbon. The major portion of the gas is liberated by heating to 2,200 degrees centigrade; no further gas is evolved by raising the temperature to 2,600 degrees. Such high anode temperatures, however, would be difficult to realize in actual manufacture of tubes.

Tungsten has 2 serious disadvantages: It is essentially a very hard metal and consequently difficult to machine and work into the desired shapes, and its cost is high. Because of these disadvantages it soon was displaced by molybdenum.

A study of the gases evolved when molybdenum is heated in vacuum has shown that to degas this material completely, it is necessary to heat it to a temperature of 1,760 degrees centigrade, as compared with 2,200 degrees for tungsten. In a typical experiment the total gas evolved was 5.6 per cent by volume, that is, approximately 10 times the volume gas content of tungsten. The various constituent gases are evolved at varying rates. Thus when degassing is carried at 800 degrees centigrade the major portion of the gas liberated is carbon monoxide, while at higher temperatures the gas evolved is

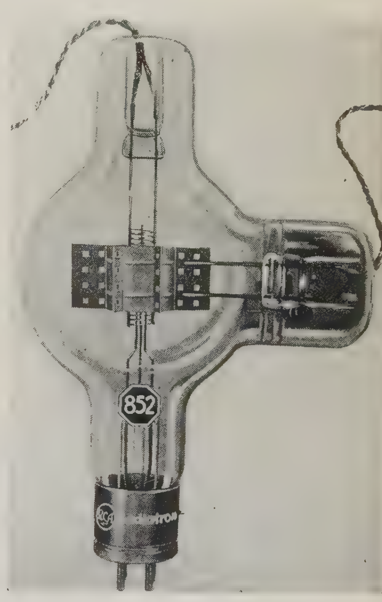

Fig. 1. A 75 watt tube with carborundum blasted molybdenum anode formed with integral cooling fins

mostly nitrogen. In heating at 800 degrees only about 15 per cent of the total gas in the sample is driven off. Further heating at 1,000 degrees contributes another 15 per cent. By raising the temperature to 1,200 degrees it is possible to drive off about 95 per cent of the gas in the sample.

In actual manufacture it is usually not desirable to use such high temperatures as the foregoing indicates are necessary for complete degassing of the anode, as the rate of evaporation of the molybdenum becomes high enough to cause deposits on the glass bulb and on insulators. Fortunately, complete degassing is not necessary because, regardless of the temperature at which degassing is carried on, the rate of gas evolution can be reduced to a negligible value by operating the anode at a sufficiently lower temperature. Long manufacturing experience has shown that molybdenum anodes that have been degassed at a brightness temperature of 1,300 to 1,400 degrees centigrade subsequently can be operated safely at temperatures as high as 1,000 degrees. Furthermore, a small amount of gas evolution can be tolerated if the tube contains active "getter" which will combine with or absorb the gases as fast as they are generated.

With molybdenum anodes ways and means soon were sought to decrease the temperature for the desired power dissipation. One way adopted was by the addition of fins which would increase the radiating area of the anode. It was found desirable to make the fins out of the same piece of metal as the anode because of the poor heat conductance between 2 pieces of metal brought together by some process such as riveting. Later it was found that the radiation emissivity of the anode could be increased by roughening the surface by carborundum blasting. Figure 1 shows a 75 watt tube in which both these methods to decrease anode temperature are employed. The anode is made of 2 strips of molybdenum folded to form radiating wings integral with the anode. The whole anode assembly then is carborundum blasted. This anode is shown at the left in figure 2.

By these means, the safe dissipation of molybdenum anodes was increased, thereby partly overcoming the low radiation emissivity characteristic of the metal. However, it is still desirable to operate the anodes at fairly high temperatures, sometimes as high as 1,000 degrees centigrade, in order to keep the anode dimensions small. These higher temperatures result in another and undesirable effect, largely in tubes employing a flat type of construction. The appearance of hot spots on the anode is common in such tubes, even when the construction is perfectly symmetrical, and is attributable to the fact that electrons attempt to travel from the cathode to the anode in lines normal to the anode surface. With anode thicknesses commonly used when the material is molybdenum, the center of the anode usually operates at a visibly higher temperature than the edges. This temperature difference produces warping or buckling, and consequently the electrical characteristics of the tube are subject to change. To overcome this in anodes of flat construction, it sometimes has been necessary to resort to elaborate constructions using channel pieces to give additional strength to those sections of the anode operating at higher temperatures. In anodes of circular construction, the heating produced is much more uniform, as the electrons flow from the central cathode in radial lines. In such anodes, the problem of warping generally is solved adequately by ridges pressed into the metal when the sections of the anodes are formed.

From a mechanical standpoint, molybdenum is a fairly satisfactory material. It does not soften or anneal even at much higher temperatures than are used during degassing.

Another material that has wide application to radiation cooled transmitting tubes is graphite. In comparison with molybdenum and tungsten, typical graphite samples suitable for use as anodes contain comparatively large amounts of adsorbed gases. The volume content of gases may be of the order of 10 times that of molybdenum; this is exclusive of the gas trapped in the cavities of the graphite. In addition, the volume of a graphite anode may be of the order of 10 times that of an equivalent metal anode, resulting in a 100-fold increase in gas that must be removed from the anode in the exhaust of the tube. By suitable pretreatment, the gas content of the anode can be reduced very greatly. One method consists among other processes of heating the anode to a high temperature in a vacuum furnace. It has been found that at 2,100 degrees centigrade it is possible to degas graphite so that subsequent heating at a higher temperature gives no further gas. As with molybdenum, the gas evolved, especially at the higher temperatures, is mainly nitrogen. An anode that has been degassed completely, reabsorbs only a fraction of the gas it originally contained if stored under proper conditions. In actual exhaust of a tube containing such a pretreated anode, the time

Fig. 2. Anode of tube shown in figure 1 (left) and a typical graphite anode used in a 50 watt tube (right)

required for completion of the degassing is not much different from that required by a molybdenum anode. Again it is found that if the anode is degassed at some given temperature, the rate of gas evolution can be reduced to a negligible value by dropping the temperature to some lower value. In a typical test the degassing was carried on at 1,300 degrees centigrade; on dropping the temperature to 1,000 de-

grees no further gas was evolved, although on increasing the temperature to 1,600 degrees considerably more gas was liberated. The total radiation emissivity of graphite depends on the treatment the surface has received. In comparison with molybdenum, graphite anodes operate at a visibly lower temperature for the same power radiation. At first thought this would seem to make graphite a much better anode material, because the energy radiated varies as the fourth power of temperature, and by operating at the same temperature as permissible with molybdenum much greater anode dissipation could be obtained. One factor militates against this, however. Glass is nearly opaque for the major portion of the radiations from anodes at the usual operating temperatures. Thus, regardless of whether the anode be metal or graphite, most of the radiated energy is absorbed by the glass and then is removed from the bulb mainly by convection air currents and reradiation. Any increase in the dissipation rating of a tube with a given bulb size must result in an increase in bulb temperature and consequent decrease in safety factors.

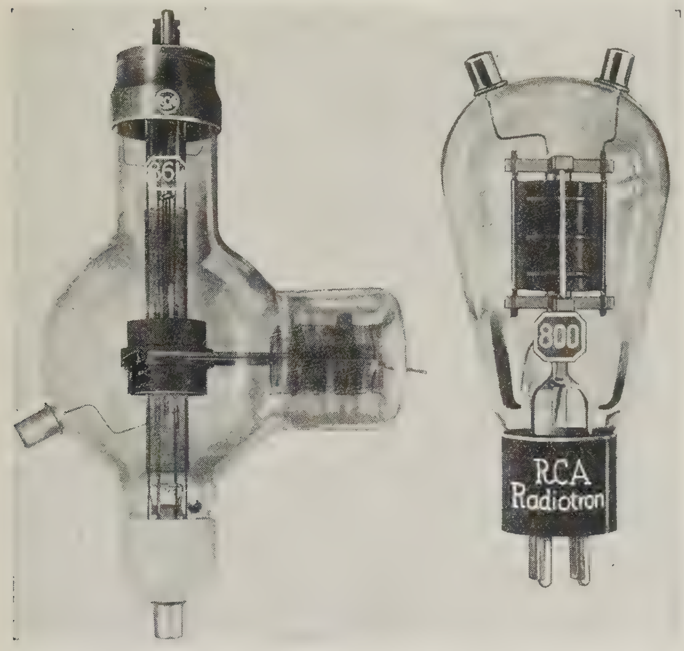

Fig. 3 (left). A typical tube with graphite anode

Fig. 4 (right). A typical tube with carbonized nickel anode

The fact that graphite anodes operate at a lower temperature than tungsten or molybdenum has been criticized. It seems that some users of transmitting tubes judge the operating efficiency of tubes by observing the color temperature of the anode. With tungsten or molybdenum anodes this is easily possible because at the normal operating temperature the anodes are distinctly orange-red in color. With graphite, however, practically no color can be seen in normal operation so that it is very difficult to judge how much energy is being lost in the anode.

One of the disadvantages of graphite as an anode material is its low tensile strength. In order to compensate for this weakness, the minimum wall thickness has been made at least $^{1}/_{16}$ inch. Consequently, graphite anodes are heavier than equivalent metal anodes. A typical graphite anode as used in the so-called "50 watt tube" is shown at the right in figure 2. Usually, more rigid supporting means are required, which may have to be rather elaborate for some kinds of service where there is a great deal of vibration such as in aircraft communication. There is also an advantage in the heavier wall thickness needed with graphite anodes. The heat conductance in the plane of the anode surface is so good that the entire anode operates at practically uniform temperature. The warping that occurs with metal anodes thus is avoided, and consequently the electrical characteristics of a graphite anode tube are more nearly constant. The absence of hot spots when graphite anodes are used means that the anode loss is radiated more uniformly over the anode surface. The glass bulb also operates at a more uniform temperature.

Mechanically, graphite does not present any serious problems. It is a soft material and therefore readily permits machining operations such as milling, grinding, and drilling.

The vapor pressure of graphite is low enough so that bulb blackening can be avoided during the exhausting of the tube. Careful selection of grades of graphite appears necessary here, because what ordinarily is termed graphite is in reality a complex mixture of a wide variety of forms of carbon, ranging from amorphous carbon to true graphite. Some of these forms produce undesirable effects, which can be eliminated partly by suitable treatment of the anodes. A typical tube with graphite anode is shown in figure 3.

Another metal that has found considerable application as an anode material, particularly in transmitting tubes of smaller sizes, is nickel. Nickel lends itself readily to a process called carbonizing. The nickel anodes are heated to a medium temperature in a hydrocarbon vapor such as natural gas. This treatment deposits a well-adhering layer of amorphous carbon or soot on the nickel. The radiating property of this layer approaches that of a black body—hence the desirability of this material.

The gas content of nickel is of the same order of magnitude as that of molybdenum. However, since it has a melting point of 1,450 degrees centigrade compared with 2,600 degrees for molybdenum, nickel anodes must be degassed at a lower temperature. The vapor pressure limits the degassing temperature to about 1,000 degrees centigrade, as this is the highest temperature at which nickel can be maintained without noticeable evaporation.

Nickel is formed very readily into the shapes desired for anodes. Care must be given to strengthening the anodes with ridges and the like in order to avoid warping during exhaust. Like other metals, nickel anodes have the advantage over graphite that they can be made very light in weight, so that elaborate supporting structures are not needed. Figure 4 shows a typical tube in which a carbonized nickel anode is used.

Other materials have found some application in transmitting tubes, resulting in some special characteristics; but the materials already mentioned have been used most widely.

Convection Cooled Anodes

So far attention has been given only to radiation cooled tubes. Anodes can be cooled also by convection air currents. In order to accomplish this, the anode is made part of the vacuum envelope. This imposes further requirements on the anode material. Obviously it must be impervious to gases, that is, vacuum tight. Since the anode can form only part of the vacuum enclosure because leads to the various electrodes must be brought through insulating seals, it must be possible to make a vacuum tight seal between the insulating material and the anode material. Copper has been found to be a very convenient anode material because it readily can be sealed to glass. In designing such a seal, the characteristics of glass and copper are taken into consideration. Two important factors are the higher coefficient of thermal expansion of copper compared with that of glass, and the fact that a copper to glass seal has a greater strength in compression than in tension. If the seal be so designed that the copper is on the outside and the glass inside, then as the seal cools from the temperature at which it is made the boundary between copper and glass is subjected to compression. In order to reduce this compressional stress, the copper is thinned out so that its thickness in the region where the copper and glass join is only a few thousandths of an inch. If the copper is thinned in this way it can stretch and thus relieve some of the stress on the seal. In making such a seal the thinned edge of the anode is heated in flames to oxidize it. At the same time the glass is heated until it is soft. The glass and copper then are brought into intimate contact and more heat is applied until the glass has dissolved the copper oxide. This solution has the characteristic orange color associated with copper to glass seals. The glass now is left in intimate contact with the copper. Such seals, when properly made, are perfectly vacuum tight.

Fig. 5 (left). A typical tube with convection cooled anode

Fig. 6 (right). A typical tube with conduction cooled anode

In degassing the anodes of tubes made in this way, considerably lower temperatures are permissible than can be used in degassing the radiation cooled anodes previously described. In the first place, copper has a melting point of only 1,083 degrees centigrade. Secondly, since the anode is now part of the vacuum enclosure, it cannot be heated to temperatures too near the softening point or there will be danger that the pressure of the atmosphere might collapse it. Keeping the temperature of the anode below this point, the degassing of a tube readily is accomplished, so that subsequent operation at temperatures as high as 300 degrees centigrade is possible.

The dissipation of heat from such an external anode tube is largely by convection currents. In order to increase the safe allowable dissipation, more area can be added to the anode by attaching fins. In order to be effective, these fins should have adequate heat conductivity and be in intimate contact with the anode. Further increase in dissipation can be secured by the use of forced cooling. It really is surprising to find how much heat can be removed from such a fin structure by means of an air stream supplied by an ordinary fan. A typical tube with a convection cooled anode is shown in figure 5.

Conduction Cooled Anodes

A third method of cooling anodes is by conduction. In order to accomplish this, the anode again is made part of the vacuum enclosure. The portion of the anode that becomes heated during operation of the tube is inserted in a water jacket, which is so constructed that a high velocity water stream can play over the anode. Heat then is conducted by the anode directly to this water stream and is carried off. It is important to have a high water velocity because otherwise steam pockets may form and result in dangerous overheating of the anode at those points. By proper design of water jacket and adequate flow of water, it is possible to carry off as much as 250 watts per square inch of anode surface.

Because of the large amounts of heat that can be carried off by conduction cooling, this method is used exclusively in high power tubes, those with ratings of a few kilowatts or more. The construction of conduction cooled anodes is the same as that

described for a convection cooled anode. The anode usually is made of drawn copper, and the open end is turned to a thin edge and sealed directly to the glass portion of the tube. A typical conduction cooled tube is shown in figure 6.

Choice of Material Depends on Requirements

In concluding this discussion, it can be seen that several materials are suitable for use in constructing transmitting tube anodes. The final choice of material must depend on all the limitations involved. For example, if the physical dimensions of the anode must be very small for operation at very high frequencies, it is helpful to construct the anodes of tungsten, as they then may be operated at high temperatures where they can radiate fairly high amounts of power. Again, if an inexpensive construction is required, carbonized nickel is a satisfactory material. Where weight must be kept at a minimum, with a high degree of strength, molybdenum is very useful. Where uniformity of characteristics is desired, graphite is a very suitable material. If high heat dissipation is required, forced convection or conduction cooled copper anodes are most satisfactory. With such a variety of materials and designs available, the engineer is enabled to make a choice that best fits the purpose at hand.

Present Status of Ferromagnetic Theory

Even though progress in the theory of ferromagnetism still lags far behind experimental progress, advances in theory have been particularly rapid during the past 5 or 10 years. The salient points brought out in these most recent advances are summarized here.

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DISCOVERY of the loadstone and some of its magnetic properties is now reputed to be some 3,000 years old. During these many years ferromagnetism has resisted very successfully the attack of theorists, and even at the present time theory lags far behind experiment. But advances in theory have been particularly rapid during the last 5 or 10 years; the author describes in this paper what he regards as the high points of this progress.

Not until the last quarter of the last century was any considerable work done on magnetic materials. During this period data were gathered rapidly until, just before the close of the century, an excellent book[2] of 400 pages, containing practically all of the important experimental and theoretical facts, was written by J. A. Ewing, now Sir James Ewing. The shape of the magnetization curves of iron, cobalt, and nickel, the existence of magnetic saturation and the magnetic transformation temperature, the existence and some of the laws of hysteresis, the simpler effects of stress and of magnetostriction, together with the important methods of measurement, all were known then, and silicon steel had just been invented.

Strangely enough, during the next 15 years there was but little advance in knowledge of magnetic materials, but there were many applications of existing knowledge by engineers to electrical machinery, including that used in electrical communication. During this period also the Heusler alloys (nonferrous alloys exhibiting ferromagnetic properties) were invented; and although these served to stimulate those interested in the theoretical aspects of ferromagnetism, still there was little progress.

Beginning between 1915 and 1920 and extending to the present, there has been a rapid development on both the experimental and theoretical sides of ferromagnetism. To illustrate the progress that has been made in the improvement of magnetic materials, table I has been prepared. The improvements made during the last 20 years have resulted from new methods of purification of the materials, new compositions (alloys), and new methods of heat treatment. Some of these figures refer only to laboratory specimens, and not to materials available in commercial quantities.

But the chief topic of this paper is the theoretical side of ferromagnetism. How is one to explain the different values of magnetic permeability, ranging from 1 to 600,000 for various materials? Or, to consider first the more fundamental questions, what is the elementary magnetic particle, and why is ferromagnetism associated with so few elements?

Origin of Ferromagnetism

It was suggested by Ampére about 100 years ago that molecules might behave as magnets because of the electric currents circulating in them. Today,

A paper recommended for publication by the A.I.E.E. committee on communication, and scheduled for discussion at the A.I.E.E. winter convention, New York, N. Y., Jan. 28–31, 1936. Manuscript submitted Sept. 14, 1935; released for publication Oct. 11, 1935.

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2. For all numbered references see list at end of paper.

Table I—Some Extremes in the Properties of Magnetic Materials Available in 1915, and in 1935

Material	Property	Value 1915	Value 1935
Iron	Maximum permeability	45,000[11]	340,000[12]
	Initial permeability	300	20,000[12]
	Coercive force in oersteds	0.3[11]	0.03[12]
Iron-nickel[13]	Maximum permeability	2,800[14]	600,000[15]
	Initial permeability	700[16]	12,000[17]
	Coercive force in oersteds	1.5[14]	0.01[15]
Silicon-iron	Initial permeability	400	2,000[12]
Iron	Hysteresis at B_M = 100 gausses, in ergs per cu cm per cycle	20	0.1[12]
Iron-cobalt-nickel "perminvar"	Hysteresis at B_M = 100 gausses, in ergs per cu cm per cycle		0.00003[18]
Iron-cobalt	Saturation value[19]	25,800	25,800
	Permeability at B = 16,000 gausses	2,100	19,000[18,12]
Tungsten steel	Coercive force[4] in oersteds	80	80
New K. S. steel	Coercive force[21] in oersteds		900

Superior numerals refer to references at end of paper.

with the advance in knowledge of atomic structure, the origin of ferromagnetism can be discussed in more specific terms. Strangely enough, the spectroscopists have supplied, so to speak, the elementary magnetic particle. It is the spinning electron. In order to explain their extensive observations on spectral lines, they found it necessary to revise the picture of the atom. For some time it has been supposed that an atom is made of a heavy nucleus with a positive charge and of electrons moving in circular or elliptical orbits around the nucleus. To this picture now must be added the idea that each electron itself is spinning about an axis that passes through its center. Each electron in an atom is then a small gyroscope, possessing a definite magnetic moment on account of its moving electrical charge and a definite angular momentum on account of its moving mass. The ratio of these 2 quantities is known from various independent lines of reasoning and evidence to possess a particular value. Electrons revolving *in orbits* also exhibit both magnetic moments and angular momenta due to their orbital motions, but for these the ratio is just half what it is for the spinning electron.

The Barnett experiment[22] shows in a very direct way the existence of these magnetic and mechanical moments of the electron and confirms the ratio between them in ferromagnetic materials (figure 1). A rod of iron is hung from a fine suspension and then is magnetized suddenly, whereupon the rod is observed to turn, twisting the suspending fiber a minute but measurable amount. The spinning electrons responsible for ferromagnetism have been turned by the applied field so that they are more nearly parallel to it; but the mechanical moment, which is also a property of those same electrons, causes the whole rod to rotate in just the way that a gyroscope would. Or, to put it differently: When the elementary magnets, pointing originally in all directions, are turned more nearly into parallelism with the axis of the rod by the applied field, they acquire a net angular momentum parallel to that axis. By the principle that action must be balanced by a corresponding reaction, the rod itself now must recoil with an equal and opposite momentum; it is this last that manifests itself by the sudden twist of the rod and may be calculated from the measured value of the twist. Its sign shows that the spinning magnetic particle is charged negatively, and its magnitude is what would be expected from the hypothesis that that particle is a spinning electron. Thus a change in magnetization is fundamentally a change in the direction of the spin of the electrons in the atom, and not a change in orientation of the whole electron orbit.

The next question is: Why is not every substance ferromagnetic? The picture of the atom of iron as now envisioned by the experts in this field, is represented by the diagram in figure 2. The 26 electrons in iron are divided into 4 principal "shells," each shell a more or less well defined region in which the electrons move in their orbits, and some of these shells are subdivided. The first (inner) shell contains 2 electrons, the next shell 8, the next 14, and the last 2. As the periodic system of the elements is built up from the lightest element, hydrogen, the formation of the innermost shells begins first. When completed, the number of electrons in the first 4 shells are 2, 8, 18, and 32, counting outward, but the *maximum* number in each shell is not always reached before the next shell begins to be formed. For example, when formation of the fourth shell begins, the third shell contains only 8 electrons instead of 18; it is the subsequent building up of this third shell that is believed to be connected intimately with ferromagnetism. The inserted numbers show how many electrons having positive and negative spins are present in each shell, and it may be noticed that in all the shells but one there are as many electrons spinning in one direction as in the other. This means that such shells are magnetically neutral and cannot have magnetic polarization. In the third shell, however, which is not yet filled to this extent,

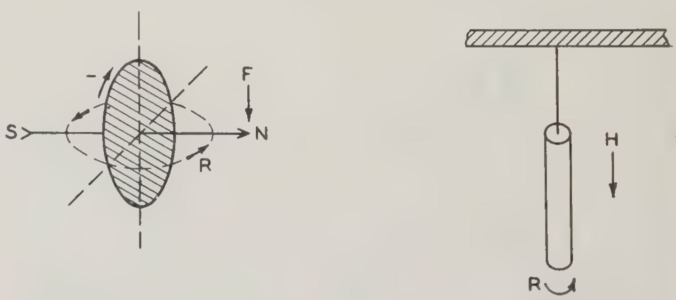


Fig. 1. Gyroscopic action (left); force F produces rotation R. Gyromagnetic effect (right); field H produces rotation R

there are 5 electrons with a positive spin and 1 with a negative spin, leaving an unbalance of 4 units of spin in one direction. If 1 more positive charge and its associated mass be added to the nucleus and 1 more electron in an outer shell, the iron is transformed into cobalt; and by repeating the process, the cobalt is transformed into nickel. These addi-

tional electrons in iron and their spins are so oriented that there is what may be called an excess spin of 4 units in iron, 3 in cobalt, and 2 in nickel. In manganese, the element just preceding iron in the periodic table, there is an excess of 5 spins. Only in incomplete shells such as this, shells that are being filled as new and heavier atoms are made, is there such excess spin. The completed shells are magnetically neutral because the spins are balanced.

The outermost electrons are those responsible for the ordinary chemical properties, and they are influenced by chemical combination. They do not contribute to ferromagnetism for reasons that will appear later.

Exchange Forces

Only in certain parts of the periodic table are electrons being added to *inner* shells, and one of these places is in the iron group; but since there are other parts, notably those occupied by the palladium, platinum, and rare earth metals, where these inner groups are being filled, there arises the further question: Why are not these other elements also ferromagnetic?

For an element to be ferromagnetic, it is necessary not only that there be uncompensated spin in the electron orbits, but also that the spins in neighboring atoms be parallel. Calculation of the energies of the electrons indicates that to align the spins in all the atoms in a small region, the diameter of an atom must bear the proper ratio to the diameter of the electron shell that has the uncompensated spin[23] (figure 3). This proper ratio is required because the electron spins and charges influence each other to an amount depending upon the distance between them; and it is only when this influence, which is known technically as the "exchange," has the right value that

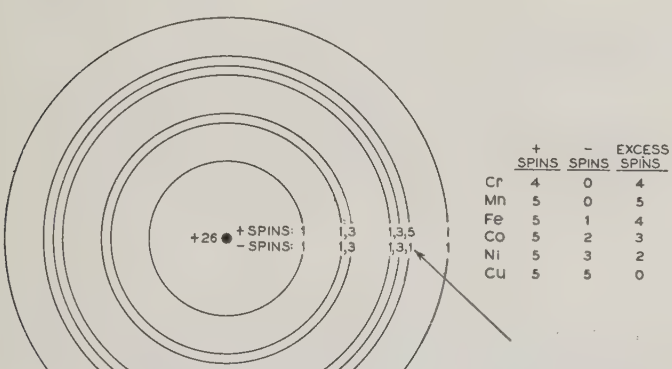


Fig. 2. Electron shells in the iron atom

the spins all can be aligned in the same direction,[24] that is, that the material can become ferromagnetic.[25]

The forces of "exchange," the existence of which has been realized only in the last few years, act to keep the spins parallel, while thermal agitation tends, obviously, to disturb this alignment. When the temperature is high enough, the temperature agitation prevails and the material ceases to be ferromagnetic. This temperature is the familiar Curie point, or magnetic transformation point, 770 degrees centigrade (or 1,043 degrees absolute) for iron. It is seen then that the height of the Curie point, θ (on the absolute temperature scale) is an indica-

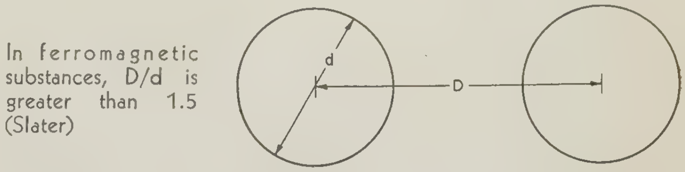


Fig. 3. "Incomplete" shells in neighboring atoms

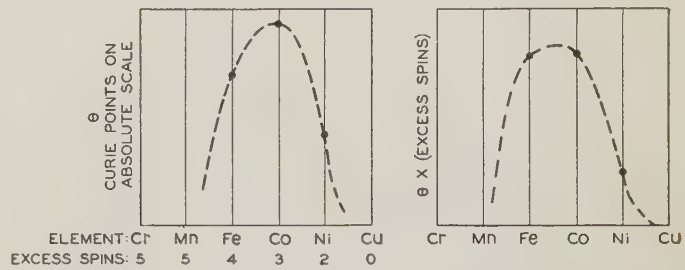


Fig. 4. Iron-cobalt alloys have the highest value of saturation magnetization

tion of the strength of the forces of exchange, which cannot yet be calculated theoretically except as to order of magnitude. These Curie points are plotted in figure 4 for the elements near iron in the periodic table; if a continuous curve be drawn through the points, it has a maximum near cobalt. Now the saturation value of magnetization depends both on the exchange and on the number of effective electron spins, that is, upon the number of electrons that can be oriented parallel to the field and the strength of the forces that hold them parallel. In a very rough way, it may be said to depend on the product of the exchange and the number, S, of uncompensated spins in the atom. Adopting θ as a measure of the exchange forces and forming the product θS, the right-hand curve in figure 4 is obtained, which indicates that the highest saturation should be attained in an iron-cobalt alloy, and that under certain appropriate conditions manganese might be ferromagnetic. Both these indications are substantiated by the data: The only known alloys having a higher saturation value than pure iron are the iron-cobalt alloys; and compounds and alloys of manganese are more magnetic than any others that do not contain iron, cobalt, or nickel. The Heusler alloys, composed of manganese, aluminum, and copper, have a saturation almost as high as nickel, and numerous compounds of manganese are ferromagnetic in a less degree.

The forces of exchange are purely electrostatic in origin. But they are not electrostatic in the classical sense of the word, they are the result of electric charges distributed in space in a definite way. It does not seem to be possible to describe them easily in words,

for it takes a great many mathematical equations to derive the result, which is a consequence of the assumptions of quantum mechanics. These forces account for the fact that it is easy to align the excess spins of neighboring atoms of some materials. In fact, when the forces of exchange are large (positive) as they are in ferromagnetic materials, the stable situation is one in which the spins are parallel, even when no magnetic field is applied. But the parallelism under such circumstances does not extend over the whole of a specimen of ordinary or even of visible size; for some reason not understood it is limited to smaller regions. On the average, those regions are found experimentally to have the volume of a cube about 0.001 inch on an edge. *An actual ferromagnetic body is composed of a great many such regions*, called "domains," *each domain being magnetized to saturation* (i. e., electron spins parallel) *in some direction.* When the material is said to be unmagnetized, the domains are oriented equally in all directions so that the magnetization of the specimen as a whole is zero.

Experimental evidence of the existence of these domains is supplied by the so-called "Barkhausen effect" (figure 5). If a small portion of a magnetization curve such as is shown in figure 5 could be magnified a billion times, it would be seen to be made up of steps, each a sudden change in magnetization as the field is increased with no further change until the field reaches a certain higher value. No known apparatus can give such direct magnification, but these sudden jumps can be detected by winding a coil around the specimen and connecting its ends to an amplifier at the output of which is a pair of telephone receivers. When the field is increased slowly, a series of clicks, or "noise," is heard in the receivers; a more quantitative method shows that the average click corresponds to the reversal of magnetization in a region the size[26] of a cube 0.001 inch on an edge, containing 10^{15} atoms. Under favorable conditions this "Barkhausen noise" can be heard without an amplifier, with the receivers connected directly to the coil.

It has been pointed out that the forces of exchange are opposed by the disordering effect of temperature agitation. As a result, the saturation value of magnetization decreases continually as the temperature is increased, until at the Curie point the ferromagnetism disappears. Data for saturation at various temperatures are shown in figure 6, plotted in such units that the saturation is unity at the absolute zero of temperature, and the Curie point is unity on the temperature axis. On such a plot it is found that the data for iron, cobalt, and nickel fall close together. The lower curve is the theoretical one calculated 30 years ago on the assumption that the elementary magnets, when they are disturbed by temperature agitation, can assume any orientation. If it be assumed, on the contrary, that the spinning electrons responsible for ferromagnetism can assume only 2 orientations with respect to the other electrons in the atom, the upper curve is the result. This is of interest because the spectroscopists and atomic structure experts came independently, each group from its own data, to the conclusion that each electron in an atom can assume only a limited number of orientations with respect to the rest of the atom.

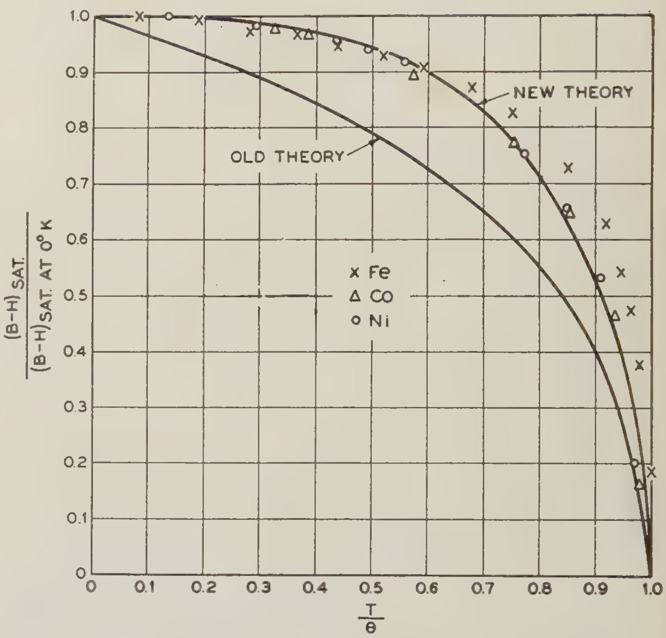


Fig. 6. Dependence on temperature of the saturation magnetization of iron, cobalt, and nickel

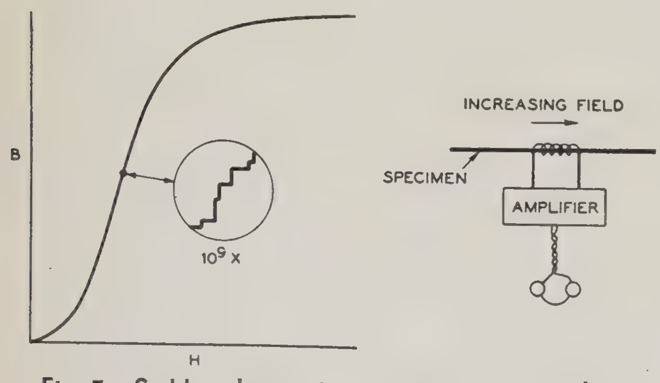
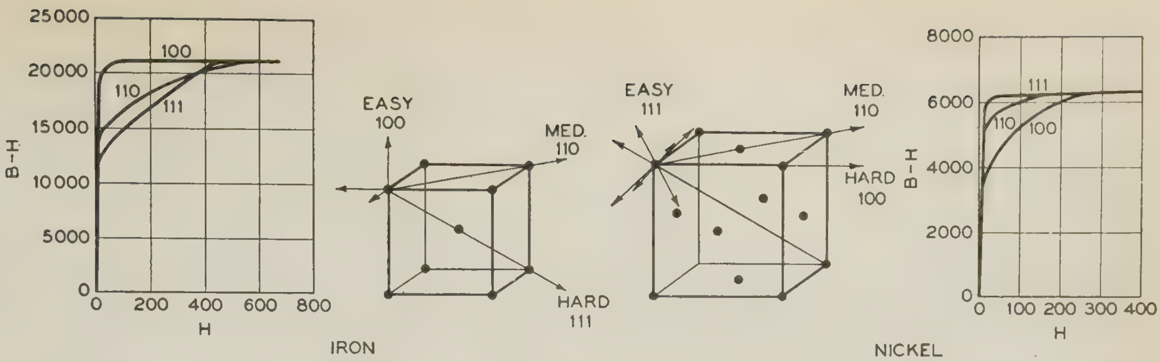


Fig. 5. Sudden changes in magnetization cause the Barkhausen effect

Effect of Crystal Structure

There is another kind of force that must be postulated in order to explain the properties of a single crystal. Because of the spinning electrons which it contains, and also because of their orbital motions, each atom may be regarded as a small magnet. These magnets will influence each other in a purely magnetic way,[27] just as a group of bar magnets will; and in a crystal it may be readily appreciated that because of these magnetic forces between atoms arranged in a regular fashion, some directions of magnetization are more stable than others. In iron the most stable direction is observed to be that of the cube edge, one of the cubic axes of the crystal. In nickel it is the cube diagonal (figure 7).

Fig. 7. Magnetic properties and crystal structures of single crystals of iron and nickel (Beck, Honda and Kaya, Webster)



Ordinarily a piece of iron is composed of crystal grains each one of which is too small to be detected by the naked eye. In recent years, however, means have been found to control the grain size of all the common metals, and single grains (i. e., single crystals) have been prepared which are so large that experiments may be performed and data collected on just one such crystal.

The structure of a single crystal of iron may be represented by a cube with an atom at each corner and one in the center, the whole crystal made up of such cubes packed together face to face. It is found experimentally that in the direction of an edge of this cube (called by the crystallographers a [100] direction) the magnetization curve labeled 100 in figure 7 is obtained.[28] In the 2 other principal directions, the direction of a face diagonal and that of a cube diagonal, the other magnetization curves shown are obtained. The difference in the initial parts of the magnetization curves is negligible, the effects being large only above half saturation.

The structure of nickel may be represented also by an assemblage of cubes, but the atoms are arranged in a different manner, being at the corners of the cubes and the centers of the cube faces (figure 7). The magnetization curves for nickel corresponding to the same 3 principal directions are shown also in figure 7, and it may be seen that the curves are reversed in order from those of iron. In iron the [100] direction is said to be the direction of easy magnetization and the [111] the direction of most difficult magnetization, whereas the reverse is true of nickel. It might be said that the electrostatic exchange forces align the spins parallel to each other and that the crystal forces determine the particular crystal direction along which they shall be aligned. The forces of exchange are so powerful that they are able to align the spins of a group of atoms, a situation that in the absence of such exchange forces could be accomplished at room temperature only by an applied field of 10,000,000 oersteds. On the other hand, the crystal forces are so feeble that it takes only 1,000 oersteds to redirect the spins of an entire group of atoms from any direction to any other direction. The ratio between these 2 equivalent fields is thus 10^7 divided by 10^3, or 10^4.

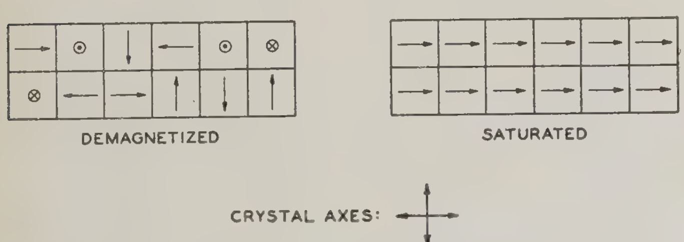


Fig. 8. Domains in a single crystal of iron

As a result of the forces of exchange and the magnetic crystal forces, in a single crystal of iron, for example, the situation is as represented in figure 8. Even when the crystal is apparently unmagnetized, or demagnetized, there are small regions, called domains, that are magnetized to saturation in one of the 6 equivalent directions of the crystal axes. Actually, the domains vary considerably in size and shape, but are represented conveniently as squares. Each of the 6 directions is equally stable and equally probable when no field is applied. The initial effect of applying a magnetic field is to change the direction of magnetization from one stable position to another, thus increasing the resultant magnetization in the direction of the field. These changes take place suddenly, and they are the cause of the Barkhausen effect; each sudden change in orientation of a domain accounts for one step in the magnified magnetization curve shown in figure 5, or for one click heard in the telephone receiver when listening to the Barkhausen effect.

Fig. 9. Powder-patterns for iron (McKeehan and Ellmore): (left) field outward; (middle) demagnetized; (right) field inward

There is even more direct evidence of the existence of domains in a piece of iron. The iron is placed under a microscope with a magnification of 500, and is covered with a colloidal suspension of iron

oxide.[29] It is found (figure 9) that the colloidal particles are concentrated along lines determined by the crystal axes, indicating that stray magnetic fields go in and out of the surface just as if some sections were magnetized differently from their neighbors. This occurs even when the iron is unmagnetized, but never occurs with materials that are not ferromagnetic.

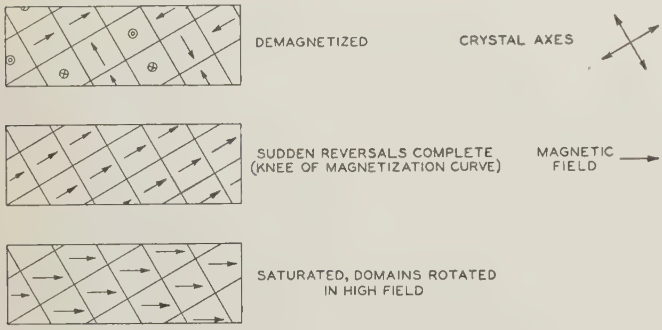


Fig. 10. As the magnetic field strength increases, domains first change direction suddenly, then rotate smoothly

Now consider in more detail by what processes changes in magnetization occur. Most changes are attributable to the reorientation of electron spins in domains, from one direction of easy magnetization to another (figure 10). These are the changes that take place over the large central portion of the magnetization curve. In general, however, it is obvious that this process is complete before the material is saturated. When all the domains are magnetized parallel to that direction of easy magnetization which is nearest to the direction of the applied field, the only way in which the magnetization can be increased further is by rotating the electron spins in each domain out of the stable position toward the field direction. Such a process is described loosely as the "rotation of the domain." This is the process that occurs in high fields, of the order of 10 to 100 oersteds; as may be seen in figure 7, its beginning corresponds to the place where the curves suddenly bend over, away from the almost vertical section. It is only when the field is applied to a single crystal in the direction of easiest magnetization that this last process is avoided. When the field is applied in the direction of most difficult magnetization, the rotational process begins at a field-strength lower than in any other case.

One other important property of single crystals is accounted for by this picture. This property is evident when a field is applied to a single crystal in a direction not parallel to a principal axis. For example, let the field be applied 30 degrees from a cubic axis of an iron crystal, as indicated in figure 11 by the longest arrow. As this field is increased from zero, the magnetization will correspond in magnitude and direction to the other arrows shown. First it is parallel to the field, but as the field increases it deviates toward a direction of easy magnetization until finally it is saturated in that direction. As the field is increased further, the magnetization approaches again the direction of the field and finally is saturated in this direction. Theory agrees with experiment in that it predicts[30] the direction and amount of the deviation of B-H from H for any given value of B-H.

The 2 ways of changing magnetization that have been described for single crystals, namely sudden changes to new directions of easy magnetization, and continuous rotation of domains, apply equally well to *ordinary polycrystalline* material, the properties of the latter being those of the former averaged for all orientations. One result of this averaging, of course, is that the specimen is now isotropic and B is parallel to H.

These last remarks must be qualified, for the magnetic materials used by engineers are not always isotropic, that is, the crystal axes are not always distributed equally in all directions. It has been known for many years that when a metal sheet is rolled, the crystals composing it tend to be oriented in special ways with respect to the direction of rolling and to the rolling plane. Even after the sheet has been annealed and recrystallized, these special orientations exist, in some metals all the way up to the melting point. Since the magnetic properties depend on the crystal direction in a single crystal, it follows that sheets composed of crystals having special orientations will not have the same magnetic properties in all directions. This was observed some years ago in iron, nickel, and iron-nickel alloys.[31] More recently, there has appeared on the market a silcon-iron[32] alloy for which the permeabilities in different directions are markedly different. Measured parallel to the direction of rolling this material has a permeability in high fields (B = 15,000 gausses) of 4,000, while measured at right angles to the direction of rolling the permeability is only 400. X ray analysis shows[33] that the crystals in this material are aligned so that most of them have a cubic axis lying

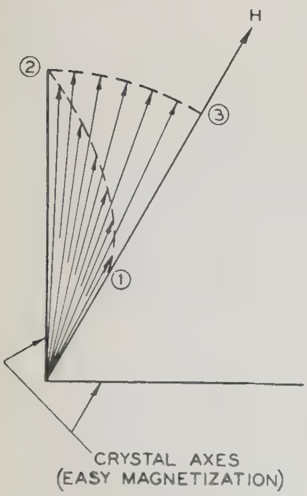


Fig. 11. Vectors represent B-H in iron, increasing in magnitude as the magnetic field (H) increases

First B-H is parallel to H (1); then as B-H increases it deviates in direction from H (2); and finally in high fields is again parallel to H (3)

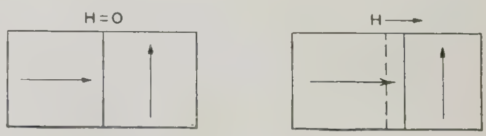


Fig. 12. Magnetization in very low fields progresses by slight displacement of domain boundaries (Becker)

within a few degrees of the direction of rolling. Thus the direction of rolling coincides with the direction of easy magnetization.

In considering the properties of single crystals, the properties in very low fields have not been mentioned, chiefly because relevant data for single crystals are very difficult to obtain. The process that occurs in this region in single crystals and polycrystals must be different from either of the 2 so far considered, because in ordinary polycrystalline material no discontinuities in magnetization are found, i. e., no Barkhausen effect occurs, and also it is known that the fields are not strong enough to rotate the domains to any significant extent against the crystal forces, out of a direction of easy magnetization.

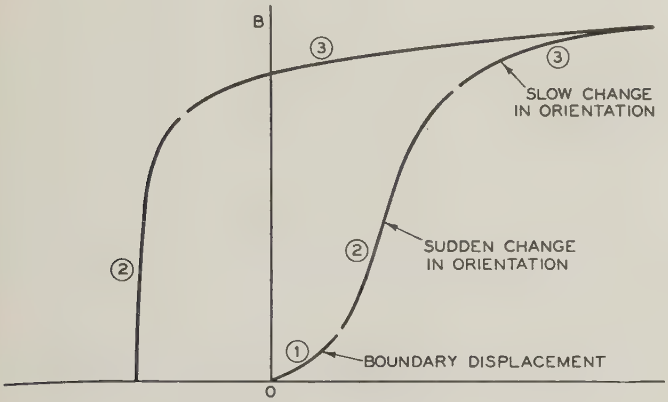


Fig. 13. Illustrating the 3 kinds of change in magnetization

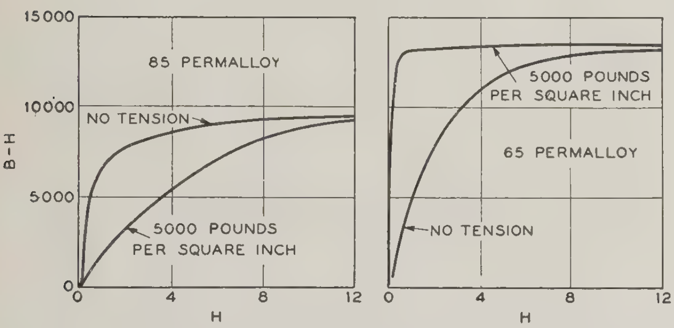


Fig. 14. Effect of tension on magnetization (Buckley and McKeehan)

Knowing as we do the relation between magnetic force and angular displacement in high fields, it is calculated that if this same mechanism applied to changes in magnetization in very low fields the highest value of initial permeability in iron would be about 20 instead of many thousands. In the past the process occurring in low fields has been the cause of much speculation, but recently a satisfactory explanation seems to have been found.[5] The changes that take place here are visualized as displacements of the boundaries of domains (figure 12); the transition region of a few atom diameters (calculated from the forces of exchange to be about 30 atom-diameters[7]) moves so as to enlarge a domain magnetized in the direction of the field at the expense of a domain pointing in a less favorable direction. Such a movement can progress for a distance that is short compared with the linear dimensions of a domain, and is limited by the small strains present in any actual material.

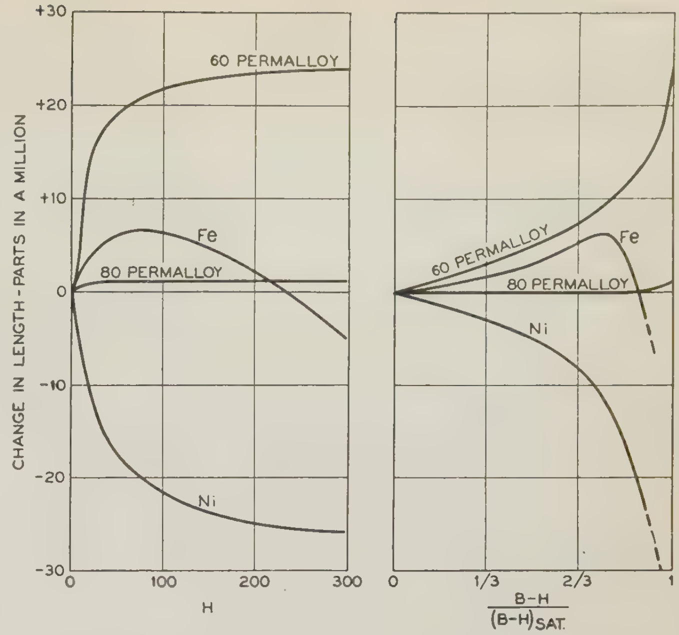


Fig. 15. Magnetostriction in iron, nickel, and 2 iron-nickel alloys ("permalloys")

Thus in the magnetization of an ordinary well-annealed ferromagnetic material there are 3 processes occurring, corresponding to the 3 well known sections of the magnetization curve (figure 13): growth of one domain at the expense of a neighboring one in the initial portion of the curve, sudden changes of direction of domains (with resulting large energy losses) in the middle portion, and continuous or smooth rotation of the domains in the upper portion. The latter 2 processes occur during the traversal of a large hysteresis loop with tips at high flux densities; the first process is important only in low fields after demagnetization.

Effect of Strain

This picture of the changes in magnetization has been made for materials that are free from any considerable strain. As a matter of fact, strain can affect magnetization in an important way, and under certain circumstances a tensile stress of 5,000 pounds per square inch may change the flux density B as much as 10,000 gausses[34]—almost from zero magnetization to saturation (figure 14). This effect is illustrated well by data for 65 and 85 permalloy (iron-nickel alloys containing, respectively, 65 and 85 per cent nickel). For 65 permalloy the effect of tension is to increase the magnetization in all fields; for 85 permalloy the effect is the opposite; and in each case the effect of compression is opposite to that of tension. For ordinary iron the effect of tension is to increase the magnetization in small fields, but to decrease it in high fields.

The effect of strain on magnetization has its counterpart in an effect of magnetization on the length of a piece of ferromagnetic material. When a rod of iron is magnetized its length increases by a

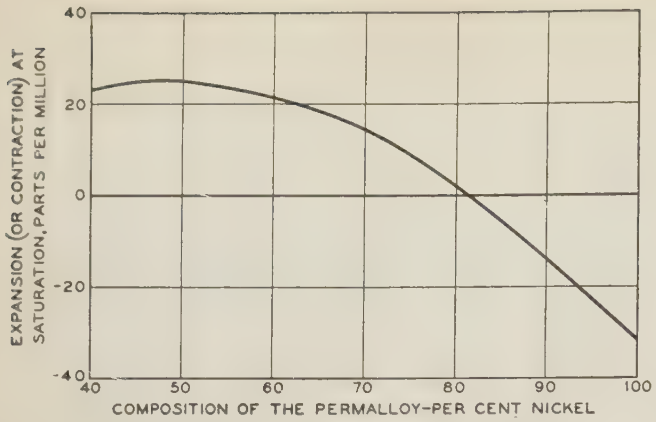
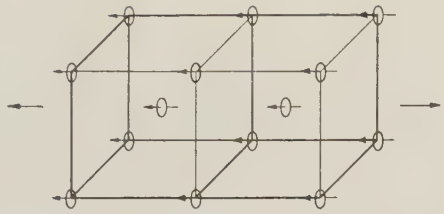


Fig. 16. Saturation magnetostriction of the permalloys (McKeehan and Cioffi, Schulze)

Fig. 17. The magnetic forces between atoms cause a slight elongation in iron (magnetostriction)



small amount. This is but one example of a large class of effects exhibited by all ferromagnetic bodies and known collectively as "magnetostriction." Figure 15 shows the data for change in length of rods of nickel, iron, and 2 alloys, plotted against the field H on the one hand and against relative B-H on the other. When saturation of magnetization is reached, the limiting value of magnetostriction, called "saturation magnetostriction," also is attained. Its values for some iron-nickel alloys are shown in figure 16. Note here that the change in length is an extension in the alloys containing less than 81 per cent nickel, a contraction otherwise. There is a close relation between magnetostriction and the effect of strain on magnetization, it being a general rule that when the magnetostriction is positive (increase in length with magnetization) the effect of tension is to increase magnetization, and vice versa (figures 14, 15, and 16).

The next question to be considered is: How much can theory say of magnetostriction and the effect of strain on magnetic properties? Figure 17 shows how the atoms are arranged in an iron crystal; each atom here is supposed to have a definite magnetic moment as a result of the spin and orbital motion of the electrons. This supposition makes it possible to calculate the magnitude of the mutual magnetic forces which are opposed by the elastic forces holding the crystal together. For iron, the calculations[5] indicate that equilibrium is reached when there has been a slight increase in length in the direction of magnetization and a decrease in length at right angles to this direction such that the volume remains practically unchanged. This calculated magnetostriction is in agreement with experiment as to sign and order of magnitude. With nickel the agreement is not so satisfactory. But in each case the theory is clear in predicting the proper qualitative relationship between magnetostriction and change in magnetization caused by strain.

Thus magnetostriction and the magnetic effects of strain are reciprocal properties, and result from the same kind of magnetic forces between atoms as those that account for the variation in magnetic properties in different directions in a crystal; and just as the crystal structure determines a direction of easy magnetization in a strain-free crystal, so the strain controls the direction of each magnetization when the strain is sufficiently great. Figure 18 shows how the domains are magnetized parallel to the crystal axes in unstrained iron, and how a sufficiently large tension will orient the magnetization parallel to the direction of tension in iron and at right angles to the direction of tension in nickel. When the stress is as large as 10,000 to 30,000 pounds per square inch, the strain effect begins to predominate over the crystal effect and the direction of magnetization is determined mainly by the strain. The calculations show also that in a material having positive magnetostriction the magnetization is increased by tension. In a qualitative way these considerations explain the increase in permeability of 65 permalloy (having positive magnetostriction) and the decrease in 85 permalloy (with negative magnetostriction). But so far the theory is quite inadequate to predict the magnitude of the effect.

In addition to unidirectional or homogeneous strains, such as those produced by stretching a wire

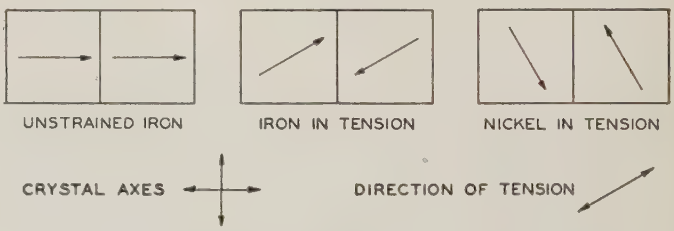


Fig. 18. Domains are oriented by crystal forces and by strain; H = 0

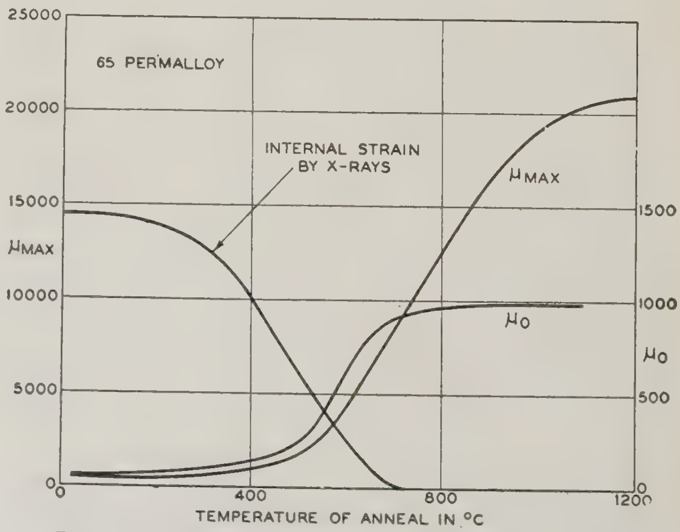


Fig. 19. Magnetic permeability rises as internal strain is diminished by annealing (Dillinger and Haworth)

in the direction of its length, random (heterogeneous) strains often are found which vary in magnitude, sign, and direction from point to point throughout a material. Such strains are produced by cold working, phase transformations, and the like. In such materials the directions of magnetization in the domains determined by the local strains, are more stable the larger the strain. So it can be appreciated that it is harder to change the magnetization of a material that is more severely hard worked. These internal strains are the same ones that contribute to the hardness of a metal—hence the parallelism between magnetic hardness and mechanical hardness, which is so well known.

This relation between internal strain and permeability is illustrated by the data[35] shown in figure 19. The permeabilities of a series of specimens of 70 permalloy tape, originally cold rolled, increase as the annealing temperature is raised. X ray data (the width of the reflected X ray beam) on these same specimens indicate the magnitude of the internal strains existing, and show that they become progressively less as the annealing temperature is increased, the most rapid change taking place in each case between 400 and 600 degrees centigrade, in which region the microscope shows recrystallization has occurred.

Following out this same idea, it may be surmised that to make good material for a permanent magnet something with very intense internal strains is required. The direct determination by X rays of in-

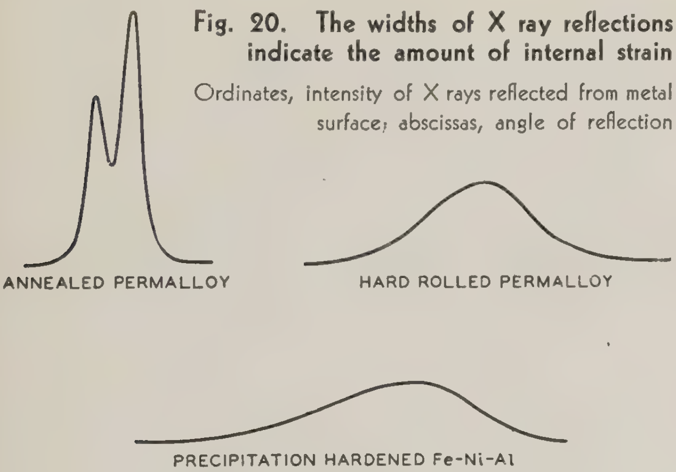


Fig. 20. The widths of X ray reflections indicate the amount of internal strain

Ordinates, intensity of X rays reflected from metal surface; abscissas, angle of reflection

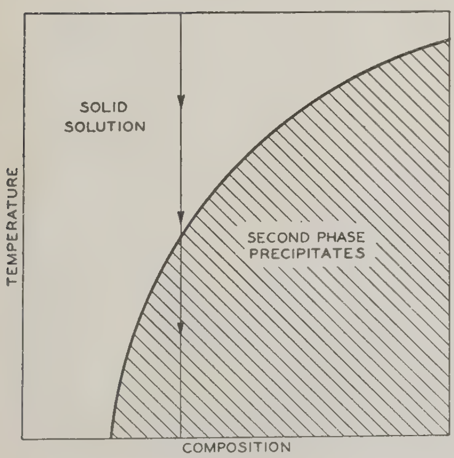


Fig. 21. Precipitation hardening of an alloy for a permanent magnet, such as an alloy of iron, nickel, and aluminum

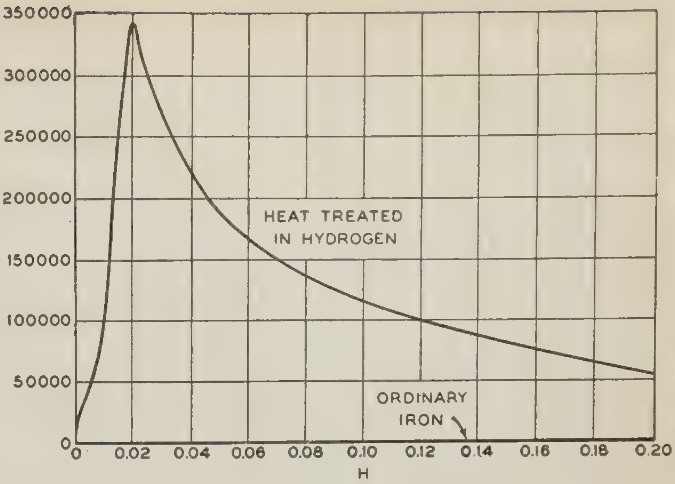


Fig. 22. Permeability curves of ordinary iron and of iron purified by heat treatment in hydrogen at 1,500 degrees centigrade (Cioffi)

ternal strain in a good permanent magnet, confirms this view (figure 20). Here the width of the reflected X ray beams are a direct measure of the internal strains. For comparison with the permanent magnet material are shown curves for other materials with less internal strain. The magnet material in this case was an iron-nickel-aluminum alloy that was precipitation-hardened, a method used more and more extensively during the last 3 or 4 years for such materials. This method is often applicable when the alloy[36] in the stable condition consists of 2 phases at room temperature (figure 21), but when at a higher temperature the one phase dissolves completely in the other to form a solid solution. In making the material, it is quenched rapidly from a high temperature and then reheated to 700 degrees centigrade, at which point the second phase precipitates slowly in very finely divided form. When the optimum amount has precipitated, the cooling is continued to room temperature, no more changes occurring. Each submicroscopic precipitated particle is a center of strain, and it is the presence of these unusually large internal strains that is responsible for the good quality of the permanent magnet.

Going now to the other extreme, where ease of magnetization is required, it is known, of course, that thorough annealing and a homogeneous structure are beneficial. Still there are at least 2 sorts of strains that annealing will not relieve. One is that attributable to the nonmetallic chemical impurities that do not fit into the regular arrangement of atoms in a pure metal or alloy. It has been found recently that by heat treating iron in hydrogen at about 1,500 degrees centigrade the nonmetallic impurities are largely removed, and that what are called "chemical strains" are much reduced. As a result it is found (figure 22) that the maximum permeability increases from 10,000 to 340,000,[12] and a large reduction in mechanical hardness occurs simultaneously.

After the chemical strains and the strains resulting from cold working have been removed, there is still another kind of residual strain—that attributable to magnetostriction. These are ordinarily random in

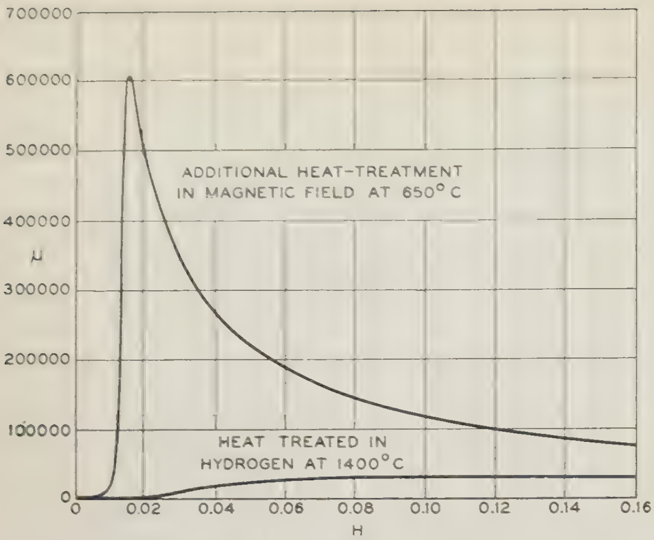


Fig. 23. Permeability curves of 65 permalloy after heat treatment in hydrogen and additional heat treatment in a magnetic field (Dillinger and Bozorth)

direction because they are associated with randomly oriented domains, but by a suitable trick they all can be oriented so as to favor magnetization in a single desired direction at the expense of ease of magnetization at right angles. This trick is heat treatment in the presence of a magnetic field. Without going into a more detailed explanation, the experimental results obtained[15] about 2 years ago will be given.

When an annealed specimen of 65 permalloy is heated for a few minutes at 650 degrees centigrade while it is subjected to a magnetic field of 10 oersteds, the maximum permeability is increased from about 20,000 to over 600,000 as shown in figure 23. This material holds the records for the highest maximum permeability, the lowest coercive force, and the lowest hysteresis loss at high flux densities. It may be compared with the most permeable material known in 1,900, iron with a maximum permeability of less than 3,000.

So far only the effects of stress on the orientation of domains in medium and high fields have been considered. But stress has an effect on the initial permeability also. It already has been said that in very weak fields a change in magnetization is attributed to a movement of the boundaries between

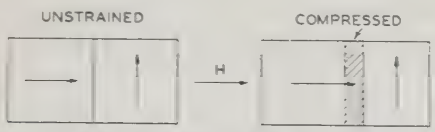


Fig. 24. Magnetostriction in the shaded region acts as a barrier to further change in magnetization

domains, the domains oriented nearly parallel to the field growing at the expense of adjacent domains oriented in less favorable directions. Such a growth obviously may be hindered by strain. A relation has been derived[37] connecting the initial perme-

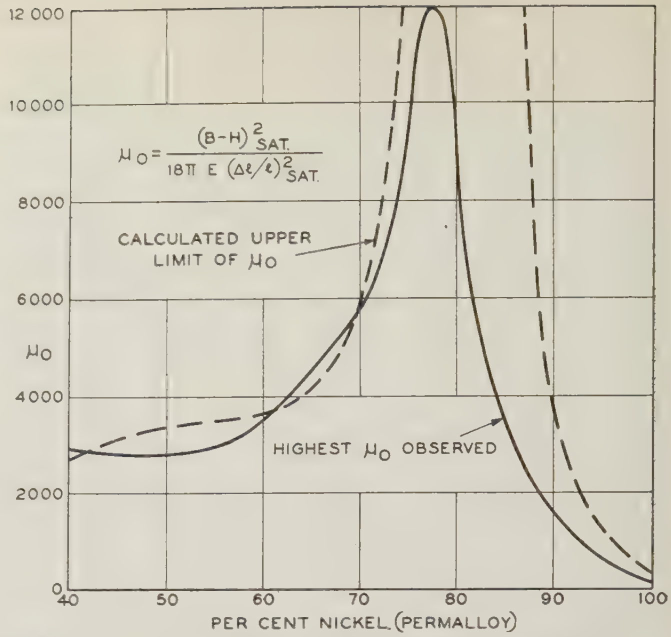


Fig. 25. Comparison of the theoretical upper limit of initial permeability (Kersten) with the highest initial permeabilities observed for iron-nickel alloys (Arnold and Elmen, Schulze)

ability with the internal stress and other magnetic quantities:

$$\mu_0 = \frac{0.018(B-H)^2_{\text{sat.}}}{(\Delta l/l)_{\text{sat.}}\ \sigma_i}$$

where μ_0 is the initial permeability, $(B-H)_{\text{sat.}}$ and $(\Delta l/l)_{\text{sat.}}$ are the (ferric) induction and magnetostriction at saturation, and σ_1 is the average value of the internal stress in dynes per square centimeter.

Even when there are no internal strains from impurities, insufficient annealing, etc., there generally will be the strains of magnetostriction itself, and these will hinder the growth of one domain at the expense of another (figure 24). In this case the stress in the foregoing equation is equal to Young's modulus, E, multiplied by the magnetostrictive strain,

$$\sigma_i = E(\Delta l/l)_{\text{sat.}}$$

and the former equation becomes

$$\mu_0 = \frac{0.018(B-H)^2_{\text{sat.}}}{(\Delta l/l)^2_{\text{sat.}}\ E}$$

This equation really gives a theoretical upper limit for μ_0. These theoretical limits and the highest observed values for iron-nickel alloys are shown in figure 25. This indicates why the composition of the permalloy having the highest initial permeability is very nearly the same as that for which the magnetostriction is zero.

The effects of strain now will be summarized briefly. The origin of the effects lies in the *magnetic* action between neighboring atoms. The magnetic action is balanced by the elastic (electrostatic) forces between atoms. The balance of these forces results in a change in shape of the magnetic body when it is magnetized (magnetostriction), and also a change in magnetization resulting from strain

(strain-sensitivity). Magnetization may be either aided or hindered by a homogeneous (unidirectional) strain, the effect depending on the magnetostriction in a way that can be estimated qualitatively but not quantitatively. But material in which local strains are directed at random is more difficult to magnetize because the strains prevent a change in magnetization; and the more intense such strains are, the harder the material is to magnetize or demagnetize. The effect of local strains upon the initial permeability can be calculated with fair success, but other magnetic quantities, such as maximum permeability, can as yet be estimated in a qualitative way only.

Summary

In concluding the author wishes to go back from here to summarize what is known about the origin of the forces responsible for the various magnetic properties and about the sizes of the various units. This information is summarized in table II.

Table II—Summary of Data Regarding Origin of Forces Responsible for Various Magnetic Properties

Unit Concerned	Property	Origin of Property	Size of Magnetic Unit
Electron	Magnetic moment	Electron spin	One unit of spin per electron
Paramagnetic atom	Magnetic moment	Uncompensated spins and orbital motions of electrons	4, 3, and 2 uncompensated spins per atom in Iron, Cobalt and Nickel, respectively
Domain	Ferromagnetism Change in properties at Curie point	"Exchange" between electrons in neighboring atoms	Volume of domain is about (0.001 inch)^3
Single crystal or region of homogeneous strain	Crystal anisotropy Magnetostriction Strain sensitivity	Magnetic forces between atoms	10^8 domains per cubic centimeter
Polycrystal	Orientation-average of single crystals and strain units	Sum of effects of single crystals and strains	Size of specimen

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Pilot Wire Relay Protection

Recent developments in relay protective schemes, using metallic pilot channels for isolating a transmission line during fault conditions, reduce considerably the magnitude of the currents which must be handled. Schemes recently developed have many other advantages over the older pilot wire schemes, being more economical to install and operate, and having excellent reliability. In addition to the advantages inherent in all pilot schemes, such as high speed tripping and absolute selectivity, the latest schemes provide indication of the condition of the pilot wire circuit at all times.

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THE idea of pilot wire protection of electric power lines and equipment has appealed to relay engineers for the last 25 years, because of the simplicity and direct action of this type of protection. Although simple and direct in theory, its practical application has until recently usually been involved and expensive, and relay engineers have generally considered it to be limited in usefulness—suitable only under special conditions. This situation is unfortunate, because pilot wire schemes of general utility and reasonable economy are now available.

The 1926 edition of the "Relay Handbook" lists advantages and disadvantages of pilot wire protection; the advantages listed are:

1. Immunity to faults outside of protected zone.
2. Independent of load currents.
3. Practically instantaneous.
4. Requires no co-ordination with other protected zones.
5. Applicable to short or long lines.
6. Provides both phase and ground protection.
7. Short circuit calculations not essential.
8. Simple construction of relays used.
9. Potential connections not required.

The disadvantages listed in the "Relay Handbook" are:

1. Cost of pilot wire channel.
2. Cost of special current transformers.
3. Necessity of uniform characteristics of current transformers.
4. Lack of supervision of pilot wire circuit continuity.
5. Induction in pilot wire circuit.

For many years prior to 1926 there had been very little change in the theory or art of pilot wire protection. In the light of changes since that time, it is interesting to note that the above advantages still apply without substantial change, but that many of the above disadvantages have since been largely removed, due to the rather recent idea of using the pilot wire channel as a remote *control* circuit, instead of a remote *metering* circuit. Attenuation thereby becomes a negligible factor in the transmission of signals.

New Principles

In the first pilot wire systems, secondary currents from the current transformers were circulated over the pilot wires. This placed heavy burden on the current transformers, required pilot wires that were short and had low impedance, and necessitated 3 pilot conductors for 3 phase systems (if both phase and ground protection were desired).

The first major development in 20 years in pilot protection occurred in March 1927 with the trial installation of a carrier current channel for pilot protection of a transmission line on the American Gas and Electric system in Ohio. This system has been described in detail by A. S. Fitzgerald.[1] The carrier channel was used to compare the instantaneous directions of fault current at the 2 ends of the protected power circuit. The idea of comparing the direction of power or reactive kilovolt-amperes instead of the instantaneous direction of current was suggested at that time, but rejected because it required potential connections.

The first installations of pilot wire protection using d-c single-channel metallic pilot-wire circuits to compare the directions of fault current at the 2 ends of a transmission line were apparently made in 1931 and 1932 by several companies. Installations were made by the Oklahoma Gas and Electric Company, Duquesne Light Company, Los Angeles Gas and Electric Corporation, Philadelphia Electric Company, and The Tennessee Electric Power Company. The first general discussion of the theory and application of this new system of pilot wire protection was presented by J. H. Neher.[2] These systems of pilot wire protection were fundamentally different from anything developed during the previous 20 years, although they resembled somewhat the carrier pilot scheme in principle.

A paper recommended for publication by the A.I.E.E. committee on protective devices, and tentatively scheduled for discussion at the A.I.E.E. winter convention, New York, N. Y., Jan. 28–31, 1936. Manuscript submitted June 26, 1935; released for publication Oct. 14, 1935.

The authors wish to acknowledge the assistance of W. E. Dent, Jr., and H. E. House of the Tennessee Electric Power Company.

1. For all numbered references see list at end of paper.

Development Work

The very satisfactory results obtained from distance relays on the Tennessee Electric Power Company system in the last 4 years were largely instrumental in removing objections to the use of potential connections. The use in distance relays of potential amplification of low voltages during fault conditions, together with the fact that the starting units of distance relays made ideal initiating units for pilot wire protection, were 2 influential items in renewing interest in pilot wire schemes.

The increased use of pilot wire protection on the Tennessee Electric Power Company system since 1931 has been due to improvements in the reliability of the pilot channel and of the pilot relay equipment, resulting from intensive development work on the generally recognized limitations of pilot wire protection.

The commercial telephone circuit, while most generally available for pilot wire service, is so completely standardized that its adaptation to pilot wire requirements presents several important engineering problems which are discussed in this paper. Power company owned telephone circuits have certain definite limitations as pilot wire channels, largely due to their more severe exposure and the general use of insulating transformers. While the power company owned telephone circuit can be handled more flexibly for protective purposes, this is more than compensated for by the technical difficulties involved. However, there seems to be a field for this type of pilot wire service where exposed telephone lines are already available without additional cost.

Power company requirements for communication are very exacting. The general recognition of this fact in the last few years by the communication companies and their study of power company protective practices has resulted in improving reliability. It seems likely that development work on pilot wire circuits may be of general advantage to the telephone as well as to the power industry, particularly in the field of protection.

Circuit Continuity

Consideration of telephone circuits as pilot channels involved conceptions of circuit continuity fundamentally different from anything used previously in communication practice. In the usual telephone circuit, an open circuit, short circuit, or ground lasting for several seconds is of small consequence, unless the circuit is busy. In many early pilot wire schemes, continuous integrity of the circuit was fundamental. Momentary short circuits of the pilot wires (or ground in the case of normally grounded battery supply) would cause incorrect tripping of oil circuit breakers.

The telephone companies were anxious to provide adequate telephone circuits for pilot wire service and made available all the recent improvements in protection designed for toll and program supply circuits. As these improvements did not seem adequate for the severe requirements of pilot wire service, it was decided to analyze the conventional requirements of pilot channels, and also to attack the problem from another side by working with the Bell Telephone System in making an analysis of telephone circuit interruptions, including their relative frequency, influence, and controllability. This analysis involved actual field operating experience, as well as general office studies.

Interruptions from electrical causes occupied the major share of attention at first, but it was soon found that the use of lead covered cable for pilot wire conductors greatly reduced interruptions from lightning, foreign contact, and induction, that rise of ground potential could be dealt with by sufficient

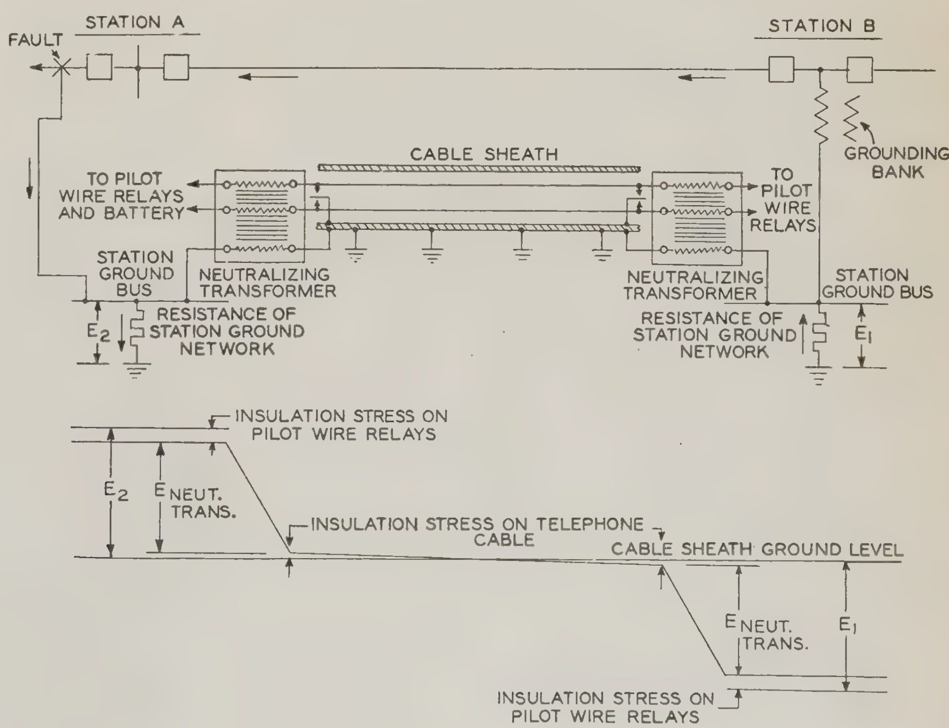


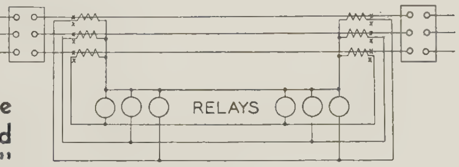
Fig. 1. Schematic diagram of recently developed pilot wire relay protective scheme using metallic channel and neutralizing transformer

See figure 6 for complete circuit showing relays and telephone protective equipment

insulation or by neutralizing transformers, and that battery failure was of such rare occurrence that the usual back-up protection could be relied upon to cover such a contingency. While progress was being made in the elimination of electrical interference affecting pilot wire protection, it was apparent that the interruptions subject to administrative control were of probably greater importance and involved greater difficulties in their reduction or elimination; for example, interruptions occurring in connection with routine maintenance and construction activities, such as testing, trouble shooting, and cable splicing.

The communication companies have made considerable progress in eliminating interference with leased circuits at cable terminals, central offices, and other junction points, by establishing distinctive markings, insulated binding post covers, etc., and by requiring authority to work on such leased circuits. Many power companies have had the same problems in connection with the maintenance of their own power circuits, but the general use of clearance orders under the dispatchers control has greatly reduced interference with service. The telephone industry has adopted in many places similar measures for the supervision of maintenance and construction on circuits where continuity is highly essential. However, some power companies still experience considerable trouble on leased circuits due to interference by cable splicers and others working at points between terminals where there is nothing to identify leased circuits except the cable color code (which is not infallible under field conditions). Another serious difficulty is that of permanent grounding of protectors. Electrical interruptions which might otherwise be momentary would often cause the protectors to ground permanently, putting the circuit out of service until the protectors were cleared. The use of fuses and carbon block protectors is such a fundamental part of telephone protection that it is very difficult to eliminate this factor as a possible source of trouble in pilot wire service. The carbon block protector has many inherent advantages as a protective device and much attention has been devoted to preventing its permanent operation, rather than to replacing it by a different type of protector. One of the newer devices is the unit type a-c relay protector which operates within one cycle after the carbon blocks, grounding and shorting the line until the source of foreign potential is removed. This device practically eliminates permanent grounding of carbon protectors but grounds both sides of the line, making it inoperative, while carbon blocks frequently operate singly. However, most electrical interference, except lightning, exists for 0.5 second or more and might occur during a few cycles when pilot wire tripping is taking place, so that it is highly desirable to prevent interference from reaching the telephone circuit, rather than to protect the circuit from permanent trouble due to the interference.

Fig. 2. Simplified schematic diagram of the earliest pilot wire scheme, often called "a-c pilot wire"



This scheme uses pilot wire conductors to connect the current transformer secondary in a differential relay scheme

Neutralizing Transformers

In February 1935 the Southern Bell Telephone and Telegraph Company, Bell Telephone Laboratories, and The Tennessee Electric Power Company made the first tests on a neutralizing transformer ever used for pilot wire circuits. The neutralizing transformer consists of a primary winding and 2 (or more) secondary windings on the same magnetic core. As shown in figure 1, the primary winding is connected between the station ground bus and the cable sheath so as to have impressed across it the voltage which would otherwise cause circuit failure. By transformer action this voltage appears in the secondary windings, which are connected in series with the communication wires, in such a direction as to oppose the disturbing voltage. The use of such an arrangement makes it possible to obtain a reliable path of relatively low transmission loss and d-c resistance into the power station for relay tripping, telemetering, dialing, etc. The neutralization is practically complete, in contrast to the limited reduction provided by "series resistance" or "series reactance" protection. The neutralizing transformer eliminates the operation of carbon block protectors, and the protectors are therefore provided only as back-up protection.

Circuit Supervision

Figure 1 illustrates the operation of the neutralizing transformers during ground faults. It is impossible to neutralize the rise of ground potential completely, due to transformer ratio errors, and capacitance of cable and other leakages. This, of course, results in an insulation stress on the pilot wire relays and telephone cable, but this stress has in all cases tested less than one per cent of the total ground potential rise. In a station with 4,000 volts rise of ground potential the neutralizing transformer would reduce the stress on telephone equipment and relay equipment to less than 40 volts, which is well within the rating of all insulation involved and less than 10 per cent of the voltage required to operate the most sensitive type of carbon block protector.

Even after taking every precaution to provide a circuit that will seldom fail, it is highly essential to know immediately when failure does occur. The first method of supervision was by the use of neon lamps at each end of the circuit. The charging current of cable circuits has been found to be a factor when d-c control voltages are applied to the cable. It was also found that neon tubes would cause undesired tripping by flashing over internally on voltages due to lightning and induction that did not affect other equipment. The vacuum contact relay was found to have the same weakness, and in certain cases it had to be replaced by a relay with a wide air gap be-

tween contacts. Another method of supervision tried was the usual ground detector scheme using ungrounded battery, and grounding the midpoint of 2 indicating voltmeters, or high resistance relays, to indicate any unbalance. This scheme failed to detect the simultaneous grounding of both sides of the circuit, but was a great improvement over grounding the supply battery, which caused tripping if the circuit was grounded by testing, maintenance activities, or protector operation.

Polar Relays

Recently much improvement has been made in the way of making pilot wire circuits self supervisory. This, of course, means continuous closed circuit operation. The most satisfactory method is to use direct current of one polarity for supervision and the other polarity for tripping. On such installations there are provided 2 relays with copper oxide rectifiers in series with their coils at one end of the circuit, using one for tripping and the other for supervision. Battery voltage is supplied at the other end of the circuit and reversed whenever the directional fault detector at that end operates. This not only takes care of the usual pilot wire scheme involving directional comparison, but also provides for transferred tripping which seemed to have considerable utility, but appeared to involve almost impossible requirements as to continuity of the pilot wire circuit. The rectifier polarized supervisory relay at the end opposite from the battery supply supervises the circuit against open circuits, double grounds, crosses, and battery failures. At some stations it is possible to bring this alarm to the attention of an operator or dispatcher directly, or over another supervisory circuit. Where this is not possible the circuit may be supervised by an undercurrent and an overcurrent relay at the battery end of the line.

The use of the plan described hereinbefore almost entirely eliminates difficulties with temporary failure of the pilot wire circuit, unless such failure should occur coincident with the fault on the protected section of the power system. If protector operation can be eliminated it is believed that there are very few other hazards to the continuity of the pilot wire circuit that will be of a permanent nature. Furthermore, with continuous supervision on the circuit it is generally possible to detect these hazards and clear them up before power trouble occurs. It will be seen from the above that adequate supervision of the circuit under all likely conditions of failure is of extreme importance. The circuit may fail by being opened, short circuited, grounded, or grounded and short circuited.

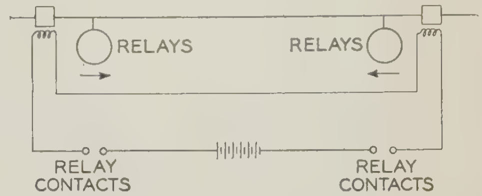


Fig. 4. Simplified schematic diagram of one of the first d-c pilot wire schemes

Three phase transmission line and relays shown by one line diagram. Pilot wire circuit shown with all conductors. Pilot wire circuit is a d-c telegraph system. Instantaneous directional fault detecting relays used at end for "directional comparison" method. Pilot wire circuit is normally open and no supervision is obtained

With these recent improvements in pilot wire tripping and supervision and the use of neutralizing transformers, experience indicates that reliability of a leased pilot wire circuit is adequate, with a reasonable amount of attention by the communication companies in the matter of designating these circuits and of educating their personnel as to the proper precautions in working on such circuits. Even with the most conscientious effort on the part of communication companies, it was difficult to maintain an adequate degree of continuity on the type of circuit first used for transferred tripping[2] and on some types of circuits used for tripping by directional comparison. The new types of circuits just described have the great advantage that any interference with the circuit is immediately reported to the operator or dispatcher, who can communicate with the telephone company, thus permitting the cause to be investigated and steps to be taken to render its repetition unlikely. This is a great improvement over former methods where the first notice any one had of the pilot wire circuit being in trouble was its failure to operate properly when power trouble occurred, and there was no way of knowing how many hours or days the pilot wire circuit had been in trouble.

Figures 2 to 5 indicate schematically some of the improvements in circuit design that have been developed for metallic pilot wire protection within the last few years. These circuits are intended to illustrate the elementary principles of circuit supervision discussed in this paper.

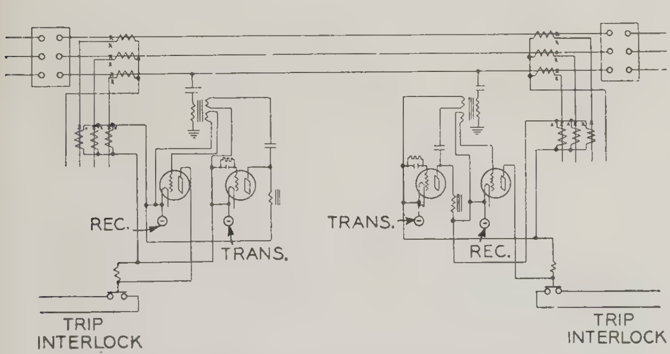


Fig. 3. Simplified schematic diagram of the earliest carrier channel pilot scheme

Current transformers in the 3 phases are of different ratios resulting in sufficient residual current under any fault condition to bias the transmitting tube and send out carrier frequency on the positive half cycles of residual current. The polarity of the residual currents at 2 ends of the line is such that the receiving and transmitting elements at either end co-operate to lock open the trip circuits in case of through faults and to close the trip circuits in case of faults in the protected section

Operating Experience

The Tennessee Electric Power Company made its first installation of metallic pilot wire protection on a

transmission tie line in Nashville in October 1932, using leased telephone cable for the pilot channel. Experience gained with the Nashville pilot wire installation was instrumental in developing the features necessary for the reliable operation of such equipment and was a deciding factor in rebuilding the distribution system in South Chattanooga. The system consists of 4 new switching substations and involves 5 short sections of overhead single circuit 11

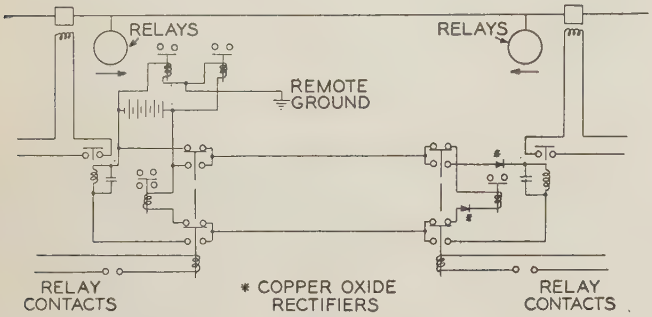


Fig. 5. Simplified schematic diagram of latest pilot wire scheme developed by the Tennessee Electric Power Company

Pilot wire circuit is normally closed but fully supervised. All supervision is obtained by relays giving an audible local alarm for attended stations. In case of nonattended stations the supervisory alarm is transmitted over other alarm circuits (or unaffected links of pilot wire in case of a tandem feed loop with all sections protected by pilot wire). By the use of copper oxide rectifiers the tripping and supervisory currents are caused to be of opposite polarity so that interference with the pilot wire circuit during normal conditions operates only the supervisory relays and not the tripping relays. With this scheme battery supply is provided in one of the stations, eliminating the disadvantages of grounded battery supply from a central office. Insulation between the station and the telephone line is maintained by the use of vacuum contact relays and other relays having high insulation between coils and contacts, as shown here, or by neutralizing transformers as shown in figure 1

kv tie lines with heavy industrial loads distributed along them. The leased circuits for the pilot wire protection of this area are all in aerial cable. Vacuum contact relays and other highly insulated relays are used throughout. One centrally located substation was selected as the heart of the pilot wire system. The pilot wire circuits have one terminal at this central substation and station storage battery voltage is supplied to all circuits through a neutralizing transformer at this station.

A new system of polarized closed circuit supervision was developed for the pilot wire equipment in the South Chattanooga area. This system gives immediate warning of failures of the leased circuits or of the battery supply. This same supervisory system is also used to transmit to the central substation of the group an alarm indication whenever any circuit breaker has opened or in case of power failure. All 4 substations are nonattended and the alarm signals received at the central substation are transmitted to the office of the system load dispatcher over a single pair of wires which are leased for that purpose. A typical pilot wire circuit is shown in figure 6.

This particular circuit has been in use on 3 pilot wire installations in South Chattanooga for 16 months. The chief difference between this and previous circuits is that ungrounded battery voltage has been used so that one permanent ground on any portion of the circuit cannot cause false operation or failure to operate unless a subsequent ground on some other portion of the circuit occurs. Continuous supervision against open circuits has been combined with supervision of oil circuit breakers at the 3 outlying stations. The code alarm sender at Alton Park substation transmits indications from all stations in the area to the dispatching office, but of course it does not distinguish between an open circuit on the pilot wire equipment and a feeder circuit in permanent trouble at one of the outlying stations. Operating experience has shown very little difficulty in determining whether an alarm signal indicates power trouble or pilot wire circuit trouble.

Instantaneous pilot wire protection is available at all times even if the circuit breaker at one end of the line section fails to reclose after an interruption. For testing a line after an interruption the scheme illustrated here utilizes an auxiliary switch on the oil circuit breaker to pick up the pilot wire auxiliary relay whenever the oil circuit breaker is open. It may be seen that the line may be tested from either end or both ends at once, the only requirement being that the pilot wire auxiliary relays and pilot wire tripping relays must have time to reset between the opening of the finger *b* on the oil circuit breaker and the closing of the finger *a*. Since some of the circuit breakers in the South Chattanooga area are quite rapid in operation a safeguard was added in the form of a small auxiliary relay with its coils in parallel with the closing solenoid of the oil circuit breaker and its normally closed contacts in series with the breaker auxiliary switch and the pilot wire auxiliary relay.

No difficulties have been experienced with the South Chattanooga scheme and no close adjustments have been found necessary.

The performance record of the South Chattanooga pilot wire scheme (see table I) may be analyzed ac-

Table I—Performance of Relay Protection in South Chattanooga Pilot Wire System

May 1, 1934 to September 1, 1935

Section	Correct Operations	Incorrect* Operations	Adjacent Operations
St. Elmo—Alton Park	11	1**	90
Rossville—Alton Park	21	1**	48
Long St.—Alton Park	5	0	30
Total	37	2	168
	95% correct		

* All unnecessary operations—no failures to operate have occurred.
** Considered incorrect but this is questionable on account of lack of information.

cording to the functions which are considered essential in a modern pilot wire installation. Of course, the primary function is to serve as a relay protective scheme. A relay protective scheme is no stronger

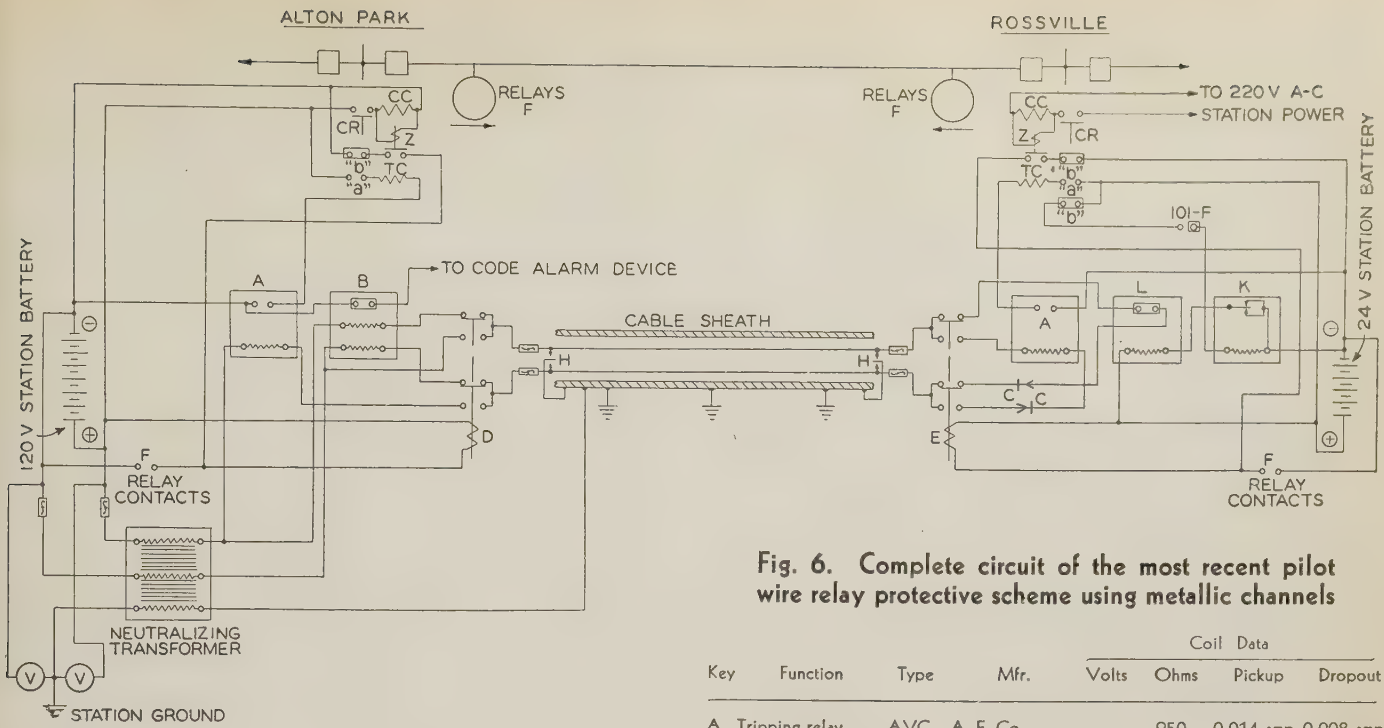


Fig. 6. Complete circuit of the most recent pilot wire relay protective scheme using metallic channels

Key	Function	Type	Mfr.	Coil Data Volts	Ohms	Pickup	Dropout
A	Tripping relay	AVC	A. E. Co.		950	0.014 amp	0.008 amp
B	Undercurrent alarm	AVC	A. E. Co.		1,000	0.014 amp	0.008 amp
C	Rectifier	Rectox	W. E. & M. Co.		200*	Forward	Direction
C	Rectifier	Rectox	W. E. & M. Co.		40,000**	Back	Direction
D	P. W. auxiliary relay		W-Leonard	120	4,000	80 volts	20 volts
E	P. W. auxiliary relay		W-Leonard	24	800	18 volts	4 volts
F	Directional fault detectors	Special	G. E.				
CR	Closing relay						
CC	Closing solenoid or motor of OCB						
TC	Trip coil of OCB						
H	Protective carbons	26–30	W. E. Co.				
K	Vibrating reed relay	AVR	A. E. Co.	24			
L	Slow release alarm relay	ASR	A. E. Co.	24			
V	Ground voltmeters	Min.	W. E. & M. Co.	150	150,000		
Z	Auxiliary lockout relay		G. E.				

* at 0.050 amp ** at 130 volts impressed

than its weakest link, which general experience in the past has shown to be the continuity of the pilot wire circuit, referring particularly to the continuity of the pair of wires connecting the stations. The South Chattanooga scheme involves a third function of great importance—giving the load dispatcher an immediate report of relay operations.

It is interesting to note that there have been no burn-downs of the tie lines since the pilot wire equipment was installed. On 88 per cent of the cases of trouble, service was restored on the instantaneous test. This record is not considered unusually high but it is considerably higher than the average "instantaneous performance" of this company, which is about 75 per cent.

The pilot wire equipment has been in almost continuous service except for routine maintenance since its installation. During the 16 months period there were 7 interruptions in the 3 leased circuits. All of these cases of trouble were detected at once by the circuit supervisory relays, reported by the code sender, and corrected, and most of them were cleared up in a few minutes with none extending for more than an hour. In every case the difficulty was corrected before the pilot wire equipment was called upon to operate.

As first designed the code sender reporting alarms from the pilot wire area did not distinguish between heavy voltage surges and successful instantaneous reclosures, because any heavy voltage surge would operate the initiating relays. Subsequently a time delay was added to the alarm sending relay on instantaneous reclosing feeders and tie lines at the pilot wire stations. This eliminated 90 per cent of the instantaneous alarm indications due to surges. Since the nonattended stations are not inspected immediately following successful instantaneous, 15 second, or 2 minute reclosures, the primary function of the alarm scheme is to report correctly all *permanent* failures of the power circuits, so that distribution maintenance men can be sent out promptly to the proper station. Between May 1, 1934 and Sept. 1, 1935, the code sender reported 122 permanent alarms, of which 120 were correct, and 2 were questionable.

The performance of the South Chattanooga pilot wire scheme has been close to 100 per cent correct. The authors consider this performance exceptionally satisfactory for a relay scheme involving newly developed equipment and methods.

The foregoing record of the South Chattanooga pilot wire system can be contrasted with the record of the South Nashville–West Nashville pilot wire installation. There have been several cases of trouble in the Nashville pilot wire which required it to be left out of service for hours at a time. The chief difference between this installation and the Chattanooga pilot wire system is that in Nashville battery supply is obtained at the middle of the loop at the telephone company exchange and accordingly has its midpoint solidly grounded and uses very light fuses and heat coils for protection. This precludes the possibility of

rectifier reverse current supervision and instead neon lamps have been used at each substation. This does not provide any audible indication of trouble, so that difficulties may not be noticed at the exact time they occur. Furthermore, the supervisory current is only about 5 per cent of the tripping current so that high circuit resistance due to loose connections, etc., will not show up.

Reference has already been made to the difficulties inherent in the use of pilot wire for transferred tripping—because the tripping relay is connected continuously to the pilot wire circuit, ready to operate, instead of being switched into the circuit by fault detector relays. The Carter Street–Long Street circuit in Chattanooga is a typical example. As originally installed, it used grounded battery voltage supplied at a telephone exchange between the 2 substations. This type of battery supply could not be eliminated in favor of substation storage battery supply until the neutralizing transformer was developed, as the rise of ground potential during power trouble was too heavy to permit direct connection of the substation storage battery to the telephone cable. This circuit had frequent incorrect operations prior to February 1935, but every operation has been correct since the installation of the neutralizing transformer and ungrounded battery in February 1935.

Until August 1933, all of the development work was concerned with pilot wire channels using leased lead covered cable in city areas. At that time there was finished an experimental installation of pilot wire protection using a power company owned exposed or "hot" telephone line of open wire construction adjacent to transmission right-of-way. It was quickly found that a carefully designed filter was necessary to prevent operation of the d-c relays on induced 60 cycle current in case of faults beyond the protected section. High voltage condensers and reactors were secured at small cost and the filter was tested successfully in the laboratory under transient and steady station conditions. Installation of the filter was not completed until after the close of the 1934 lightning season, but in 1935, false operations continued to occur, due to insufficient contact spacing on the battery supply relays, and inadequate grounding of the protective tubes on the line side of the telephone insulating transformers.

Another installation using an exposed telephone line was completed early in 1935. This is between Nashville and Murfreesboro and uses transferred tripping to open the 110 kv bus breaker at Murfreesboro in case the 110 kv breaker opens at South Nashville. This installation shows promise of very satisfactory operation.

Both exposed telephone lines have insulating transformers at all stations, and the pilot wire equipment is connected on the line side of the insulating transformers. No supervision is provided on these telephone lines except their almost continuous use for dispatching and commercial traffic.

General Utility

The availability of adequate channels for pilot wire protection opens the way toward greatly improved protection at much less cost than anything heretofore available. The saving in investment is due not only to the fact that the relay and protective equipment for pilot wire installations is relatively simple and inexpensive, but also to the fact that pilot wire protection permits savings in distribution, transmission, and substation expense, because it allows a simpler fundamental system plan, that is, it makes feasible for the first time the satisfactory use of short single circuit tie lines of any voltage.

The advantages of metallic channel pilot protection may be summed up as follows:

Low Capital Investment. If distance relays or other adequate directional relays are available to serve as instantaneous directional fault detectors, the additional equipment required for pilot wire operation may be less than $75 per terminal. On new jobs the total cost per terminal including instantaneous directional fault detecting relays and back-up protection might run around $600 per terminal, which is materially less than the cost of a standard distance relay installation that would still not give the remarkable advantages of pilot protection. This comparison is made on the basis of distance relays because it is the only type the authors would consider for tie line protection if pilot protection was not available, and because the starting units of distance relays are considered by them to be the best type of fault detector unit for pilot protection. Furthermore, in many cases there is a substantial saving in transmission or distribution cost, because short single circuit tie lines can be used—and relayed selectively.

Reasonably Low Pilot Channel Cost. Thirty dollars per route mile per year is a typical rental figure for metallic circuits in the base rate areas of cities. This covers everything including maintenance, an item sometimes underestimated in studying cost of carrier current or privately owned telephone circuits. In fact, the maintenance alone of privately owned single circuit open wire telephone lines may easily exceed $20 per mile per year. Leased routes between cities have been quoted at $48 per mile per year for a full metallic circuit. This is less than the carrying charges of a single circuit telephone line. The rates for the distances involved in protecting tie lines in or around city areas seem to be low enough to attract some business of this kind, especially in connection with new tie lines. Lower leased rates would undoubtedly extend the field of pilot wire systems to the point where it would be standard protection for city tie lines, especially with those companies which have used protection of this kind sufficiently to have confidence in it. The leased rates between cities seem to be an obstacle to the general use of pilot wire protection for such service, judging from the very limited amount of such protection in use today. Some progress toward more favorable rates has been made recently by the communication companies, such as the use of air line mileages, lower rates for single wire circuits with ground return, and a differential in favor of ground return channel circuits using low frequency as compared with rates for circuits suitable for voice frequency. In some localities telephone companies are already quoting more attractive rates for pilot wire service, low frequency

round return channels in some cases being as low s $18 per air line mile per year, and there is a general endency to encourage development of pilot wire chemes that will not require high cost facilities.

Low Maintenance Cost. Maintenance of the pilot hannel has been discussed in the preceding paraaph. Maintenance of the terminal equipment is lso cheap as the relays are simple and more or less tandard. No elaborate calculations are necessary n planning the scheme or in making the settings, and nstallation and testing involve no unusual diffiulties.

High Speed. Tripping speeds of 1 to 3 cycles can e obtained with metallic pilot wire protection. This ompares with 10 to 40 cycles with other methods on hose portions of a tie line where sequential clearing s necessary. To any relay time must be added the ircuit breaker time of 15 to 25 cycles with the older reakers, 8 to 12 cycles with more recent breakers, and cycles or less with breakers now under development. This remarkable reduction in breaker time makes eduction in relay time all the more desirable and ermits real advantages to be taken of the high speed nherent in pilot protection.

Absolute Selectivity. Being a differential scheme here is no question of selectivity if proper equipment as been chosen and structural defects do not deelop. Furthermore, pilot protection does not depend upon constant characteristics of instrument transformers, properly calculated settings or constant generating conditions, but is inherently selective regardless of these conditions.

Independence of Other Zones of Protection. The zone protected by metallic wire is completely independent of all other protected areas.

Metallic Pilot Wire Protection Can Be Used on Tie Lines With Tapped Loads. The only precaution is that the load should not be large enough to operate the initiating relays. No special equipment at the tap connection is necessary.

Excellent Reliability. The first metallic pilot wire schemes were deficient in this respect but development work to date has resulted in great improvement in reliability. This is due to several factors, such as more suitable relay equipment, co-operation on the part of the communication companies in improving circuit reliability, circuit design for immunity from momentary circuit failures (unless coincident with power trouble), and circuit design for continuous supervision.

References

1. A Carrier Current Pilot System of Transmission Line Protection, A. S. Fitzgerald. A.I.E.E. Trans., v. 47, Jan. 1928, p. 22–8.

2. The Use of Communication Facilities in Transmission Line Relaying, J. H. Neher. A.I.E.E. Trans., v. 52, June 1933, p. 595–602.

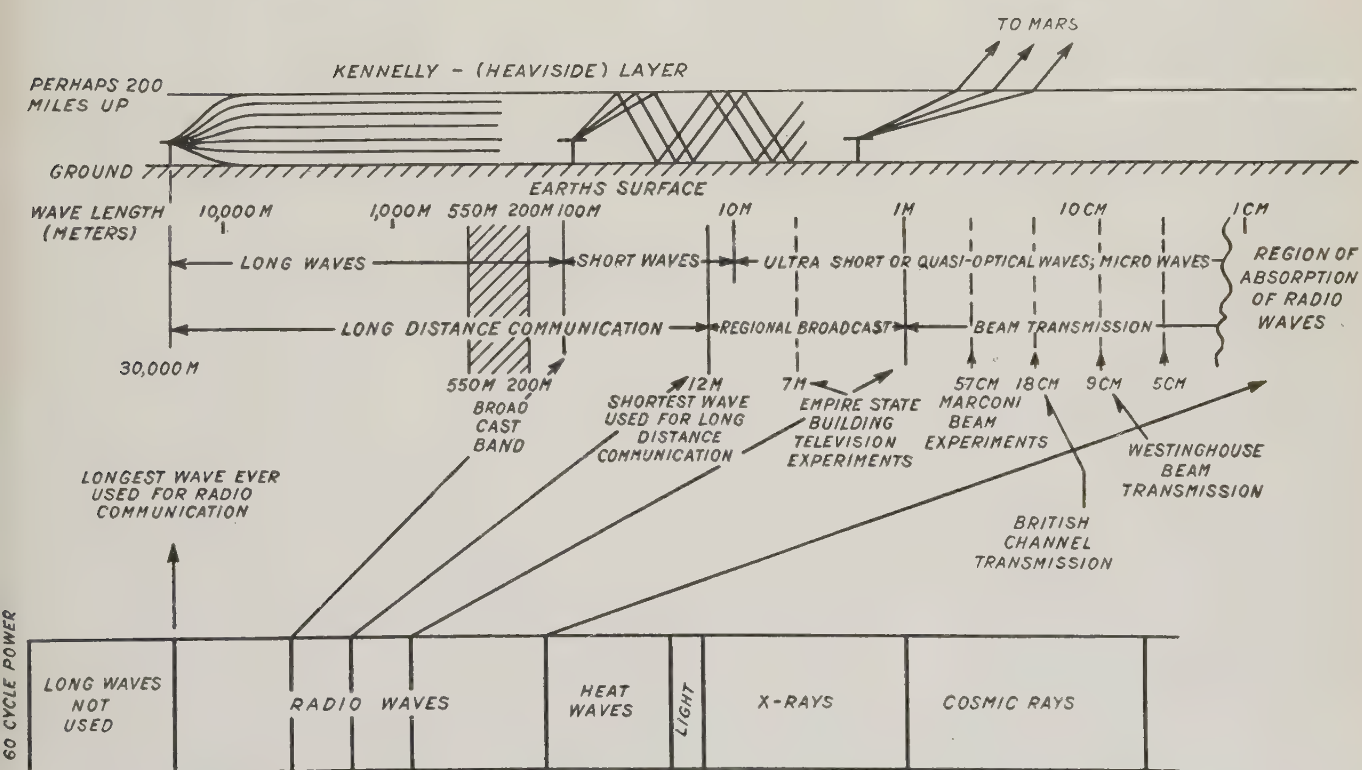


Diagram showing how the various wave lengths are used in radio communication, and their relation to radiation of other wave lengths. The "Westinghouse beam transmission" on a wave length of 9 centimeters, indicated at the right of the diagram, refers to a demonstration that formed part of that company's exhibit at the "Century of Progress" exposition in Chicago, Ill., during 1933–34. The "British channel transmission" on 18 centimeters refers to the commercial radiotelephone circuit between Lympne, England, and St. Inglevert, France (see "Production and Utilization of Micro-Rays," November 1933 issue of ELECTRICAL ENGINEERING, pages 739–40)

News

Of Institute and Related Activities

Nomination of A.I.E.E. Officers for 1936 Election; Members Invited to Submit Suggestions by Dec. 15

FOR THE nomination of national officers to be voted upon in the spring of 1936, the A.I.E.E. national nominating committee will meet during the winter convention, January 28–31, 1936. To guide this committee in performing its constituted task, suggestions from the membership are, of course, highly desirable. To be available for the consideration of the committee, all such suggestions must be received by the secretary of the committee at Institute headquarters, New York, N. Y., not later than December 15, 1935.

As reported on page 892 of the August 1935 issue of ELECTRICAL ENGINEERING, the sections of the constitution prescribing the election procedure were amended by vote of the membership. Corresponding amendments to the by-laws were adopted by the board of directors on October 22, 1935. In accordance with the provisions in the constitution and by-laws, as amended, quoted herewith, actions relative to the organization of the national nominating committee are now under way.

Constitution

28. There shall be constituted each year a national nominating committee consisting of one representative of each geographical district, elected by its executive committee, and other members chosen by and from the board of directors not exceeding in number the number of geographical districts; all to be selected when and as provided in the by-laws. The national secretary of the Institute shall be the secretary of the national nominating committee, without voting power.

29. The executive committee of each geographical district shall act as a nominating committee of the candidate for election as vice president of that district, or for filling a vacancy in such office for an unexpired term, whenever a vacancy occurs.

30. The national nominating committee shall receive such suggestions and proposals as any member or group of members may desire to offer, such suggestions being sent to the secretary of the committee.

The national nominating committee shall name on or before January 31 of each year, one or more candidates for president, national treasurer, and the proper number of directors, and shall include in its ticket such candidates for vice presidents as have been named by the nominating committees of the respective geographical districts, if received by the national nominating committee when and as provided in the by-laws; otherwise the national nominating committee shall nominate one or more candidates for vice president(s) from the district(s) concerned.

By-Laws

SEC. 22. During September of each year, the secretary of the national nominating committee shall notify the chairman of the executive committee of each geographical district that by December 15 of that year the executive committee of each district must select a member of that district to serve as a member of the national nominating committee and shall, by December 15, notify the secretary of the national nominating committee of the name of the member selected.

During September of each year, the secretary of the national nominating committee shall notify the chairman of the executive committee of each geographical district in which there is or will be during the year a vacancy in the office of vice president, that by December 15 of that year a nomination for a vice president from that district, made by the district executive committee, must be in the hands of the secretary of the national nominating committee.

Between October 1 and December 15 of each year, the board of directors shall choose 5 of its members to serve on the national nominating committee and shall notify the secretary of that committee of the names so selected, and shall also notify the 5 members selected.

The secretary of the national nominating committee shall give the 15 members so selected not less than 10 days' notice of the first meeting of the committee, which shall be held not later than January 31. At this meeting, the committee shall elect a chairman and shall proceed to make up a ticket of nominees for the offices to be filled at the next election. All suggestions to be considered by the national nominating committee must be received by the secretary of the committee by December 15. The nominations as made by the national nominating committee shall be published in the March issue of ELECTRICAL ENGINEERING (Journal of A.I.E.E.), or otherwise mailed to the Institute membership not later than the first week in March.

INDEPENDENT NOMINATIONS

Independent nominations may be made in accordance with provisions in Section 31 of the constitution and Section 23 of the by-laws, which are quoted below:

Constitution

31. Independent nominations may be made by a petition of twenty-five (25) or more members sent to the national secretary when and as provided in the by-laws; such petitions for the nomination of vice presidents shall be signed only by members within the district concerned.

By-Laws

SEC. 23. Petitions proposing the names of candidates as independent nominations for the various offices to be filled at the ensuing election, in accordance with Article VI, Section 31 (constitution), must be received by the secretary of the national nominating committee not later than March 25th of each year, to be placed before that committee for the inclusion in the ballot of such candidates as are eligible.

On the ballot prepared by the national nominating committee in accordance with Article VI of the constitution and sent by the national secretary to all qualified voters during the first week in April of each year, the names of the candidates shall be grouped alphabetically under the name of the office for which each is a candidate.

(*Signed*) H. H. HENLINE,
National Secretary

November 1, 1935.

Annual Meeting of A.S.M.E. in December. The technical program of the annual meeting of The American Society of Mechanical Engineers to be held in the Engineering Societies Building, New York, N. Y., December 2–6, 1935, has been announced. A total of 35 different sessions has been scheduled. Among these sessions, those which may be of interest to electrical engineers include 3 power sessions, 2 hydraulic sessions, 2 railroad sessions, an oil and gas power session, 2 radiant heat sessions, 3 psychology sessions, and 2 cost sessions, at one of which a paper on power distribution costs will be presented.

Membership—

Mr. Institute Member:

Though applications for membership are being received substantially, we are not quite equaling our work of last year. The figures are as follows:

	September		May to October	
	1934	1935	1934	1935
Applications received	55	50	232	230

Will you therefore do all you can to submit desirable names to the chairman of your Section membership committee as requested in our letter of October 1, 1935. It is your participation in this work that maintains the membership.

Chairman National Membership Committee

Lehigh Valley Section Sponsors Joint Meeting

Approximately 550 persons attended a highly successful engineers' dinner meeting held September 28, 1935, at the Hotel Mallow-Sterling, Wilkes-Barre, Pa., under joint sponsorship of the Lehigh Valley Section of the A.I.E.E., the local sections of The American Society of Mechanical Engineers and American Institute of Mining and Metallurgical Engineers, the Wilkes-Barre Chamber of Commerce, and the Engineers' Club of the Lehigh Valley. The meeting marked the culmination of an all-day program which earlier in the day included inspection trips to anthracite coal mines and other points of interest in and near Wilkes-Barre.

After dinner, the attendants listened to 3 interesting addresses. In the first, F. H. Wagner, vice president and general manager, Lehigh Valley Coal Company, Wilkes-Barre, Pa., outlined briefly some of the high points in the history of mining engineering in the anthracite region. He said that although the mining engineer might have been regarded as a necessary evil in the early days, he was considered indispensable today. In the second address, James H. Pierce, president, James H. Pierce and Company, Scranton, Pa., pointed out the various factors responsible for the steady decline in anthracite tonnage since 1926, and outlined measures being taken to increase the consumption of anthracite. In the third address, N. G. Reinicker (M'18) vice president and general manager, Pennsylvania Power and Light Company, Allentown, Pa., discussed some of the problems with which the electric utility is confronted in serving the anthracite industry and some of the measures taken to preserve continuity of service. W. H. Lesser (M'24) was chairman of the committee on arrangements.

Wilson Dam Substation, Being Erected in Alabama

Photo courtesy Delta-Star Electric Company

A VIEW of the 44,000-volt steel-tower substation which is now nearing completion in Alabama at Wilson Dam. This substation is a T.V.A. project. The 3-pole and 6-pole disconnecting switches are shown on the right-hand tower and on the center bay structure. Note also the ground wires for lightning protection strung on the tops of the 4 towers outside the substation fence.

A.I.E.E. 1936 Winter Convention

Committees are making arrangements for the program of the A.I.E.E. 1936 winter convention, which will be held in the Engineering Societies' Building, 33 West 39th St., New York, N. Y., January 28–31, 1936. Similarly to the plan of the past 2 years the convention will convene on a Tuesday with technical sessions both mornings and afternoons of the first 3 days while Friday, the fourth day, will be devoted entirely to inspection trips. The evenings will be given over to social functions—the smoker, the Edison medal presentation, and the dinner-dance. Details as soon as they become available will be announced in ELECTRICAL ENGINEERING.

A technical program, which will advance the theory and practice of electrical engineering in many specialized fields, is being developed. Two well-rounded symposiums, one on modernization of distribution systems, the other on magnetic materials and structures, should provide unusual interest for many engineers. The former symposium is a normal outgrowth of the trend of the times. The latter symposium, while mainly from the communication point of view, nevertheless should be of value to many others as the magnetic properties of the materials are important in electrical designs. In addition to the symposiums a number of other papers will be presented at sessions on subjects as follows: instruments and measurements, electrical machinery (2 sessions), automatic stations, power transmission and distribution, communication, electrochemistry and electrometallurgy, transportation, electrophysics, and protective devices.

Some of the papers on the program already have been published while others appear in this issue and still others will appear in subsequent issues up to the time of the convention. For scheduling, see the footnote on the first page of each published paper.

WINTER CONVENTION COMMITTEE

The personnel of the general committee making the arrangements is as follows: C. R. Beardsley, *chairman;* T. F. Barton, C. O. Bickelhaupt, A. F. Dixon, E. E. Dorting, C. R. Jones, W. R. Smith, George Sutherland, and R. H. Tapscott. The following have been appointed as chairmen of subcommittees: C. S. Purnell, *dinner-dance;* S. A. Smith, Jr., *inspection trips,* and George Sutherland, *smoker.*

Russia Joins Illumination Commission. The U.S.S.R. has been admitted to membership in the International Commission on Illumination according to an announcement by G. H. Stickney (A'04, F'24) president of the United States National Committee of the I.C.I. With the addition of Spain, which was admitted to membership during the convention in Germany last July, there are now 17 countries affiliated with the commission.

E.C.P.D. Holds Annual Meeting

Charles F. Scott (A'92, F'25, HM'29, past-president and member for life) professor of electrical engineering emeritus, Yale University, New Haven, Conn., and chairman of the board appointed by Governor Cross to administer the recently enacted engineers' registration law in the State of Connecticut, was elected chairman of the Engineers' Council for Professional Development at the third annual meeting of that body held in New York, N. Y., on October 8, 1935. Professor Scott succeeds C. F. Hirshfeld (A'05) chief of research of the Detroit (Mich.) Edison Company, who has served as chairman of the E.C.P.D. since its formation.

The Engineers' Council for Professional Development is a conference of engineering bodies organized to enhance the professional status of the engineer through the cooperative support of the national organizations directly representing the professional, technical, educational, and legislative phases of the engineer's life. The participating bodies are: American Society of Civil Engineers, American Institute of Mining and Metallurgical Engineers, The American Society of Mechanical Engineers, American Institute of Electrical Engineers, Society for the Promotion of Engineering Education, American Institute of Chemical Engineers, and National Council of State Boards of Engineering Examiners.

George T. Seabury, secretary of the American Society of Civil Engineers, was re-elected secretary of the E.C.P.D. New members of the executive committee also elected were C. F. Hirshfeld, representing The American Society of Mechanical Engineers, Harrison P. Eddy, of Boston, Mass., past-president of the American Society of Civil Engineers, representing that society, and L. W. W. Morrow (A'13, F'25, and

director) editor of *Electrical World*, New York, N. Y., representing the American Institute of Electrical Engineers. Other members of the executive committee who continue in office and the societies they represent are: F. M. Becket, of New York, past-president, American Institute of Mining and Metallurgical Engineers; H. C. Parmelee, of New York, American Institute of Chemical Engineers; R. I. Rees, of New York, Society for the Promotion of Engineering Education; and D. B. Steinman, of New York, National Council of State Boards of Engineering Examiners.

After the meeting adjourned the annual dinner, at which the chairman, C. F. Hirshfeld presided, was held at the Engineers' Club, and was attended by about 30 representatives of the participating bodies and guests of the E.C.P.D. A résumé of the work of the 4 principal committees and of the actions of the annual meeting was reported by Dr. Hirshfeld. Professor Scott spoke on the significance of the activities of E.C.P.D. in carrying forward its program in a spirit of understanding and co-operation.

At the annual meeting reports of the committees were presented and approved. These reports will be made public when printed. The 4 committees under whose guidance the principal activities are conducted have been set up to cover the 4 phases of the engineer's training. These are: committee on student selection and guidance, charged with the pre-collegiate phases; committee on engineering schools, whose interests at present are directed toward carrying out a program of accrediting engineering curricula; committee on professional training, which gives its attention to the problem of the young engineer and recent graduate; and committee on professional recognition, whose concern is with procedures by which the individual becomes a recognized member of the engineering profession.

Because of misunderstandings of the purposes and objectives of the committee on professional recognition which had arisen since the 1934 annual meeting, the council of E.C.P.D. adopted a resolution in which it regretted and disclaimed any statement that had been made in its behalf that might have been construed as unfriendly toward the engineers' registration movement, and any intention to set up a certifying agency in conflict with the legal registration of engineers by state boards or to parallel or supplant the National Bureau of Engineering Registration.

At the request of the committee on professional recognition its "program of certification," adopted by the E.C.P.D. in October, 1934, was referred back to the committee for further study and clarification.

Recent Views of Columbia River Projects

Something of the tremendous magnitude of the Government's Grand Coulee power and irrigation project and Bonneville power and navigation project may be gleaned by study of the several illustrations on the facing page. These snapshots were taken early in September 1935 upon the occasion of the visit of President E. B. Meyer and party to these projects, and indicate the status of progress as of that time.

The Grand Coulee project is on the upper Columbia River in the state of Washington, about 100 miles west of Spokane, and as at present scheduled, according to the latest information available from the U.S. Bureau of Reclamation, embraces in this initial phase the excavation for and the construction of the foundation for the proposed 500 foot dam and related structures. According to reports, an estimated $60,000,000 will be required to carry this initial phase of the project to completion during the next 2 years. The final project contemplates a 150 mile reservoir providing some 5,200,000 acre-feet of usable irrigation water, together with provisions for a power plant expected to be capable of generating some 1,890,000 kva.

The $32,500,000 Bonneville project on the Columbia River boundary line between the states of Washington and Oregon some 40 miles east of Portland and 145 miles from the sea, is expected to open the Columbia River to navigation by flooding the Cascade Rapids, and also to create power development possibilities amounting to some 600,000 horsepower. Work on the project was launched in November 1933, and is scheduled to be completed some time in 1937.

(1, across the top) Cofferdam and foundation excavation at Bonneville. Size of project and equipment used may be indicated by noting the men spotted in the white circles.

(2) Downstream view of the Bonneville power house structure as it appears from the construction road through the construction railroad trestle.

(3) Looking down into the west abutment excavation for the Grand Coulee Dam from a point some 1,200 feet above it at the top of the slide that has imposed construction difficulties. (At this point the river flows north.) Mason City, contractors' camp, may be seen across the river in the upper bac ground, the north end of the cofferdam e tends into the picture from the right and sligh above center, and the main excavation do approximately to bedrock may be se toward the right inside of the cofferda

(4–5) These 2 together, give something o panoramic view of the Grand Coulee proj from the upper west bank. The river flows to t north (left). Contractors' camp may be not again at the left. The cofferdam appe through the upper center, with the main e cavation in the foreground toward the le where magnitudes again may be compar by noting the 5 yard shovel in the lower e treme foreground toward the left of (5). Al the famous conveyor system may be trac from the main excavation leading toward t spoil dump more than a mile away toward t right, together with its feeder line extendi across the river to the excavation for the e bank abutment.

(6) Conveyor system extending from main e cavation more than a mile over the hill to Rattl snake Canyon spoil dump. This belt co veyor system comprised of successive sectio each dumping on to the following one, i volves a total of some 4,000 horsepower electric motive drives, together with associat controls and automatic safety features; operates at 620 feet per minute, and has a rat capacity of 2,500 cubic yards per hour (1 tons per second).

(7) Conveyor distributor head discharging e cavated material at the rate of more than o ton per second after it has been raised 350 fe or more from the excavation.

(8) Composite panoramic view indicating t general character of the terrain adjacent to t Grand Coulee project and indicating somethi of the extent to which speculative towns a town sites have sprung up. Mason City, t contractors' camp, may be noted again at t extreme right in the background; ceme storage plant may be noted slightly to the le also spoil dump in Rattlesnake Canyon may seen over the hill separating the lower fo ground at the right from the central bac ground. This is the view of the project th greets the eye as the visitor comes over t crest of the hill on the highway.

Another Light-Weight Stainless-Steel Train. The "Mark Twain" a 4-car light-weight stainless-steel train has been added to the fleet of streamlined Diesel electric trains of the Burlington Railroad. It will run between St. Louis, Mo., and Burlington, Iowa. The new train is similar in structure and appearance to the 3 "Zephyrs" now in operation on the Burlington. Like its predecessors, the "Mark Twain" is powered by a 600-horsepower Diesel electric engine which operates on common fuel oil. The train, supplied with fuel, water, and sand, weighs 287,245 pounds, and has seats for 92 passengers. It contains a kitchen, dinette, baggage car, and railway post office. The train was built by the Edward G. Budd Manufacturing Company, Philadelphia, Pa. The stainless steel body structure was fabricated by means of the "shot weld" process used in the building of railroad trains, elevated cars, airplanes, and truck bodies.

Draftsmen Organize. The American Society of Draftsmen, with national headquarters at Room 911, 424 South Broadway, Los Angeles, Calif., has recently been organized to take an active interest in the welfare of the large number of men who are in the drafting phase of engineering.

Future AIEE Meetings

Winter Convention,
New York, N. Y., Jan. 28–31, 1936

North Eastern District Meeting,
New Haven, Conn., May 1936

Summer Convention,
Huntington Hotel, Pasadena, Calif., June 22–26, 1936

Middle Eastern District Meeting,
Pittsburgh, Pa., part of week of Oct. 12, 1936. Transferred from Akron.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Thomas Alva Edison Foundation Organized

Owen D. Young will serve as national chairman of the country-wide movement to honor Thomas Alva Edison by the establishment of a program to keep alive the ideals of the "wizard of Menlo Park," it was recently announced by William S. Barstow (A'94, M'99, F'12, and Life Member), president of The Thomas Alva Edison Foundation. National vice-chairman will be George B. Cortelyou.

The Edison Center for the Advancement of Youth in Science will comprise the physical setting from which a wide educational program will be developed with emphasis on the encouragement of youth in the pursuit of scientific knowledge, to be an ever-living memorial in keeping with Edison's life-long services to mankind, and particularly to youth.

The basic memorial will include facilities for the permanent care of Edison's personal library, which is probably one of the most valuable scientific libraries in existence, his records and papers, and originals or replicas of each of the more than 1,000 inventions which he developed. The plans also include provision for a final resting place for the great inventor. Menlo Park, the scene of Edison's early activities, will be marked by a simple and impressive shaft, as a part of the general memorial program.

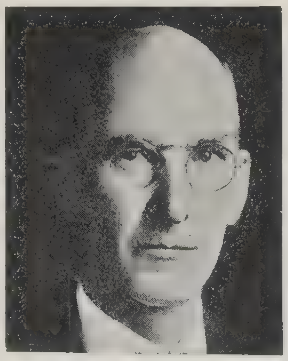
Officers of The Thomas Alva Edison Foundation are: William S. Barstow (A'94, M'99, F'12 and Life Member) president; W. H. Meadowcraft, honorary vice president; L. W. W. Morrow (A'13, F'25, and director) and F. A. Scheffler (A'93, F'12, and member for life) vice presidents; and W. S. Mallory, secretary and treasurer. These, with Dr. A. E. Kennelley (A'88, M'99, F'13, HM'33, Life Member and past-president), H. H. Barnes, Jr. (A'00, F'13, and past vice president), W. J. Orchard, Dr. C. F. Scott (A'92, M'93, F'25, HM'29, member for life, and past-president), Dr. C. H. Sharp (A'03, F'12, and member for life), W. H. Taylor, P. B. Millar (A'03, M'13) and Arthur Walsh, comprise the board of directors.

Chemical Industries Exposition. What is reported to be "the greatest industrial pageant of chemical achievement since the depression" is the reward in store for those planning to attend the 15th Exposition of Chemical Industries, which will be held in Grand Central Palace, New York, N. Y., December 2–7, 1935. The exposition this year is divided into the following classifications: chemicals and chemical products; plastics, molded products, and lacquers; laboratory equipment and supplies; general equipment; instruments of precision; containers; brewing equipment; materials handling equipment; raw materials; natural resources; and educational exhibits.

In Memoriam

THE death of J. Allen Johnson, on October 4, 1935, removed from the American Institute of Electrical Engineers its forty-seventh president and one of its most loyal and active members.

After his graduation from the Worcester Polytechnic Institute, in 1905, with the degree of bachelor of science in electrical engineering, Mr. Johnson embarked upon his life-work in the generation, transmission, and distribution of electric power. He quickly rose to positions of considerable responsibility, and in 1929 was appointed chief electrical engineer of the Buffalo, Niagara and Eastern Power Corporation. He contributed several important technical papers to the Institute publications.



J. ALLEN JOHNSON

Mr. Johnson joined the Institute in 1907, and was transferred to the grade of Fellow in 1927. He was the first chairman of the Worcester Polytechnic Institute Branch, in 1905, and was organizer and first chairman of the Niagara Frontier Section, 1925–26. He served as a member of many Institute committees and as chairman of several, as director 1928–32, vice president 1932–34, and as president 1934–35. Throughout these activities he was loyal at all times to the highest ideals of the Institute.

His ability as an engineer, his keen perception of the human values involved, his high ideals, and the breadth and thoroughness of his thought upon all matters affecting his field of work won him not only an outstanding reputation in his chosen profession, but also the respect and admiration of all who knew him.

RESOLVED: That the board of directors of the American Institute of Electrical Engineers hereby expresses, upon behalf of the membership, its deepest regret at the death of Mr. Johnson, and its sincere appreciation of his many important contributions to the development of Institute activities, and be it further

RESOLVED: That these resolutions be entered in the minutes and transmitted to members of his family.

—A.I.E.E. Board of Directors, October 22, 1935

Columbia University Scholarship Awarded

The special tuition scholarship at Columbia University, New York, N. Y., which is placed at the disposal of the A.I.E.E. each year has been awarded for 1935–36 to Joseph Leo Dalton (A'35) of Glenside, Pa. Mr. Dalton received the degree of bachelor of science in electrical engineering from Pennsylvania State College in 1934, and is now a candidate for the degree of master of science in electrical engineering.

This scholarship was originally founded by a group of Columbia alumni. Each year a committee of alumni, members of the A.I.E.E., considers the applications of graduate students for this scholarship. This year the committee, H. C. Carpenter, Francis Blossom, and Prof. W. I. Slichter, chose the recipient as the best from a group of 8 applicants. Availability of this scholarship was announced in ELECTRICAL ENGINEERING for February 1935, page 248.

Recent Progress in Welding Research

At the request of the committee on research of the Institute, the technical committee on welding has made available to the members of the Institute a report entitled "Recent Progress in Welding Research." It was prepared collectively by the fundamental research committee, a committee of the American Welding Society of which H. M. Hobart (A'94, F'12, past vice president, and member for life) is chairman. The availability of this report is, therefore, due to the informal collaboration between several committees and societies.

The report, notwithstanding its length (containing over 20,000 words plus a bibliography of 283 references) is considered to be preliminary to a final report which it is anticipated will be a more critical review of the recent literature on research in the welding fields. As much more work is necessary before this final revised report can be completed, a limited supply of mimeographed copies of the report in its present form has been prepared and copies may be had without charge by members of the A.I.E.E. and the A.W.S. (as long as the supply lasts) on application to either H. H. Henline, national secretary of the A.I.E.E. or William Spraragen, secretary of the A.W.S., both at 33 West 39th Street, New York, N. Y.

The report deals with *recent* researches on welding subjects and the references rarely relate to any publication of earlier date than 1933. The policy followed in preparing this compilation was to utilize the experience of specialists in each branch of welding and to include in the bibliography only those articles and papers which were considered by the specialists to be the most notable and valuable contributions to the literature of the subject.

Notwithstanding the recognized incompleteness of the report in its present form, it is believed that any approach to an equivalent is not yet available in any language and that, therefore, until the revision is completed, it will be of much value to welding specialists and others interested in the technical phases of welding and its applications.

Engineering Foundation

Election Held by Engineering Foundation

The 21st annual meeting of The Engineering Foundation was held in the Engineering Societies Building, New York, N. Y., on Thursday, October 10, 1935. The meeting opened as the final meeting of the 1935 board and was followed immediately by the first meeting of the 1935–36 board. Since, during the past year, the annual meeting of the board of The Engineering Foundation was advanced from January to October, the 1935 board held office only from January to September. The 1935–36 board holds office until the annual meeting in October 1936.

The Engineering Foundation is a department of United Engineering Trustees, Inc., organized in 1904, and now joint agency of the 4 national societies representing the civil, mining and metallurgical, mechanical, and electrical engineers. The Engineering Foundation, founded by Ambrose Swasey (HM'28) in 1918, is entrusted with the expenditure of the income from endowments and other funds, its present preferred activity being engineering research.

The feature of the annual meeting of October 10, 1935, was the election of officers. H. P. Charlesworth (M'22, F'28, and past-president) assistant chief engineer, American Telephone and Telegraph Company, New York, N. Y., was re-elected chairman. Mr. Charlesworth is a representative of the A.I.E.E. on the board. D. Robert Yarnall, a representative of The American Society of Mechanical Engineers, and a member of the firm of Yarnall-Waring Company, Philadelphia, Pa., was re-elected vice chairman. The executive committee, the members of which were elected or re-elected, is composed of these 2 members, together with the following 3 persons: A. L. Queneau, *trustee*, metallurgist, United States Steel Corporation, New York, N. Y., and a representative of the American Institute of Mining and Metallurgical Engineers; Edwards R. Fish, *member-at-large*, chief engineer, boiler division, Hartford Steam Boiler Inspection and Insurance Company, Hartford, Conn., and a representative of The American Society of Mechanical Engineers; and O. E. Hovey, *trustee*, consulting engineer, American Bridge Company, New York, N. Y. (Mr. Queneau succeeds J. V. N. Dorr, as a member of the executive committee.) Doctor A. D. Flinn was re-elected director and secretary.

These officers and the executive committee of The Engineering Foundation are elected by The Engineering Foundation board from among its own members. The Engineering Foundation board is itself elected by the board of trustees of United Engineering Trustees, Inc. Members of The Engineering Foundation board who were elected at the meeting of United Engineering Trustees, Inc., on June 27, 1935, and who serve for terms of 4 years expiring at the annual meeting in 1939, or upon expiration of terms as trustees, are as follows:

H. P. Charlesworth, trustee, to succeed himself
George E. Beggs, nominated by A.S.C.E. to succeed himself
Albert E. White, nominated by A.S.M.E. to succeed himself
F. Malcolm Farmer, nominated by A.I.E.E. to succeed C. E. Skinner
John V. N. Dorr, member-at-large, to succeed himself
Edwards R. Fish, member-at-large, to succeed himself.
G. L. Knight (A'11, F'17), *ex officio*, president, U.E.T., Inc.

The complete membership of The Engineering Foundation board, consisting of the above mentioned newly elected members, and the members holding over from previous elections, is as follows:

Name		Term Expires
Four Trustees of U.E.T., Inc.		
Otis E. Hovey	A.S.C.E.	1937
A. L. Queneau	A.I.M.E.	1938
D. Robert Yarnall	A.S.M.E.	1936
H. P. Charlesworth	A.I.E.E.	1939
Eight Members Nominated by Founder Societies		
George E. Beggs	A.S.C.E.	1939
Langdon Pearse	A.S.C.E.	1938
George D. Barron	A.I.M.E.	1936
Everette DeGolyer	A.I.M.E.	1938
Albert E. White	A.S.M.E.	1939
W. H. Fulweiler	A.S.M.E.	1936
F. M. Farmer	A.I.E.E.	1939
W. I. Slichter	A.I.E.E.	1936
Three Members-at-Large		
Frederick M. Becket		1938
John V. N. Dorr		1939
Edwards R. Fish		1939
Ex-Officio, President, U.E.T., Inc.		
G. L. Knight (A'11, F'17)		

At this annual meeting, members of the research procedure committee were appointed as follows:

D. Robert Yarnall, *chairman*, representing The Engineering Foundation

F. M. Farmer (A'02, F'13, and director) second member representing The Engineering Foundation, Thaddeus Merriman, representing the A.S.C.E.

F. F. Colcord, representing the A.I.M.E.

W. H. Fulweiler, representing the A.S.M.E.

L. W. Chubb (A'09, F'21) representing the A.I.E.E.

No changes were suggested in the membership of the other committees, except for the appointment of C. L. Eksergian, chief engineer, Budd Wheel Company, to the welding research committee. H. P. Charlesworth was re-elected representative on National Research Council.

Among other business transacted at the annual meeting was the adoption of the condensed annual report of The Engineering Foundation for the period January 1 to September 30, 1935, for transmittal to United Engineering Trustees, Inc., for the annual meeting of said corporation which was held October 24, 1935. Further details will be given in succeeding issues of ELECTRICAL ENGINEERING.

American Engineering Council

Federal Relief Projects

Excerpts from the current "news letter" of the American Engineering Council follow:

The approval of projects under the federal works relief program has been speeded up in recent weeks to make work available against the winter peak in operations. The jurisdiction of federal agencies over portions of the program has been clarified and funds have been shifted, especially to the Works Progress Administration, whose position is strengthened as a means of providing quick jobs.

Discontinuance of the division of applications and information, and clarification of functions of the advisory committee on allotments, seem to place broad discretionary power under the state administrators and directors as to the carrying out of approved projects and the advancement of projects not yet approved.

The Works Progress Administration, the largest spending and employing agency of the program, has been given a high degree of flexibility through the approval of approximately twice as many projects as there is money to finance. In other words, each State has a pool of projects from which to draw in placing the jobless in work suited to their individual skills, with latitude as to the selection of projects at the time and place most needed. The expectation is that W.P.A. will handle work aggregating more than $1,000,000,000.

Of primary importance is the entrace of the Corps of Engineers of the U.S. Army into the W.P.A. program. Lt. Col. F. C. Harrington has been appointed chief engineer of W.P.A. Officers of the corps have been appointed consulting field engineers in 11 areas of the United States. The consulting field engineers soon will visit their areas to secure full information as to the works program in each state, with "particular regard to those obstacles or difficulties which are hampering the speedy development of the program."

The Public Works Administration is going

forward with a new program involving between 3,000 and 4,000 projects in which federal grants from new money and loans from the P.W.A. revolving fund amount to about $500,000,000. With the addition of funds which the project sponsors have secured by private borrowings, the program amounts to about $750,000,000 of new construction. In addition, numerous projects are in various stages of construction under the old P.W.A. program.

Indefinite continuation of the Civilian Conservation Corps is contemplated in plans which call for a tapering off of enrollment from a peak of about 600,000 to 300,000 next July. Of approximately 750,000 young men who finish their formal education each year, some 600,000 are employable. On the basis discussed, the corps could offer half of these a year of out-of-door work at some time in their young manhood. The age limit for enrollment, originally 18 to 25, has been widened from 17 to 28. W.P.A. also plans to employ young men and women from 16 to 25, especially those from large relief families, to supplement the primary earnings of heads of families carried under the work program.

Other A.E.C. Items

"The Engineer's Duty as a Citizen," a pamphlet prepared as an outline for a discussion course at the Newark College of Engineering, is now in its third edition. The author, Roy V. Wright, editor of the *Railway Age* and other trade publications, has made revisions during the summer. New material added includes an account of the political experiences of Joshua D'Esposito, builder of Chicago's $100,000,000 Union Station, who ran for office on a reform ticket. The work of Col. William J. Wilgus, railroad engineer and executive, who came out of retirement to direct work relief in New York City, also is described. The pamphlet seeks to outline, with specific examples, some of the ways in which engineers can discharge their civic responsibilities, as individuals or by group action. Copies are obtainable at 25 cents each from the book store of the Newark College of Engineering, 367 High Street, Newark, N. J.

American Engineering Council at the present time, through its member organizations, represents 38 engineering bodies composed of 7 national societies and 31 state and local engineering organizations. New members of Council, since the new membership plan of nominal dues to State and local societies went into effect in January 1935, include the following:

State Societies: Arkansas Engineers' Club, Little Rock; Engineering Societies of New England, Boston; Florida Engineering Society, Gainesville; Louisiana Engineering Society, New Orleans; Maryland Association of Engineers; Michigan Engineering Society, Detroit; and South Carolina Society of Engineers, Columbia.
Local Societies: Associated Engineers of Spokane, Wash.; Cleveland Engineering Society, Ohio; Engineers' Club of Baltimore, Md.; Engineers' Club of Columbus, Ohio; Engineers' Club of Philadelphia, Pa.; Engineers' Club of St. Louis, Mo.; Engineers Society of Western Pennsylvania, Pittsburgh, Pa.; and Providence Engineering Society, R. I.

Letters to the Editor

CONTRIBUTIONS to these columns are invited from Institute members and subscribers. They should be concise and may deal with technical papers, articles published in previous issues, or other subjects of some general interest and professional importance. ELECTRICAL ENGINEERING will endeavor to publish as many letters as possible, but of necessity reserves the right to publish them in whole or in part, or to reject them entirely.

ALL letters submitted for consideration should be the original typewritten copy, double spaced. Any illustrations submitted should be in duplicate, one copy to be an inked drawing but without lettering, and other to be lettered. Captions should be furnished for all illustrations.

STATEMENTS in these letters are expressly understood to be made by the writers; publication here in no wise constitutes endorsement or recognition by the American Institute of Electrical Engineers.

The Status of Television in Europe

To the Editor:

Further developments have recently taken place regarding the operation of television in England and the following comments are given as a supplement to the recent article by A. W. Cruse under the above title (ELECTRICAL ENGINEERING, Sept. 1935, pages 966–9).

The article mentions the public service operating on a low definition, offered by the British Broadcasting Corporation. This service was stopped on September 11, 1935, after very short notice, so that no public service is now operating at all.

Regarding the proposed experimental television service from Alexandra Palace the B.B.C. was authorized on August 12 to complete the arrangements for the provision of transmitting apparatus on the lines given in the above-mentioned article. March 1936 has been given as the date of inception for this service, but actually work on the contract is at present held up.

The opinions of the membership of The Television Society are worth quoting on the technical aspect of the scheme. In answer to a recent questionnaire the majority of replies stated that "interlaced scanning gave no advantages that were not offset by greater disadvantages. "Further, it was agreed that 240 lines were quite sufficient for a public service.

Very truly yours,

G. V. HARRAP (A'34)
58 Wellington Road,
Dunston, Gateshead, England

Overcompounded D-C Generators in Parallel Without an Equalizer

To the Editor:

"Letters to the Editor" on the subject of "Overcompounded D-C Generators in Parallel Without an Equalizer" have appeared in ELECTRICAL ENGINEERING intermittently for the past 2 or 3 years. No unanimity of opinion is shown.

The question to be discussed may be fully stated as follows. If 2 overcompounded generators, driven by prime movers with drooping speed-load characteristics, are in momentarily stable parallel operation without an equalizer connection and the speed of one generator changes, which generator assumes a greater share of the load? I have investigated this question by carefully controlled experimental tests. Two points of the experimental procedure are very important. First, the temporary stable condition, without equalizer, may be produced in only one way: by putting the machines in parallel with an equalizer connection, adjusting the circuit constants until the current through the equalizer is zero, and then opening a switch in the equalizer. This switching was performed by 1 blade of a 2 pole switch, the second blade of which enforced a speed change on 1 of the prime movers (see *S*, figure 1). Secondly, it is essential that the speed of each generator be known accurately at every instant so that we will not confuse cause and effect; the first change in

Fig. 1. Connection diagram

G_1—Rated 5 kw, 125 volts, 40 amperes, 1450 rpm
G_2—Rated 4 kw, 125 volts, 32 amperes, 1350 rpm

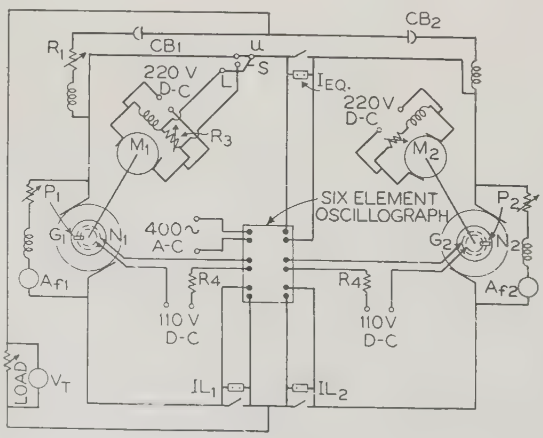
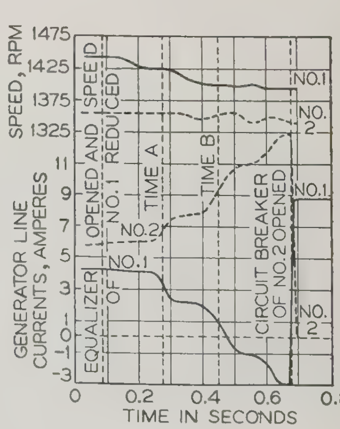


Fig. 2. Typical results of tests



speed is to be the *cause* and *not the result* of a change of load. The speed recording devices used (a 400 cycle wave and a shaft contact) gave an accurate value of speed for each single revolution of each generator.

The oscillograms taken by this method show the opening of the equalizer and the subsequent changes in speed and line current of each generator. Figure 2 shows typical results. Note that, as a *result* of lowered speed, generator 1 *drops* its load and finally shifts to motor operation. In over 100 tests, with various load divisions and with speed either increased or decreased, the results were entirely consistent; increased speed causes the generator to take more load and *vice versa*.

It is possible (*e. g.*, at time *A* or *B* of figure 2) to determine the rate of change of current in each generator circuit and to substitute these values with measured resistance and inductance in the Kirchhoff's loop equation involved. The checks are as close as could be expected and the experimental results thus check the theory.

The purpose of this study was to settle the controversy mentioned in the first paragraph. The following conclusions were reached.

1. The presentation of Prof. Harold Pender in his book "Direct Current Machinery" is contrary to both fact and theory. It should not be expected to be correct because it is the result of applying a steady state analysis to a transient condition.

2. The oscillograms given by Professor J. G. Brainerd (ELECTRICAL ENGINEERING, volume 51, Oct. 1932, page 745) in support of Professor Pender's view, are valueless because they fail to show when the transient condition began, and because the boundary condition of a change in relative speed of the generators was not enforced. Thus, his oscillograms show the incontrovertible fact that if a prime mover has a drooping speed characteristic, the generator has a drooping speed characteristic. The change of speed is the result and not the cause of the change of load.

3. Prof. D. W. Walter (ELECTRICAL ENGINEERING, volume 53, Nov. 1934, pages 1553–4) points out the correct theory.

4. The work of E. H. Nelson and Sidney Rock (ELECTRICAL ENGINEERING, volume 54, March 1935, pages 347–8) points out certain obvious results obtained by starting the transient condition while current is flowing in the equalizer connection and checks the oscillograms of Professor Brainerd with the definite admission that the speed change is the result and not the cause of the change in load division.

5. The letter of J. O. Reid (ELECTRICAL ENGINEERING, volume 54, March 1935, pages 348–9) gives a nicely worded, correct analysis of the problem, particularly in the statement. "If the transfer (of load) is the result of a momentary speed variation, this variation will be in the opposite direction from that resulting from the load transfer." In other words, the only dissenting opinions on this problem are due to a failure to differentiate between cause and effect. This failure causes improper experimental control and produces erroneous experimental results.

Very truly yours,

RICHARD R WHIPPLE (A'30)
Asst. Prof. of Elec. Engg.,
State Univ. of Iowa, Iowa City

Personal Items

J. L. WILSON (A'25) assistant chief surveyor, American Bureau of Shipping, New York, N. Y., is chairman of the Institute's committee on applications to marine work, for the year 1935–36. Mr. Wilson was born May 29, 1894, at Elizabeth, N. J. After graduating from Webb Institute of Naval Architecture in 1914, he undertook postgraduate work in mechanical engineering at the Polytechnic Institute, New York. Previous to graduation he did work in connection with shipbuilding. After graduation, he became assistant editor of *Marine Engineering*. In 1915 he became a designer of submarines for the Lake Torpedo Boat Company, later undertaking aviation research in the U.S. Navy Department, Washington, D. C. He entered the scientific section of the New York Navy Yard in 1916, and during 1916–17 was chief engineer of the Talbot Boiler and Engine Corporation. He entered the employ of the American Bureau of Shipping in 1917, being at that time surveyor at Philadelphia, Pa. In 1920 he was transferred to the technical staff at New York, N. Y., and in 1926 was appointed to his present position as assistant chief surveyor. Mr. Wilson has for a number of years supervised all the electrical engineering work of the bureau, including the construction, installation, and maintenance of electrical apparatus on vessels of the Merchant Marine under the cognizance of the American Bureau of Shipping. The work also has involved modification of electrical standards for marine work. He has been active on a number of sectional committees relating to electrical installations on shipboard, and has been a member of the Institute's committee on applications to marine work since 1925.

C. F. SCOTT (A'92, F'25, HM'29, past-president and member for life) professor of electrical engineering emeritus, Yale University, New Haven, Conn., was elected chairman of the Engineers' Council for Professional Development at the third annual meeting of that body, held in New York, N. Y., October 8, 1935. Dr. Scott has been active on the E.C.P.D. since its inception, having been a member of its executive committee, as well as A.I.E.E. representative in the E.C.P.D. Dr. Scott, who rose to prominence in the Westinghouse Electric and Manufacturing Company during his term of service with that company from 1888 to 1911, and who since the latter year has been professor of electrical engineering at Yale University, has also had a long term of service with the Institute. He has served on many of its committees and other groups, contributed several technical papers to it, and served as its president 1902–03. He was instrumental in securing funds for the Engineering Societies Building in New York, N. Y., and was chairman of the building committee. A biographical sketch outlining Doctor Scott's career was given in the May 1934 (50th anniversary issue of ELECTRICAL ENGINEERING, page 795.

A. L. POWELL (A'13, F'26) supervising engineer, Atlantic division, Incandescent Lamp Department of the General Electric Company, New York, N. Y., is chairman for 1935–36 of the Institute's committee on production and application of light. Mr. Powell was born in Brooklyn, N. Y., April 6, 1889. Graduating from the school of electrical engineering of Columbia University in 1910, he entered the employ of the Edison Lamp Works, Harrison, N. J., that same year. After a year on the test course, he entered the commercial engineering department in 1911, later that year entering the lighting service department where he remained as first assistant to the illuminating engineer until 1924. During this period he was engaged in the design of lighting installations and equipment, tests, and experimental and research work. He also gave many lectures on lighting throughout the country, and was a frequent contributor of technical articles. During 1924–25 he was special representative in Europe, of the International General Electric Company, teaching and working with local lighting industries. He was a delegate of the U.S. Committee of the International Commission on Illumination at Geneva, 1924, and a member of the U.S. Commission to the Paris Exposition, 1925. In 1925, he became manager of the engineering department of the Edison Lamp Works at Harrison, N. J., and in 1932, was transferred to his present position. In the latter year, Mr. Powell made another trip to Europe to investigate lighting conditions, and in 1935 was a member of the United States delegation to the

J. L. WILSON

C. F. SCOTT

A. L. POWELL

meeting of the International Commission on Illumination held in Germany. Mr. Powell has served the Institute as a member of its board of examiners since 1929, and a member of the committee on production and application of light since 1933. He was president of the Illuminating Engineering Society, 1934–35, is now vice president of the Montclair (N. J.) Society of Engineers, is a member of the Architectural League, New York (N. Y.) Electrical Association, and Sigma Xi, and has served on many committees dealing with lighting.

F. H. HOLLISTER

C. T. SINCLAIR

T. F. BARTON

F. H. Hollister (A'08, M'30) chief electrical engineer, Sargent and Lundy, Inc., Chicago, Ill., is chairman of the Institute's committee on power generation for the year 1935–36. Mr. Hollister was born at Cambridge, N. Y., Oct. 5, 1882. After attending Northwestern University, he transferred to the University of Michigan, where he graduated in 1906 with the degree of bachelor of science in electrical engineering. Following graduation he entered the test department of the General Electric Company at Schenectady, N. Y., and spent 2 years each in the test, construction, and lighting engineering departments of the company. In 1912, he was transferred to the district engineer's office of the General Electric Company at Atlanta, Ga., where he remained for 4 years. In 1916, Mr. Hollister entered the employ of Sargent and Lundy, Inc., consulting engineers, Chicago, to specialize particularly in steam-electric power production design and associated engineering problems. In 1930, he was made chief electrical engineer and a director of Sargent and Lundy, Inc., having charge and supervision of the electrical department. Mr. Hollister has served the Institute as a member of the executive committee of the Chicago Section, and has been a member of the national committee on power generation since 1929.

T. F. Barton (A'12, M'18, F'30) district engineer, New York, (N. Y.) district, General Electric Company, is chairman of the Institute's committee on legislation affecting the engineering profession, for the year 1935–36. Mr. Barton was born at Orangeburg, S. C., December 25, 1885. He graduated from Clemson Agricultural College with the degree of bachelor of science in 1906, and a few years later received the degree of electrical engineer from that institution. Upon graduation in 1906, he entered the test department of the General Electric Company at Schenectady, N. Y., and has been employed by the General Electric Company continuously since that time. He was at first in the direct current engineering department at Schenectady, and in 1911 was transferred to the engineering department of the company's New York district. In 1917 he returned to Schenectady where he entered the central station engineering department, being section head in charge of application work in connection with power generating, transmission, and distribution systems. In 1927 he became engineer of the New York district, in charge of engineering for the state of New York, northern New Jersey, and western Connecticut. Mr. Barton has been particularly active in the Institute's New York Section, having been chairman of this Section 1932–33, and a member of its executive committee 1933–34. He also has served on 5 of the Institute's committees, the first being the membership committee 1917–18, at present is a member of 3 committees.

C. T. Sinclair (A'19, M'29) electrical engineer, Byllesby Engineering and Management Corporation, Pittsburgh, Pa., is chairman of the Institute's committee on power transmission and distribution for the year 1935–36. Mr. Sinclair was born in Towson, Maryland, December 16, 1894, and graduated from Lehigh University, 1917, with the degree of electrical engineer. Following brief employment with the Western Union Telegraph Company after graduation, he enlisted in the Field Artillery and was later commissioned in the corps of engineers of the U.S. Army. In 1919 he entered the employ of the Pennsylvania Water and Power Company at Holtwood, Pa., in their test department, and later in their operating department in Baltimore. In 1922 he went to New York, N. Y., with the United Electric Light and Power Company in charge of their planning division. He became assistant superintendent and later the superintendent of the transmission and distribution department of that company, in charge of the engineering, construction, and operation of the transmission and distribution system, including the pioneer low voltage a-c network system in New York City. In 1925 he went to Pittsburgh with the Philadelphia Company as assistant electrical engineer, responsible for the design of the transmission and distribution system. He later became electrical engineer for the Byllesby Engineering and Management Corporation in the same type of work. Mr. Sinclair has written numerous articles for the technical press and presented a number of papers before the Institute, largely on distribution subjects. He has been active in the Institute for a number of years, having been chairman of the subcommittee on distribution and vice-chairman of the committee on power transmission and distribution. He was a member of the executive committee of the Pittsburgh Section, chairman of several of its committees, and chairman of the Section in 1930–31. He has served as chairman of the underground systems committee of the National Electric Light Association and of the underground section of the transmission and distribution committee, Edison Electric Institute. He is a member of the sectional committee on wires and cables, American Standards Association, and is chairman or member of various committees of that association. He was editor-in-chief of the "Underground Systems Reference Book," issued in 1931 by the National Electric Light Association.

H. S. Warren (A'03, F'13) director of protection development, Bell Telephone Laboratories, Inc., New York, N. Y., is chairman of the Institute's committee on safety codes for 1935–36. He was born in Oldtown, Maine, and studied electrical engineering at Stanford University, graduating in 1898 with the degree of bachelor of arts. Immediately after, he entered the employ of the Standard Electric Company of California, making a study for a projected 50,000 volt transmission line from Amador County to San Francisco. His next work was with the California State Board of Harbor Commissioners, and in 1898, he took a position with the Nevada County Electric Power Company, Nevada City, Calif. In 1899, he began work with the American Bell Telephone Company (subsequently the American Telephone and Telegraph Company), in Boston, Mass., in connection with the commercial realization of the loaded-line principle, to which art he made many contributions. He also was in charge of divisions responsible for development work on equipment and on transmission and protection. More recently, Mr. Warren's work has been connected with problems of inductive co-ordination and protection. He has had an active part in all major inductive co-ordination problems of the Bell Telephone System involving electrified railways since his early work in connection with the single-phase electrification of the New Haven Railroad. The department of which he is now head is in charge of development work for the Bell System on this and other phases of inductive co-ordination. In 1912, he was instrumental in starting the pioneer investigations of the Joint Committee on Inductive Interference in California. He has been an active participant in the work of the Joint Subcommittee on Development and Research of the Edison Electric Institute and the Bell Telephone System, beginning with the formation of the subcommittee in 1923. He is at present Bell System chairman of the subcommittee. For many years he has had

charge in the Bell System of development work on electrolysis and on structural co-ordination. Mr. Warren acquired his present position in 1934, when the Bell Telephone Laboratories and the department of development and research of the American Telephone and Telegraph Company were consolidated. He has served the Institute as a member of its committee on safety codes since 1915, and on other committees.

T. D. Yensen (A'09, M'23) engineer in charge of the magnetic division of the research department of the Westinghouse Electric and Manufacturing Company, East Pittsburgh, Pa., has been awarded the Henry Marion Howe medal by the American Society for Metals for a paper of which he was a co-author, on properties of iron, adjudged the best paper published in the *Transactions* of the society during 1934. Doctor Yensen is a native of Norway, and is a graduate of the University of Illinois, where he received the degrees of bachelor and master of electrical engineering and electrical engineer. In 1927 he received the degree of doctor of philosophy in physics at California Institute of Technology. A discovery credited to Doctor Yensen is "hyperrik," a magnetic alloy of high permeability.

A. J. Althouse (A'11, M'29) assistant general manager, Metropolitan Edison Company, Reading, Pa., has been elected president of the Pennsylvania Electric Association. Mr. Althouse has been employed in various positions in Pennsylvania electric companies since 1906, having been employed at Bethel, Birdsboro, and Hamburg before becoming general superintendent of the Metropolitan Edison Company at Reading in 1925. In 1929 he was made assistant general manager. Mr. Althouse has been a member of the Pennsylvania Electric Association for 25 years, and recently served as a vice president. He is a past-chairman of the Lehigh Valley Section of the Institute.

F. B. Jewett (A'03, F'12, and past-president) vice president, American Telephone and Telegraph Company, New York, N. Y., has accepted the chairmanship of a committee on warning signals to lessen hazards of collisions at sea, formed by the Science Advisory Board. Doctor Jewett is at present serving the Institute as chairman of the Iwadare foundation committee, as a representative on the Hoover medal board of award, and as a member of the committees on the Lamme medal and on the code of principles of professional conduct, of which he is a former chairman.

H. S. WARREN

M. D. Engle (A'21, M'26) who has been assistant superintendent of the station engineering department of the Edison Electric Illuminating Company of Boston, Mass., has been appointed superintendent. Mr. Engle, a graduate of the University of Michigan, came to the company in 1925 as assistant superintendent, having previously been assistant to the electrical engineer of the Consolidated Gas, Electric Light and Power Company, Baltimore, Md., and assistant to the chief mechanical engineer with McClellan and Junkersfeld, consulting engineers at New York, N. Y.

James Burke (A'93, F'13, and member for life) whose retirement from the Burke Electric Company, Erie, Pa., was announced in Electrical Engineering for August 1935, page 913, has been elected president of the International Electrotechnical Commission. He has been a member of the United States National Committee of the commission for many years. He served on the committee as a representative of the National Electrical Manufacturers' Association from 1914 to 1930, and as a representative of the Institute from 1930 to 1932.

G. A. Riley (A'28, M'29) who since 1924 has been assistant superintendent of electric distribution of the Los Angeles Gas and Electric Corporation, Los Angeles, Calif., has been appointed superintendent. Mr. Riley graduated from the University of Illinois in 1904, and entered the employ of the company the following year. He became general foreman in the electric distribution department in 1919, holding this position until his appointment as assistant superintendent.

H. E. Dexter (A'14, M'17) general commercial manager, Central Hudson Gas and Electric Corporation, Poughkeepsie, N. Y., has been elected to the board of directors of the company. Mr. Dexter has been with the company since 1927, and has been responsible for the administration of the commercial and sales policies of the company, which have included a progressive plan of co-operation with appliance dealers.

H. C. Dean (A'12, F'30) vice president of the New York and Queens Electric Light and Power Company, Flushing, N. Y., has been elected a director of the company, and also a member of its executive committee. Mr. Dean has been a member of the Institute's board of examiners since 1933 and of the committee on legislation affecting the engineering profession since 1934, and was a member of the power transmission and distribution committee 1929–32.

C. W. Koiner (A'04, F'12) city manager of Pasadena, Calif., has been appointed a consulting engineer for the new power division of the Public Works Administration. He has had considerable experience in the management of electric plants, and is a former district manager of the Southern California Edison Company. Mr. Koiner, a past-president of the International City Managers Association, has been city manager of Pasadena since 1933, having previously served in this office from 1921 to 1925.

W. N. Clark (A'05) president, Southern Colorado Power Company, Pueblo, has been presented with a diamond studded medal chain by the Rocky Mountain Electrical Association. The award, established a year ago, is presented "for the most outstanding result or material contribution toward the advancement of the electrical industry by an individual or company in the Rocky Mountain region."

Harry Reid (A'22, F'31) president of Harry Reid and Company, Inc., New York, N. Y., is now with the Associated Gas and Electric Company, being engaged in operating management of that company, taking with him the staff of the company of which he has been head.

Roy Wilkins (A'16, F'29) former vice president in charge of manufacturing, Pacific Electric Manufacturing Corporation, San Francisco, Calif., is now with K. P. F. Electric Company, San Francisco. He served as a member of the Institute's committee on instruments and measurements 1923–24 and 1925–27.

F. T. Hicks (A'26) patent attorney for the Kelvinator Corporation who has been at Cleveland, Ohio, now has opened an office at Detroit, Mich., and is specializing in prosecutions before the United States Patent Office, of which he is a former member of the examining staff.

Gothard Sargl (A'17) former superintendent of construction for Stone and Webster Engineering Corporation, Boston, Mass., has engaged in a general contracting business at Beaumont, Tex. He was recently engaged on the construction of plants at Beaumont and Baton Rouge, La.

M. F. Beall (A'32) former assistant to chief engineer, Michigan Gas and Electric Company and Michigan Public Service Company, Holland, is now with the Timken Roller Bearing Company at Canton, Ohio.

R. D. Miller (M'32) former chief engineer in the Oregon area for The Pacific Telephone and Telegraph Company at Portland, Ore., has been made assistant vice president and is now at San Francisco, Calif.

E. L. Gaines (A'22, M'30) former traffic superintendent of the Home Telephone and Telegraph Company, Fort Wayne, Ind., has become sales engineer for the American Automatic Electric Sales Company, Chicago, Ill.

Whitworth Ferguson (A'25, M'34) former vice president and electrical engineer of the Robertson Electric Construction Company, Buffalo, N. Y., is now associated with the Ferguson Electric Construction Company, Buffalo.

E. E. Althouse (A'28, M'34) who has been division distribution superintendent of the Central Hudson Gas and Electric Corporation at Poughkeepsie, N. Y., is now division operating superintendent at Kingston.

E. L. Holmgren (A'19) former transformer engineer with Westinghouse Electric and Manufacturing Company, Sharon, Pa., is now with the Kuhlman Electric Company, Bay City, Mich., as transformer design engineer.

E. J. Verrier (A'28) formerly steam plant superintendent, Anglo-Newfoundland Development Company, Grand Falls, N. F., is now building superintendent and chief engineer at the Montreal General Hospital, Montreal, Can.

E. B. Whitman (M'34) formerly with the West Penn Electric Company, Pittsburgh, Pa., recently engaged in consulting engineering with the firm of Whitman, Requardt and Smith, Baltimore, Md.

M. C. Winetsky (A'25) recently with Empreza Forca e Luz de Ribeirao Preto, Campinas, Brazil, S. A., is now an engineering assistant with the Public Service Electric and Gas Company, Elizabeth, N. J.

Carl Whitmore (A'18) former general manager of installation with the Western Electric Company, Inc., New York, N. Y., is now at Albany, N. Y., with the New York Telephone Company.

W. K. Graves (A'33) former relay inspector with the Potomac Electric Power Company, Washington, D. C., is now with the Cerro de Pasco Copper Corporation at La Oroya, Peru, S. A.

W. J. Kenline (A'35) formerly employed by The Southern Sierras Power Company, El Centro, Calif., is now with the San Diego (Calif.) Consolidated Gas and Electric Company.

F. E. Swift (A'16, M'30) who has been with The McKinnon Industries, Ltd., St. Catherines, Ont., Can., is now with the Howell Electric Motors Company, Howell, Mich.

P. L. Morton (A'33) recently employed by Ford, Bacon and Davis, Inc., at Seattle, Wash., is now employed in the meter department of the Puget Sound Power and Light Company, Seattle.

C. T. Lindstrom (A'26) formerly with the Blaw-Knox Company, Pittsburgh, Pa., is now development engineer with American Cyanamid and Chemical Company, Bridgeville, Pa.

L. A. Whitsit (M'19) former hydraulic engineer with United Engineers and Constructors, Inc., Philadelphia, Pa., is now principal engineer in the U.S. Engineers Office at Eastport, Me.

R. C. Kemp (A'31) former engineer in charge of electrical sales with Reginald Aitken, Kingston, Jamaica, W. I., has engaged in private practice as an electrical engineer.

H. R. Nelson (A'32) who has been employed by the Wisconsin Gas and Electric Company at Kenosha is now employed by the Waukesha Motor Company, Waukesha, Wis.

C. S. Lumley (A'23, M'29) former chief executive engineer of Longwood Towers Company, Brookline, Mass., is now with Smith, Hinchman and Grylls, Inc., Detroit, Mich.

S. O. Cowan (A'31) former manager of the Columbia Blue Print Company, Columbia, S. C., is now an assistant electrical engineer with Underwriters Laboratories, Inc., Chicago, Ill.

V. R. Bacon (A'19) engineer formerly with United Engineers and Constructors, Inc., Philadelphia, Pa., is now with the American Gas and Electric Company, New York, N. Y.

C. N. Rice, Jr. (A'28) former valuation engineer with Byllesby Engineering and Management Corporation, Chicago, Ill., is now with the Northern States Power Company at Eau Claire, Wis.

G. A. Mills (M'18) former field executive of the Middle West Utilities Company, Chicago, Ill., is now at Lawrence, Kan., as executive vice president of The Kansas Electric Power Company.

G. W. Baker (A'33) who has been an assistant in the department of electrical engineering at Columbia University, New York, N. Y., is now an engineer with Tung-Sol Radio Tubes, Inc., Newark, N. J.

H. A. Campbell (A'35) who has been a technician with the Consolidated Aircraft Corporation at Buffalo, N. Y., is now in the engineering department of the corporation at San Diego, Calif.

P. A. Robert (A'35) former serviceman at New York, N. Y., for International Business Machines Corporation, is now in the Company's educational department at Endicott, N. Y.

E. E. Steinert (A'30) formerly with the General Electric Company at Schenectady, N. Y., is now an electrical engineer in the bureau of engineering of the U.S. Navy Department at Washington, D. C.

A. E. Torrance (A'34) formerly in the electricity department of the city of Port Elizabeth, South Africa, is now in the engineering department of Johnson and Phillips at Johannesburg.

J. C. McClung (A'28) former foreman in the underground distribution department of the Los Angeles Gas and Electric Corporation, Los Angeles, Calif., has been made overhead electrical engineer.

R. U. Berry (A'29) air conditioning department, General Electric Company, Bloomfield, N. J., has been placed in charge of special applications in the recent consolidation of all divisions of the department.

A. W. Robertson (A'27) chairman of the board, Westinghouse Electric and Manufacturing Company, Pittsburgh, Pa., has been elected president of the Electrical Manufacturers Club.

D. G. Evans (A'20) electrical engineer, Wisconsin Gas and Electric Company, Racine, has been appointed assistant manager. He has been with the company since 1919.

K. J. Granbois (A'31) formerly with the Pennsylvania Water and Power Company at Holtwood, is now test engineer for the Safe Harbor Water Power Corporation at Safe Harbor, Pa.

F. M. Ryan (A'19, M'26) Bell Telephone Laboratories, Inc., New York, N. Y., has been asked to serve on a radio technical committee for aeronautics recently organized by the U.S. Department of Commerce.

J. A. Roseborough (A'34) recently with Continental Can Company of California, Oakland, has become chief engineer of radio station KRE, Berkeley, Calif.

W. D. Weidlein (A'31, M'32) consulting engineer formerly with Black and Veatch, Kansas City, Mo., is now with Wm. D. Weidlein and Company at Kansas City.

L. F. Pries (A'33) formerly with the Colonial Radio Corporation, Buffalo, N. Y., is now with the Wurlitzer Grand Piano Company, De Kalb, Ill.

F. S. Himebrook (A'32) recently with the Master Electric Company, Dayton, Ohio, has accepted a position with The Hoover Company, North Canton, Ohio.

R. J. Weesner (A'35) formerly with the Taylor-Winfield Corporation, Warren, Ohio, is now employed by the American Sheet and Tin Plate Company at Gary, Ind.

P. W. Kuhlman (A'33) is now in the communication department of the Air Corps Technical School, Chanute Field, Rantoul, Ill.

C. A. Schneider (A'04, M'13) engineer with Ford, Bacon and Davis, Inc., has been transferred from Seattle, Wash., to New York, N. Y.

R. M. Hull (A'26, M'32) division distribution engineer, Pennsylvania Power and Light Company, has been transferred from Allentown to Harrisburg.

T. M. HAUER (A'22) engineer formerly with the Empire Management Company, Philadelphia, Pa., is now with the Central Ohio Light and Power Company at Findlay.

J. E. MILLIKAN (A'34) is now employed in the sales department of the Ideal Electric and Manufacturing Company, Mansfield, Ohio.

L. H. RUHLEN (A'34) electrician formerly with Libbey-Owens Ford Glass Company, Toledo, Ohio, is now with the Interlake Iron Company, Toledo.

RICHARD SMITH (A'35) former meterman at Wewoka for the Oklahoma Gas and Electric Company is now in the commercial department at Shawnee.

C. K. BEYETTE (A'34) former student engineer with the Texas Electric Service Company at Eastland is now district operator at Wichita Falls.

E. A. SKONBERG (A'30) formerly with the Ekholm Corporation, Boston, Mass., is now with the Mengel Body Company, Louisville, Ky.

J. W. DAVIS (A'33) division plant engineer of the Southern Bell Telephone and Telegraph Company who has been at Atlanta, Ga., is now at Jackson, Miss.

G. M. BABCOCK (A'26) former assistant foreman for the Los Angeles Gas and Electric Corporation, Los Angeles, Calif., has been made electrical protection engineer.

C. E. PARKS, JR. (A'34) has been transferred from substation operation to relay maintenance in the Public Service Company of Indiana, Greenwood.

W. P. BEAR (M'22) is now a sales engineer in the general sales department of the Pacific Gas and Electric Company, San Francisco, Calif.

K. M. KLEIN (A'35) formerly an inspector, Oregon State Highway Commission, Waldport, is now resident bridge engineer at Salem.

H. A. TANKERSLEY (A'30) former student engineer with the Texas Power and Light Company at Hillsboro is now district dispatcher at Temple.

P. B. TAYLOR (A'25, M'34) who has been at Washington, D. C., is now in the aircraft radio laboratory at Wright Field, Dayton, Ohio

E. A. MOYLE (A'26) recently with S. W. Scales Electric Company, Greenville, Tex., is again with the Texas Power and Light Company, Dallas.

E. O. LUNN (A'32) recently with Bradian Mines, Ltd., Bralorne, B. C., Can., is now an engineer with Peterson and Cowan Elevator Company, Ltd., Vancouver.

V. G. DODDS (A'26) Aluminum Company of America, recently was transferred from Albany, N. Y., to San Francisco, Calif.

P. C. CLARK (A'23) recently became field engineer for A. T. Jergins Trust, oil producers at Bakersfield, Calif.

R. C. THORN (A'34) of Tutbury, England, is now at Columbo, Ceylon, India, with the Columbo Commercial Company.

KURT SCHOENFELD (A'31) is now an engineer with the Kurman Electric Company, New York, N. Y.

L. J. MULBACH (A'28) is now employed as machine designer by E. W. Bliss Manufacturing Company, Salem, Ohio

M. W. SMEDBURG (A'33) is now employed by the Rural Electrification Administration at Washington, D. C.

R. J. MCGONEGLE (A'34) is now connected with the Michigan Electric Power Company at Lapier.

W. G. ESCHMANN (A'26) is now employed as aeronautical engineer by Glenn L. Martin Company, Middle River, Md.

E. A. FREEDMAN (A'34) is now in the construction department of the Consolidated Gas Company, New York, N. Y.

THOMAS HOWELLS, JR. (A'31) is now an electrician with the Stanton Operating Company, Pittston, Pa.

C. R. ISEMINGER (A'30) is now an engineer in the rectifier section of the Union Switch and Signal Company, Swissvale, Pa.

E. E. PIEPHO (A'27) is now an electrical engineer with the Detroit Edison Company, Detroit, Mich.

HARRY WILKIE (A'30) is now an assistant radio engineer with the Signal Corps at Fort Sam Houston, San Antonio, Tex.

W. H. TREADWAY (A'34) is now assistant electrical engineer with the Illinois Commerce Commission, Springfield.

E. B. ETCHELLS (A'33) is now employed by General Motors Research, Detroit, Mich., as project engineer.

C. E. HOUSTON (A'32) is now in the geophysics department of the Humble Oil Company at Houston, Tex.

F. J. SAFFORD (A'35) is now enrolled in the training course of the Harnischfeger Corporation at Milwaukee, Wis.

H. H. HAYES (A'33) has been appointed a junior engineer for the Works Progress Administration at Pierre, S. D.

F. W. SUHR (A'32) is now employed by the Wisconsin Hydro-Electric Company at Amery.

Obituary

JOSEPH ALLEN JOHNSON (A'07, F'27, and junior past-president) chief electrical engineer, Buffalo, Niagara and Eastern Power Corporation, Buffalo, N. Y., died October 4, 1935. Mr. Johnson was born at Northboro, Mass., June 21, 1882. Following graduation from the Worcester Polytechnic Institute in 1905, he was employed by the Ontario Power Company at Niagara Falls, and was appointed electrical engineer in 1912; when this company was purchased by the Hydro-Electric Power Commission of Ontario in 1917, Mr. Johnson became assistant engineer. In 1918 he was appointed electrical engineer of the Cliff Electrical Distributing Company, Niagara Falls, N. Y., which in a consolidation later that same year with the Hydraulic Power Company and the old Niagara Falls Power Company into The Niagara Falls Power Company, became electrical engineer of the enlarged company. In this capacity he was responsible for many important features of the electrical design of the Niagara development during the period of their rapid growth from 1918 to 1924. In 1929, he was appointed chief electrical engineer of the Buffalo, Niagara and Eastern Power Corporation, which position he held until the time of his death. Perhaps the most important of the many developments in the art of power generation and transmission in which Mr. Johnson took a prominent part was that of the system of generator voltage control by individual regulators, which he originated and pioneered in the plant of the Ontario Power Company about 1910, and which he described in a technical paper shortly thereafter. His contributions to the art of generator testing by means of the retardation method are also well known. Mr. Johnson had contributed to the Institute several important papers. In 1933 he received the A.I.E.E. national prize for best paper in engineering practice, and in 1934, with his co-author, again received this prize. Mr. Johnson began his term of service with the Institute when he became first chairman of the Student Branch at Worcester Polytechnic Institute in 1905. He was also organizer and first chairman of the Niagara Frontier Section, serving as chairman 1925–26. Mr. Johnson had served the Institute on a large number of its committees. He had been a director of the Institute 1928–32, vice president 1932–34, and president 1934–35. He had served as Institute representative on other organizations, including among these American Engineering Council. He also was active on the technical committees of the Edison Electric Institute, and was a member of the American Association for the Advancement of Science and of the Electrical Standards Committee of the American Standards Association. One of the last honors which Mr. Johnson received was the honorary degree of doctor of engineering which was conferred upon him by Worcester Polytechnic Institute, June 14, 1935. In his absence, caused by illness, the diploma and doctor's hood were received for him by his son, a member of the graduating class. The citation for Mr. Johnson's degree was, in part: "A leader in the development and applica-

tion of electrical energy from Niagara Falls, he has for years directed power production at that grand center. From a humble private in the army of electrical engineers .... he has risen to high command, to be an authority in his chosen field ...." Resolutions adopted by the Institute's board of directors at its meeting on October 22, 1935, commemorating Mr. Johnson's career, are given on page 1274 of this issue.

Samuel Reynolds Pritchard (M'13) professor of electrical engineering at Virginia Polytechnic Institute, Blacksburg, died September 30, 1935. Doctor Pritchard was born at Bascombville, S. C., October 30, 1863, and received the degrees of bachelor of arts and master of arts from South Carolina University in 1885 and 1889, respectively. He served for a time as instructor at the university and at Wofford College, Spartanburg, S. C., and in 1893 became head of the department of electrical engineering at Virginia Polytechnic Institute, serving as dean of the college of engineering for the past 10 years. Doctor Pritchard was also a member of the American Association for the Advancement of Science and of the Society for the Promotion of Engineering Education.

Samuel Dunlap Collett (A'96, and member for life) retired, Brooklyn, N. Y., died recently. He was born at Newport, Ind., October 25, 1868, and received the degree of bachelor of science in mechanical and electrical engineering at Rose Polytechnic Institute in 1890. The following year he received a bachelor of science degree in civil engineering, and in 1894 the degree of master of science. In 1891 he became connected with Thomson-Houston Electric Company, being assigned to several branch offices, and in 1897 became eastern manager for the Elevator Supply and Repair Company, New York, N. Y. Later he was with the Elevator Supplies Company, Inc., at Hoboken, N. J.

Louis G. Carpenter (M'28) consulting engineer, Denver, Colo., died September 12, 1935. He was born at Orion, Mich., March 28, 1861, and was graduated from the University of Michigan in 1879, later taking graduate studies there and at The Johns Hopkins University. In 1888 he became professor of physics and engineering at Colorado State Agricultural College, later becoming professor of engineering and director of the experiment station. In 1911 he engaged in a consulting practice, principally in matters relating to hydraulics. He served as state engineer for Colorado during 1903–05, and undertook considerable work in foreign countries in his consulting practice.

Clarence Frederick Norberg (A'28), head of the service department of The Washington Water Power Company, Spokane, died August 31, 1935, as the result of injuries received in an automobile accident. He was born at Kenora, Ont., Can., June 14, 1903, and graduated from the University of Washington in 1926 with the degree of bachelor of science in electrical engineering. Following graduation he was employed in the engineering department of The Washington Water Power Company, and in 1931 joined the service department, of which he became head in 1934.

James Clark, Jr. (A'10, M'28) president, James Clark, Jr., Electric Company, Louisville, Ky., died October 9, 1935. He was born August 29, 1869, and was graduated from the electrical engineering course at Massachusetts Institute of Technology in 1890. Two years later he engaged in business as a manufacturer of electrically driven tools, motors, and generators. He served as a member of the Institute's committee on general power applications during the period 1930–34, and was a member of a number of other organizations.

Shanker Sadashiv Tatrey (A'32) electrical engineer, Belgaum Electricity Company, Ltd., Belgaum, Bombay, India, died August 9, 1935. He was born at Navasari, Bombay, June 30, 1890, and was graduated from the Victoria Jubilee Technical Institute, Bombay, in 1917. He had since been employed by a number of electrical companies in India, designing and constructing several distribution systems.

Membership

Recommended for Transfer

The board of examiners, at its meetings held September 25 and October 26, 1935, recommended the following members for transfer to the grade of membership indicated. Any objection to these transfers should be filed at once with the national secretary.

To Grade of Fellow

Sleeman, Hector, chief engr., Rangoon Elec. Tramway & Supply Co. Ltd., Rangoon, Burma, India.
Tate, Thomas R., director, National Pwr. Survey, Federal Pwr. Comm., Washington, D. C.
Williams, Allison R., rate specialist, Elec. Rate Survey, Federal Pwr. Comm., Washington, D. C.

3 to Grade of Fellow

To Grade of Member

Amsden, Burton R., Elec. Rate Survey, Federal Pwr. Comm., Washington, D. C.
Atlas, Louis, asst. engr., Brooklyn Edison Co., Brooklyn, N. Y.
Bailey, J. F., div. mgr., Fla. Pwr. Corp., Ocala.
Davis, R. L., cons. engr., Grand Rapid, Mich.
deKay, R. D., member of technical staff, Bell Tel. Labs., Inc., New York, N. Y.
Eastom, Frank A., assoc. prof. of E.E., Univ. of Colo., Boulder.
Faucett, M. A., associate in E.E., Univ. of Ill., Urbana.
Fletemeyer, Louis H., Jr., power engr., R & H Chemicals Dept., E. I. du Pont de Nemours & Co. Inc., Niagara Falls, N. Y.
Goldner, Henry A., equipment maintenance engg., Amer. Tel. & Tel. Co., New York, N. Y.
Gordon, Walter S., Jr., gen. foreman catenary constr., Pennsylvania R.R. Co., Baltimore, Md.
Hanners, William H., outside plant facilities engr., Amer. Tel. & Tel. Co., New York, N. Y.
Harrell, Frederick E., asst. chief engr., Reliance Elec. & Engg. Co., Cleveland, O.
Heafer, Harold P., div. plant engr., Southwestern Bell Tel. Co., Houston, Tex.
Herring, Edgar J. C., director and gen. mgr., Jost's Engg. Co. Ltd., Bombay, India.
Ienger, N. N., E.E., Tata Hydro Electric Companies, Bombay House, Fort Bombay, India
Jenkins, Paul R., div. equipment engr., Amer. Tel. & Tel. Co., Denver, Colo.
Kirkwood, Arthur C., assoc. engr., Burns & McDonnell Engg. Co., Kansas City, Mo.
Lewis, F. A., asst. editor, A.I.E.E., New York.
Lindvall, F. C., asst. prof. of E.E., Calif. Inst. of Tech., Pasadena.
Malan, Stephanus A., E.E., Electricity Supply Commission, Johannesburg, So. Africa.
Meador, Barclay F., asst. to chief engr., Elec. Assignments, Great Lakes Pipe Line Co., Kansas City, Mo.
McKinley, John L., gen. engr., Public Service Co., of Colo., Denver.
Moody, Dwight L., dial toll switching engr., Bell Tel. Labs., Inc., New York, N. Y.
Page, Harold F., div. transmission engr., Amer. Tel. & Tel. Co., Chicago, Ill.
Palmer, H. B., assoc. prof. of E.E., Univ. of Colo., Boulder.
Peirce, W. T., chief E.E., Am. Steel & Wire Co., Worcester, Mass.
Rights, H. T., E.E., in charge of oscillograph design, Westinghouse Elec. & Mfg. Co., Newark, N. J.
Riley, James R., tel. engr., N. Y. Telephone Co., New York.
Schnure, Fred O., elec. supt., Bethlehem Steel Co., Sparrows Point, Md.
Schurch, E. C., E.E., U.S. Bureau of Reclamation, Denver, Colo.
Torok, Julius J., E.E., Corning Glass Works, Corning, N. Y.
Wiewall, Miguel J., instructor in E.E. and physics, Univ. of Porto Rico, Mayaguez, P. R.
Wilcox, Bertram W., statistical dept., Buffalo, Niagara & Eastern Pwr. Corp., Buffalo, N. Y.
Wilson, Richard I., engr., Habirshaw Wire & Cable Co., New York, N. Y.
Wyman, M. B., mgr., engg. dept., Westinghouse Elec. & Mfg. Co., St. Louis, Mo.
Young, H. P., head, elec. pwr. & machy. section, The Polytechnic, London, Eng.

36 to Grade of Member

Applications for Election

Applications have been received at headquarters from the following candidates for election to membership in the Institute. If the applicant has applied for direct admission to a grade higher than Associate, the grade follows immediately after the name. Any member objecting to the election of any of these candidates should so inform the national secretary before Nov. 30, 1935, or Jan. 31, 1936, if the applicant resides outside of the United States or Canada.

Baker, R. M., Westinghouse E. & M. Co., E. Pittsburgh, Pa.
Barnes, D. M., Associated Telephone Co. Ltd., Long Beach, Calif.
Birge, K. R., 302 City Hall Bldg., Pasadena, Calif.
Brown, D. L., Portland Gen. Elec. Co., Ore.
Burns, A. E., Pacific Tel. & Tel. Co., San Francisco, Calif.
Burrell, R. W., N. Y. & Queens Elec. Lt. & Pwr. Co., Flushing, N. Y.
Carpenter, A. G., Jr. (Member), Federal Power Comm., Washington, D. C.
Challgren, C. F., National Sugar Refining Co., Long Island City, N. Y.
Crawford, C. H., Gen. Elec. Co., Schenectady, N. Y.
DeNagy, B. (Member), Gibbs & Hill, New York, N. Y.
Eastin, M. R., Harmon Elec. Shops, Harmon-on-Hudson, N. Y.
Engel, E. D., Univ. of Wash., Seattle.
Evans, F. E. (Member), 8577—98th St., Woodhaven, L. I., N. Y.
Evans, J. L., Metropolitan Edison Co., Reading, Pa.
Finlaw, J., Phila. Elec. Co., Pa.
Friedman, A., Elec. Generator & Motor Co., Cleveland, Ohio.
Geddes, G., Buffalo Gen. Elec. Co., N. Y.
Gettinger, R. F. (Member), Westinghouse Elec. & Mfg. Co., St. Louis, Mo.
Gibbons, J. P., Goodman Mfg. Co., Chicago, Ill.
Gidlund, H. F., Pub. Serv. Co. of Colo., Denver.
Giesecke, H. W., 306 W. 109th St., New York, N. Y.
Gleichman, R. C., Federal Pwr. Comm., Washington, D. C.
Grane, M. M., 194 West 4th St., New York, N. Y.
Guseck, F., Canadian Lt. & Pwr. Co., Montreal, Que., Can.
Haefner, S. J., Union Col., Schenectady, N. Y.
Hamlin, E. W. Univ. of Kan., Lawrence.
Hammerstrom, A. P., Westinghouse Elec. & Mfg. Co., St. Louis, Mo.

Harvey, A. J., Jr., 2627 Idlewood Rd., Cleveland, Ohio.
Hierath, D. C., Gen. Elec. Co., New York, N. Y.
Johnson, C. H. (Member), Idaho Maryland Mines Co., Grass Valley, Calif.
Keever, L. M., N. C. State Col., Raleigh.
Kipp, E. B., Westinghouse Elec. & Mfg. Co., Los Angeles, Calif.
Koerner, C. T., Mackay Radio & Telegraph Co., New York, N. Y.
Kuenzler, E. A., Bell Tel. Lab., Inc., New York, N. Y.
Lord, H. W., General Elec. Co., Schenectady, N. Y.
Martin, J. R., Case Sch. of Applied Sci., Cleveland, O.
Martinez de la Terre, A. M., Cayey Hydro Elec. Plant, Cayey, Puerto Rico.
McMurray, W. K., St. Joseph Ry., Lt., Ht. & Pwr. Co., Mo.
Meadows, C. W., Brooklyn Edison Co., Inc., Brooklyn, N. Y.
Meyer, L. W., Federal Pwr. Comm., Washington, D. C.
Miller, A. A., Buffalo Gen. Elec. Co., N. Y.
Montgomery, E. G. (Member), Am. Tel. & Tel. Co., New York, N. Y.
Montgomery, L. McD., Okla. Gas & Elec. Co., Shawnee.
Mosman, C. R. (Member), Am. Tel. & Tel. Co., Chicago, Ill.
Neiman, W. E., N. Y. C. Pwr. Survey, New York, N. Y.
Ozment, K. J., Camp Twanoh, Belfair, Wash.
Paul, R. D., Gibbs & Hill, New York, N. Y.
Peterson, C. H., N. Y. Tel. Co., New York, N. Y.
Peterson, R. M. (Member), Am. Tel. & Tel. Co., New York, N. Y.
Prentice, B. R., Gen. Elec. Co., Schenectady, N. Y.
Quarles, L. R., Univ. of Va., University.
Raps, F. J. (Member), Buffalo Gen. Elec. Co., N. Y.
Rees, M. M., Union Oil Co., Oleum, Calif.
Rinke, A., Brooklyn Edison Co., Inc., Brooklyn, N. Y.
Ritchie, D. R., 1518 Addison St., Berkeley, Calif.
Roddy, F. T., Minnesota Utilities Co., Minneapolis.
Roll, E. P., Jr. (Member), Federal Power Comm., Washington, D. C.
Schindler, R. W., 4106 Spokane Ave., Cleveland, Ohio.
Schmidt, H. R., 345 Adelphi St., Brooklyn, N. Y.
Scott, J. P., Union Elec. Lt. & Pwr. Co., St. Louis, Mo.
Sills, A. L., Federal Pwr. Comm., Washington, D. C.
Simmons, L. C., Harmon Electric Shops, Harmon, N. Y.
Sinclair, J. H., 400–2 Public Bldg., Regina, Sask., Can.
Smith, R. W., Robbins & Myers Inc., Springfield, Ohio.
Stockwell, G. A., Los Angeles Bureau of Pwr. & Lt., Calif.
Taugher, F. P., Westinghouse Elec. & Mfg. Co., St. Louis, Mo.
Thompson, C. St. C., Western Union Telegraph Co., New York, N. Y.
Toltesey, P. J., Brooklyn Edison Co., Inc., Brooklyn, N. Y.
Trekell, H. E., General Elec. Co., W. Lynn, Mass.
Wilder, G. L., Locke Insulator Corp., Salt Lake City, Utah.

70 Domestic

Foreign

Milner, R. C., Macintosh Cable Co. Ltd., Derby, England.
Minchin, C. W. H., Mawdsley's Ltd., Dursley, Gloucester, Eng.
Mukerjee, G. C. (Member), Hindu Univ., Benares, India.

3 Foreign

Addresses Wanted

A list of members whose mail has been returned by the postal authorities is given below, with the addresses as it now appears on the Institute record. Any member knowing of corrections to these addresses will kindly communicate them at once to the office of the secretary at 33 West 39th St., New York, N. Y.

Brune, Otto, 214 White St., Waverly, Mass.
Chiofalo, J., 203 Graham Ave., Brooklyn, N. Y.
Crite, Mitchel, 32 E. 126th St., New York, N. Y.
Ghosh, K. C., c/o Compagnia Generale Di Elettricita, 34 Via Borgognone, Milan, Italy.
Golikoff, A., Main P. O. Gen. Del., Moscow, U. S. S. R.
Kimball, Gordon S., 154 Elmer Ave., Schenectady, N. Y.
Nelson, Charles J., 1515 N. Lotus Ave., Chicago, Ill.
Rozelle, P. M., 2018 Chestnut St., Harrisburg, Pa.
Soskin, Samuel B., 1141 S. Central Park, Chicago, Ill.
Spiegel, William F., 7 Stegman Court, Jersey City, N. J.
Vance, Paul E., c/o Marietta Mfg. Co., Point Pleasant, W. Va.
Whittemore, Geo. W., 151 Ridgewood Ave., Glenwood, N. J.

12 Addresses Wanted

Engineering Literature

New Books in the Societies Library

Among the new books received at the Engineering Societies Library, New York, recently, are the following which have been selected because of their possible interest to the electrical engineer. Unless otherwise specified, books listed have been presented gratis by the publishers. The Institute assumes no responsibility for statements made in the following outlines, information for which is taken from the preface of the book in question.

BOSCH KRAFTFAHRTECHNISCHES TASCHENBUCH. 4ed. of "Technischen Tabellen." Stuttgart, Robert Bosch A.G.; Berlin, VDI-Verlag, 1935. 6x4 in., lea., 2.50 rm. A pocketbook containing a well-chosen collection of data for the automotive engineer. Includes physical, electrical, chemical, and mechanical data, information about fuels, lubricants, internal-combustion engines, electrical equipment for automobiles and aircraft, and general data on racing records, etc.

FUNDAMENTALS of INDUSTRIAL MARKETING. By R. F. Elder. N. Y. and Lond., McGraw-Hill Book Co., 1935. 317 p., illus., 9x6 in., cloth, $3.00. An attempt to present comprehensively the problems connected with the marketing of goods used in manufacture or trade.

ELEMENTS of ELECTRICAL ENGINEERING, a Textbook of Principles and Practice. By A. L. Cook. 3 ed. N. Y., John Wiley & Sons, 1935. 603 p., illus., 9x6 in., lea., $4.00. A textbook dealing with the fundamentals and their application in practice, intended for an introductory course for engineering students.

FANS. By T. Baumeister. N. Y. and Lond., McGraw-Hill Book Co., 1935. 241 p., illus., 9x6 in., cloth, $3.50. A book to assist the fan user in the selection of the best equipment for any given service conditions. Chapters are devoted to design and testing, and to the flow of fluids.

GENERATING STATIONS, Economic Elements of Electrical Design. By A. H. Lovell. 2 ed. N. Y. and Lond., McGraw-Hill Book Co., 1935. 438 p., illus., 9x6 in., $4.50. Describes the application of economic principles to the problems of the design of generating stations and transmission systems. The selection of apparatus, the proportioning of details of the assembly, the balancing of initial and subsequent costs, and related topics are considered. The new edition has been revised in the light of recent developments.

HEATING and AIR CONDITIONING. By J. R. Allen and J. H. Walker. 4 ed. N. Y. and Lond., McGraw-Hill Book Co., 1935. 444 p., illus., 9x6 in., cloth, $4.00. A comprehensive account of fundamental principles and working methods, intended as a text for students.

INTRODUCTION to QUANTUM MECHANICS. By L. Pauling and E. B. Wilson. N. Y. and Lond., McGraw-Hill Book Co., 1935. 468 p., illus., 9x6 in., cloth, $5.00. A textbook of practical quantum mechanics for chemists, experimental physicists, and beginning students of theoretical physics.

Die KRITISCHEN DREHZAHLEN WICHTIGER ROTORFORMEN. By K. Karas. Vienna, Julius Springer, 1935. 154 p., illus., 10x7 in., paper, 18 rm. Derivation of methods and formulas to calculate the critical speeds of rotors with more exactness than is obtainable by the usual graphical methods.

MECHANICAL and ELECTRICAL EQUIPMENT for BUILDINGS. By C. M. Gay and C. D. Fawcett. N. Y., John Wiley & Sons; Lond., Chapman & Hall, 1935. 429 p., illus., 9x6 in., $5.00. The essentials of building equipment, embracing statements of the fundamental theories involved and their broad applications, but not covering the engineering design.

NEON SIGNS, Manufacture, Installation, Maintenance. By S. C. Miller and D. G. Fink. N. Y. and Lond., McGraw-Hill Book Co., 1935. 288 p., illus., 9x6 in., cloth, $3.00. A practical description of the manufacture, installation, and maintenance of these signs.

NOISE, a Comprehensive Survey From Every Point of View. By N. W. McLachlan, with a foreword by Sir Henry Fowler. Lond., Oxford Univ. Press, 1935. 148 p., illus., 7x5 in., cloth, $2.25. An introduction to the subject for both lay and technical readers interested in the prevention of noise.

PLANE and SPHERICAL TRIGONOMETRY. By L. M. Kells, W. F. Kern, and J. R. Bland. N. Y. and Lond., McGraw-Hill Book Co., 1935. 269 p., illus., 9x6 in., cloth, $2.50. A textbook to present the subject simply and comprehensively.

PRINCIPLES of EXPERIMENTAL and THEORETICAL ELECTROCHEMISTRY. By M. Dole. N. Y. and Lond., McGraw-Hill Book Co., 1935. 549 p., illus., 8x6 in., cloth, $5.00. A book intended for graduate students with a grounding in physical chemistry.

DIESEL LOCOMOTIVES & RAILCARS. By B. Reed. Lond., Locomotive Pub. Co., 1935. 190 p., illus., 9x6 in., cloth, 6s. A review of the present situation in the development of Diesel rolling stock in America and Europe, including data on costs, efficiencies, etc.

MEASUREMENTS in RADIO ENGINEERING. By F. E. Terman. N. Y. and Lond., McGraw-Hill Book Co., 1935. 400 p., illus., 9x6 in., cloth, $4.00. A comprehensive engineering discussion of the measuring problems commonly encountered by radio engineers, intended to serve the practicing engineer and the student.

PHOTO-ELECTRIC and SELENIUM CELLS, Their Operation, Construction and Uses. By T. J. Fielding. Lond., Chapman & Hall, 1935. 140 p., illus., 8x5 in., cloth, 6s. An elementary presentation of the subject intended for beginners and amateur experimenters interested in the construction and uses of light sensitive cells.

POWER OPERATOR'S GUIDE (1001 Practical Helps), a Compilation of Ideas, Methods, and Tools for Saving Time, Labor, and Money in the Power Plant and Along the Line of Power Services. Compiled and edited by E. J. Tangerman. N. Y. and Lond., McGraw-Hill Book Co., 1935. 568 p., illus., 9x6 in., cloth, $4.00. A compilation of practical hints upon the operation and maintenance of power plant machinery, chosen from *Power*.

PRACTICAL RADIO COMMUNICATION, Principles, Systems, Equipment, Operation, including Short-Wave and Ultra-short-wave Radio. By A. R. Nilson and J. L. Hornung. N. Y. and Lond., McGraw-Hill Book Co., 1935. 754 p., illus., 9x6 in., lea., $5.00. A textbook designed for those preparing for licenses as radio operators.

PUBLIC UTILITY QUESTION. By H. G. Hendricks. Pub. by author at 5629 Kansas Ave., Washington, D. C.; on sale Amer. News Co., 131 Varick St., New York, 1935. 148 p., tables, 8x5 in., cloth, $2.00. Discussion by a former member of the staff of the house committee on interstate and foreign commerce of the part played by the holding company in the public utility industry.

REPORTS on PROGRESS in PHYSICS. Lond., The Physical Society; printed at the University Press, Cambridge, 1934. 371 p., illus., 10x7 in., cloth, 12s6d. Detailed accounts of recent advances in physics up to the end of 1933, prepared by specialists in the various fields.

STORY of RADIO. By O. E. Dunlap, Jr. N. Y., Dial Press, 1935. 326 p., illus., 9x6 in., cloth, $2.75. A popular history of the development of radio, from the work of Maxwell and Hertz to television.

Engineering Societies Library

29 West 39th Street, New York, N. Y.

MAINTAINED as a public reference library of engineering and the allied sciences, this library is a cooperative activity of the national societies of civil, electrical, mechanical, and mining engineers.

Resources of the library are available also to those unable to visit it in person. Lists of references, copies or translation of articles, and similar assistance may be obtained upon written application, subject only to charges sufficient to cover the cost of the work required.

A collection of modern technical books is available to any member residing in North America at a rental rate of five cents per day per volume, plus transportation charges.

Many other services are obtainable and an inquiry to the director of the library will bring information concerning them.

Industrial Notes

Increased Sales for General Electric.—Orders received by the General Electric Co., for the third quarter of 1935 amounted to $54,400,819, compared with $40,458,901 for the third quarter of 1934, an increase of 34%. Orders received for the nine months amounted to $158,943,765, compared with $132,613,543 for the first nine months of last year, an increase of 20%.

High-Output Photronic Cell.—A photoelectric cell of the "dry-plate" type which provides an increased current output has been developed by the Weston Instrument Corp., Newark, N. J. Known as the type 2 photronic cell, it is intended primarily for use at levels of illumination so low that the regular photronic cell will not provide sufficient output for the purposes intended. Current output of the new cell is about 3 times that of the regular photronic cell for the same illumination.

New High Voltage Switch.—The Delta-Star Electric Co., Chicago, Ill., announces its "FMR-239" motor or manually operated, balanced-blade disconnecting switch for inverted mounting. There are three stacks of insulators per pole, two of which rotate, one raising and lowering the blade, the other rotating a contact which engages the forked end of the main blade. In operation, the contact entirely releases before the blade is removed, thus reducing cantilever stresses to a minimum. The new switch is available in voltages from 115 to 287 kilovolts, and in capacities up to 1,200 amperes.

New Portable Test Units.—The Westinghouse Electric & Mfg. Co., East Pittsburgh, Pa., announces a new line of compact lightweight portable volt-ohmmeters and test units, suitable for general testing, laboratory and radio work. The volt-ohmmeter is particularly useful for exploration and trouble shooting in complex circuits such as radios, public speech amplifiers, signal systems, control systems and other applications where a detailed knowledge of circuit conditions and values must be obtained. Several types are available including simple ohmmeters, volt-ohmmeters, d-c test units and the more elaborate multi-scale a-c and d-c test units. The instruments weigh approximately 2 pounds, have a 2.4 inch scale length, an accuracy within 2% for d-c volts and milliamperes and within 5% for alternating current.

Tin Consumption Increases.—According to the October issue of the Bulletin of the International Tin Research and Development Council published by The Hague statistical office, the world's apparent consumption of tin during the first eight months of the current year totaled 90,910 tons, showing an increase of 11,692 tons, or 14.8%, compared with the corresponding period of 1934. The total production of tin in the first eight months of 1935 is given as 84,929 tons, an increase of 14,306 tons over the corresponding figure for 1934. In the year ended August 1935 the consumption of tin in the United States of America increased by 11.9% to 54,160 tons. Tin consumption in Russia continues to expand, a new record of 6,345 tons having been reached for the year ended August 1935. Consumption in Italy has also reached a record of 5,362 tons for the year ended August, representing an increase of 34.4% over the figure for the previous year. France and Belgium show decreases of 16.7% and 25.0%, respectively.

Big Furnaces for Ford.—Thirteen big bell-type electric furnaces, larger than any of this type heretofore constructed, will soon be installed by the General Electric Co., in the Rouge plant of the Ford Motor Co., where, in conjunction with twelve other bell-type furnaces built by another manufacturer, they will be used to anneal steel strip for the Ford V-8. Each is of sufficient capacity to bright-anneal, in one charge, two 16,000-pound coils of steel strip 48 inches wide, 52 inches outside diameter and 30 inches inside diameter. A number of the furnaces had already been shipped to the Ford plant by late September, while those still in the course of construction were inspected with interest by Henry Ford on the occasion of his recent visit to the G-E Schenectady Works. The purchase of the large furnaces is based on results of tests made in a G-E furnace of this type which was installed in the Ford plant early this year. The test furnace demonstrated the ability of the bell-type electric furnace to produce steel of superior qualities in a short annealing time and at low cost. The annealing cycle is in the neighborhood of 45 hours—much shorter than with conventional fuel-fired equipment.

Trade Literature

Texrope Drives.—Bulletin 1259, 32 pp. Describes texrope drives for machine tools; profusely illustrated. Allis-Chalmers Mfg. Co., Milwaukee, Wis.

Mica Undercutter.—Bulletin, 4 pp. Describes the improved Ideal undercutter, electrically driven, for servicing and undercutting commutator mica. Ideal Commutator Dresser Co., Sycamore, Ill.

Cathode-Ray Oscillograph. Bulletin GEA-1768A, 4 pp. Describes type HC-10 B1 cathode-ray oscillograph, suited for the study of recurring waves from power frequencies to several million cycles per second. General Electric Co., Schenectady, N. Y.

Farm Line Accessories. Bulletin 513-H 28 pp., "O-B Materials for Low-Cost Farm Lines." Describes hardware, insulators and fittings applicable in low-cost distribution line construction. Ohio Brass Co., Mansfield, O.

Arc Welders.—Bulletin HW-4, "Weld It Well." Describes a complete line of arc welders, from 50 to 800-ampere units, as well as welding fixtures and accessories. Condensed specifications and performance data are included. Harnischfeger Corp., Milwaukee, Wis.

Water Heating Load Control.—Bulletin GEA-1669D, 4 pp. Describes type T-11 time switches for controlled water heating, for indoor and outdoor service, and especially designed for the off-peak control of water-heater loads. General Electric Co., Schenectady, N. Y.

Gear Motors.—Bulletin 1870, 4 pp. Describes a new line of gear motors, including helical gear and worm gear units, either direct connected or coupled types, in single, double and triple reductions for horizontal, vertical and right angle drive. Diehl Mfg. Co., Elizabethport, N. J.

Engine-Type Alternators.—Bulletin 1153-A, 24 pp. Describes engine type alternators for use with any kind of reciprocating prime movers; available for standard engine speeds from 100 rpm to 600 rpm in standard ratings from 25 kva to 10,000 kva. Numerous installations are illustrated. Allis-Chalmers Mfg. Co., Milwaukee, Wis.

Pumps.—Bulletin 166, 12 pp. Describes class 7-AT and 8-AT air cushion valve vacuum pumps, single-stage and two-stage horizontal bearing type. Bulletin 224, 4 pp., describes class SHD, SMD and SLD double suction, single stage, sleeve bearing centrifugal pumps. Pennsylvania Pump & Compressor Co., Easton, Pa.

Fuse Links.—Bulletin, 16 pp. Describes "Unifit" (universal) fuse links for every service requirement, adaptable to all standard makes of cutouts of new and old designs; available in three types—strain, spring and silver links of spring type with silver fusible element, and in all standard ratings. Line Material Co., So. Milwaukee, Wis.

Circuit Breaker Arc Extinguisher.—Bulletin 45, 4 pp., "Pacific Expulsion Chamber." This device has been developed for use in oil circuit breakers to insure early arc extinction with a minimum of oil carbonization, without oil throw and without stress to the breaker in which it is installed. Pacific Electric Mfg. Co., 5815 Third St., San Francisco, Calif.

Arc Welding.—Bulletin, 42 pp. Describes automatic arc welding by the "electronic tornado" system, for use in industries where products of relatively uniform character are produced in large quantities. The various types of welds suitable for automatic welding are illustrated by line drawings; typical applications are pictured. Lincoln Electric Co., Cleveland, O.

Magnetic Contactors.—Bulletin 1145, 4 pp. Describes a complete line of new "Line-Arc" magnetic contactors, for use on mill, crane machinery controllers, and other d-c applications, up to 800-ampere capacity in which the arc, formed as the circuit is opened, is ruptured harmlessly in a narrow path or line centered between, but not touching the arc shields. The Electric Controller & Mfg. Co., 2700 E. 79th St., Cleveland, O.